Nonlinear Dynamical Systems
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Lecture - 28
Describing: Optical Gain

Welcome everyone to this next lecture on describing functions. So, the certain describing functions, for that purpose we define what is the meaning of gain of an operator, possibly non-linear. And hence we define a notion of quasi linearization and quasi word refer to the linearization depending on the input and there amongst various linearization's. We decided a notion of optimality.

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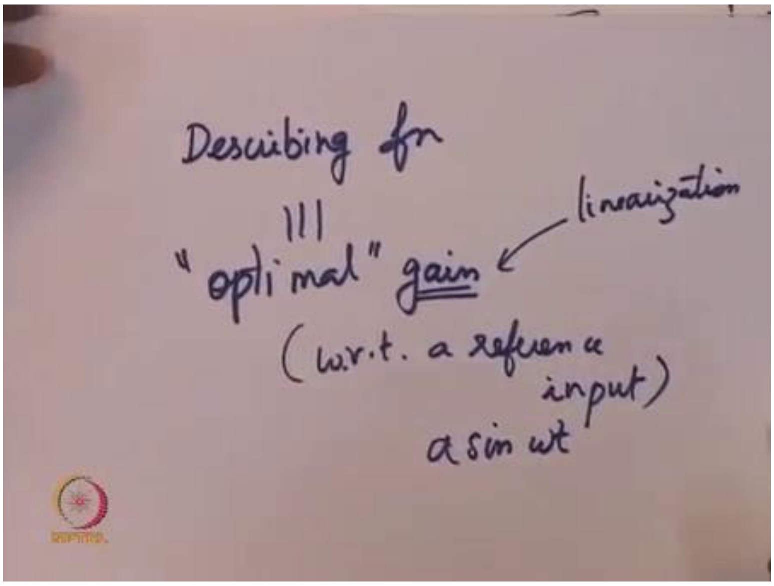

So, we said that describing function is an optimal gain, optimal gain with respect to reference input. This reference input a sin omega t, because the linearization depends on the input. This is called quasi linearization and the word gain is what makes it a linearization. So, the describing function is nothing but an optimal quasi linearization of a non-linear system. We will recapitulate this is a little more detail.

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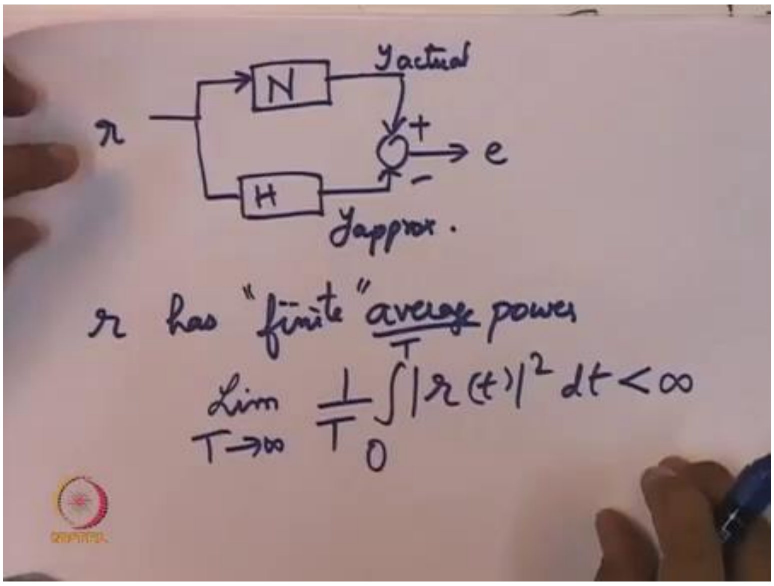

So, what we did last time is we let a r come here this also one non-linear system, this was the actual output. We are going to compare it with some output of another linear system y approx, we called it. Then we said r is a signal of finite average power, r has finite average power. What is finite about it, what is average about it? We integrate it from 0 to capital T, r of t square d t. So 0 to T when we integrate, then it would give us energy, we divide it by T and now this is power, the average power and we let limit as T tends to infinity because if you do not let T tend to infinity, this will always be finite.

Hence, there is no way it will become infinite for a function r, for a function r that is bounded at every time instant. So, we the finite word refers to a as this limit tends to infinity, as this limit T tends to infinity. This quantity is not unbounded, it is finite. To say that this limit exists when capital T tends to infinity means that this signal r is finite average power. So, we took a r like this, we assume that this N is bounded input, bounded output table that means that the actual output y actual is also of finite average power.

Then we decided that what H should we fit here, so that the error e has a least average power, the if y, actual y is finite average power. Then you can always take H equal to 0 and you get the error also to be finite average power, why because error e will be equal to y actual, so one can try to minimize this. So, e the best, at the best approximation H, e will also be of finite average power and now we want to find H such that this finite average, the average power in this is the least.

So, that minimization problem is what makes describing function the best solution and it turns out that the optimum H, the optimal value is not unique. The optimal value is unique, but the optimizer H is not unique. Any linear system, any stable linear system, whose which evaluates to the, which evaluates to the Fourier coefficients, first harmonics at this particular sinusoid also turns out to be a optimal quasi linearization.

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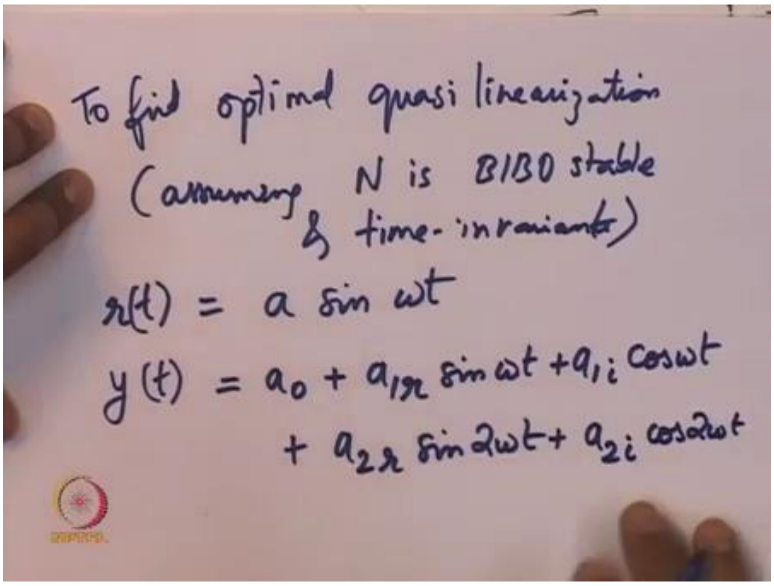

So, let us see this in little more detail to find optimal quasi linearization. Assuming N is BIBO stable and time invariant, time invariance is required because we want y, y actual to also be a periodic signal when r of t equal to a sin omega t, the time we calculate y of t. We analyze y into its various Fourier coefficients a 1 r sin omega t plus a 1 i cos omega t plus a 2 r sin 2 omega t plus a 2 i cos 2 omega t and so on. So, we are going to take a 1 r and a 1 i, this we will use to define the so called describing function.

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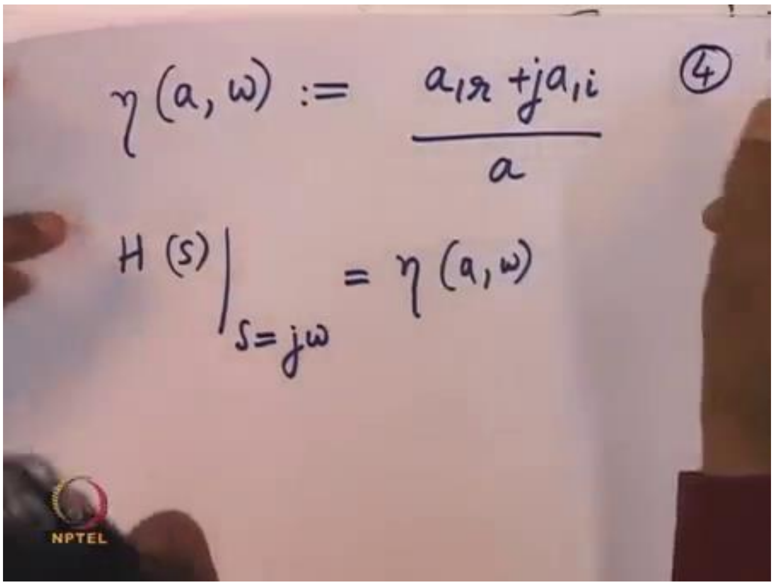

A describing function, a function that describes on both a, and omega of that particular non-linear system is defined as a 1 r plus a1 i j times of that divided by the amplitude of the input sinusoid, this is the definition of the describing function. Now you can take any stable transfer function H has all it is course on the left half complex plane and evaluate at S is equal to j omega, if that is equal to eta a comma omega then this any such H is in, is an optimal quasi linearization for that non-linear system.

So, one can notice that there is a lot of non uniqueness here. In other words you are specified with this particular point on the complex plane any transfer function H, any stable transfer function whose Niquous plot passes precisely this point, it has to pass through this point precisely at S is equal to j omega and not at any other frequency. Any such transfer function will all qualify as an optimal quasi linearization. So, now we are going to see some more examples of describing functions or describing functions of some more non-linearity.

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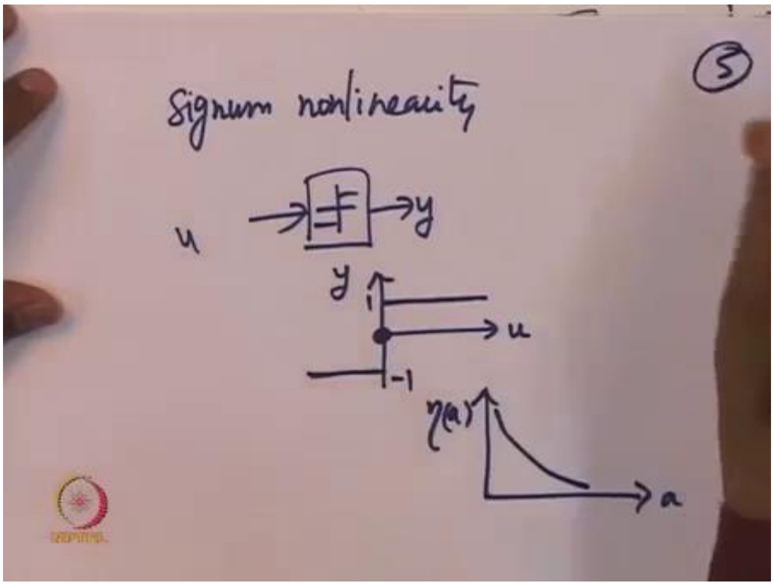

We already evaluated from first principle by using the Fourier coefficients of the signum, a non-linearity, whose input output graph depends like this. We had some debate whether at u equal to 0, it should be equal to 0 or plus 1 or minus 1, as I said the Fourier coefficients do not depend on the value of y at just one point, but more on a aggregate sense for the purpose of calling this particular non-linearity as a odd non-linearity. It is already memory less, because its memory less, we could express why as a graph of u instead of the dependence on time t.

In addition to it being time invariant, it is already memory less, in addition it is also odd an odd function, that is what was helpful in saying that the describing function in such a case is a real function of a and omega, because its memory less its only a function of a the amplitude a. So, we already saw that the describing function graph looks like this in terms of a, so for very small amplitude of the incoming signal. The amplification is very high, on the other hand for large amplitude the amplification is very low. So, we are going to see some more examples, for example of the saturation non-linearity.

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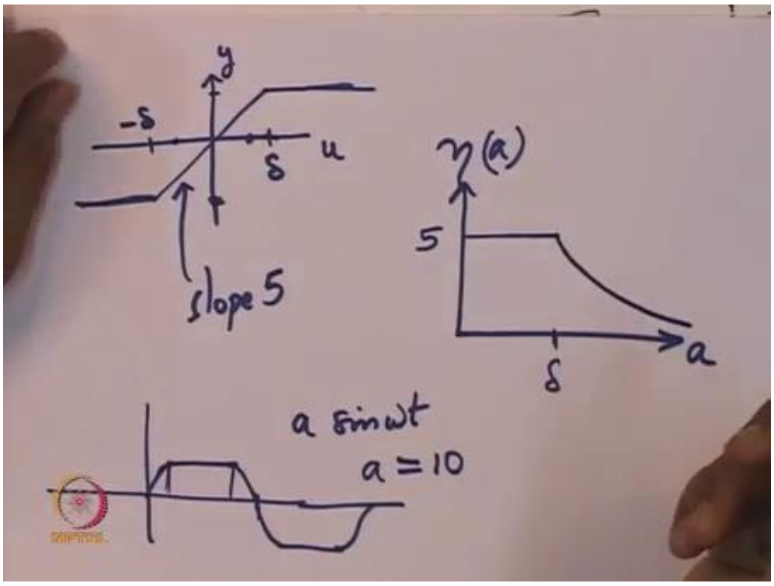

Now, for example, consider the non-linearity, which is saturation. How do we expect the graph of this, let us say the it is equal to slope 5 as long as input u varies within the range delta. For plus minus delta, the slope of this is equal to slope 5 and for beyond that it has saturated, so it has saturated to 5 times of delta. So, let us try to already draw the graph of the describing function as a function of a, do we expect the describing function to be real. Yes, we expect that to be real because this is the odd function of u, if we change u to change any value here to its negative and the value of y becomes just the negative.

This is why we can call this function an odd function. In addition to it being time invariant and memory less, it is also odd. Because it is memory less the describing is a function of only a, because it is odd nonlinearity, there is only a real part the imaginary part is equal to 0. So, because it is slope 5, for all amplitudes up to delta. If you give a sin omega t as input and amplitude is less than or equal to delta, then the output is just scaled by 5, because of that we expect that it will be equal to a constant 5, but beyond that you see there is more and more clipping going on, what exactly is the clipping.

For a let us say equal to 10, it has got saturated, this is what we saw briefly in the previous lecture. This clipping amount is what for more and more fraction of the period of this signal, it will be clipped if the amplitude is high, that is why we can say that describing function is decreasing, monotonically decreasing for amplitude larger than delta, for amplitude greater than delta. So, we will see an exact formula for this, we will see a formula for slightly more general with a little more generality even though the derivation is pretty cumbersome, but then I think that with lots of careful manipulation by keeping track of at what value of time t saturates. By keeping track of this, one should be able to integrate explicitly and find this out.

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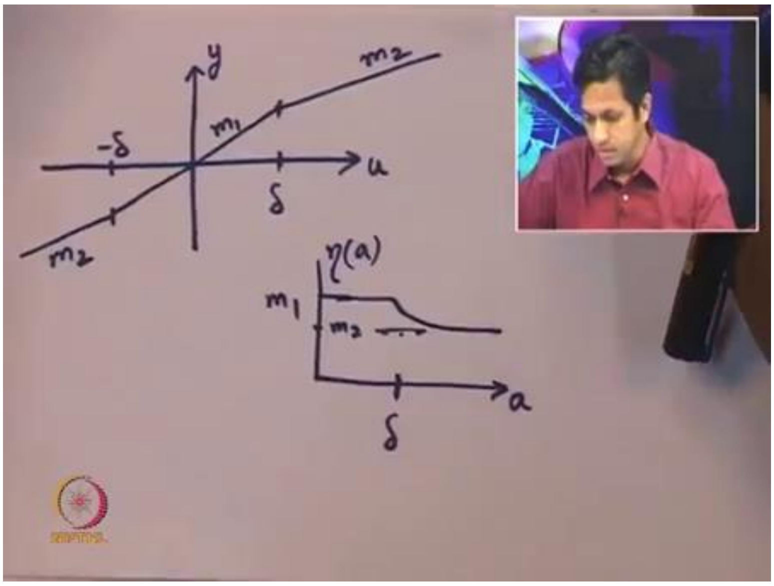

So, let us just reproduce the formula from Vidhyasagar of a non-linearity that looks like this up to delta it is equal to, and beyond that it is slope m 2, here it is slope m 1. This is an odd non-linearity, these two lines have different slopes, these are slope m 2 these are slope m 2 and in the middle there is slope m 1. For such a function, for such a input output map, the saturation non-linearity is a special case of this in which m 2 is equal to 0 and the dead zone is another special case of this, in which m 1 slope is equal to 0.

So, we can see that this particular, this can also be thought of as like a hardening spring, a spring whose spring constant goes on decreasing or can or goes on increasing. If the slope m 2 is larger than slope m 1 and if m 1 has a interpretation that it is a spring constant, then one can think of this as a spring that hardens when it is extended, as n when it is extended more and more this spring hardens.

So, of course, these are approximations there is hardening is gradual, here suddenly for amplitude larger than delta, there is some aspect of the signal that encounters an amplification of this m 2, but for other aspects for all lower values of amplitude, the amplification is just m 1. So, how do we expect the describing function to be, expect the describing function to be m 1 and if m 2 is lower then up to delta it is equal to this, after which it comes down to m 2. Either comes down or comes up, depending on whether m 2 is larger or smaller than m 1 and what is exactly this, what is the close form expression for this, for this particular thing we will reproduce a formula that has been calculated carefully. So, let me just write it here. So, we decided to reproduce the formula for the describing function, so this involves a good amount of calculation, a careful calculation, but then it is the graph is not very, formula is not very unexpected.

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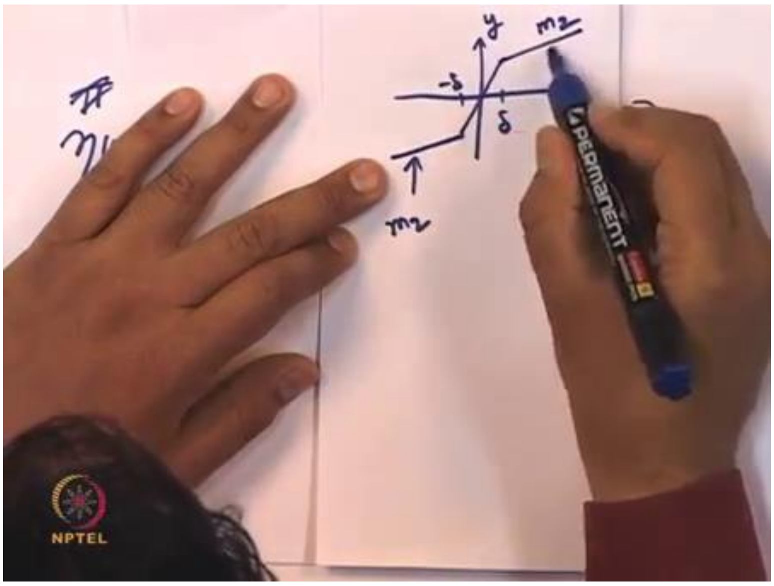

So, let us, it is for this that we are trying to find the describing function. For input u, outside the range minus to delta, the slope is m 2 and m 2, m 2 here, m 2 here and for slope inside this range minus delta to delta the slope is just m 1. So, for this particular example, the formula goes like this.

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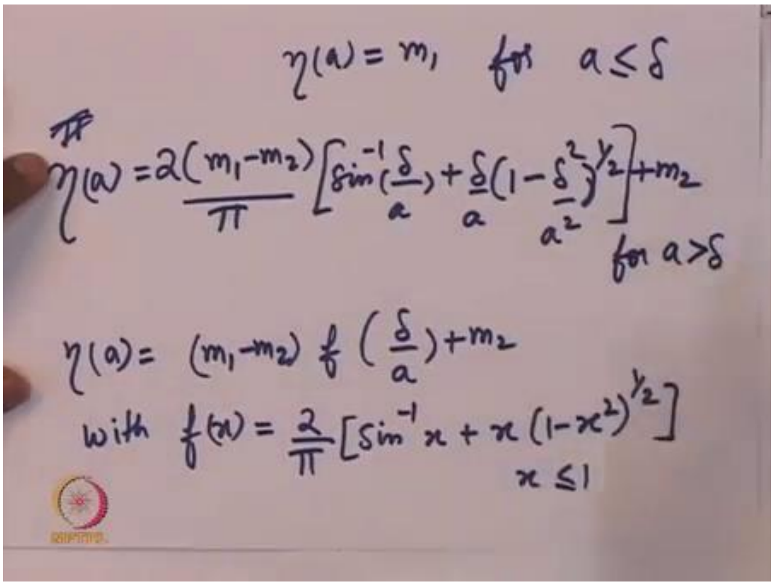

So, it has been a practice to use a look up table, where we use the readymade formula and apply it to our example, and this formula is what takes a good amount of labor to prove. But once it is proved it is extremely handy, one often uses a look up table of describing function of various of many standard non-linearity's and this one I have taken from Vidhyasagar's book on non-linear systems analysis.

So, this is the formula of course, one would ask is this formula what range of a, that is not difficult to answer because when amplitude is less than or equal to delta, that time describing function of is just equal to m 1, for a less than or equal to delta. And for amplitude larger than for a greater than delta is when this formula is applicable. One could check for a equal to delta whether the two describing functions give the same value, we do not expect that the describing function for this non-linearity becomes dis-continuous as a function of amplitude a, why because? When a is equal to delta there is 0 amount that gets by slope m 2 and for a slightly more than delta.

There is an infinite decimally small amount that gets amplified by m 2 and hence we expect some continuity act a equal to delta and that is what one indeed can verify by putting a equal to delta inside this formula and checking whether it is indeed equal to m 1. Of course, this formula looks pretty complicated, so it turns out that it gets simplified if we use another function f of x in this particular way, this is also for x less than or equal to 1 and for x greater than 1, this is just to be equal to 1.

So, one can use this formula and then this entire difficult part gets absorbed into just function f of delta over a. It is expected that delta over a will play a role because at what value the new slope starts acting depends will affect the amplitude at which the formula will also change. So, this is the formula if we use an expression for f of x for this intermediate part. So, we will see what this evaluates to for the saturation non-linearity, saturation non-linearity is a case where m 2 is equal to 0, m 1 equal to 1.

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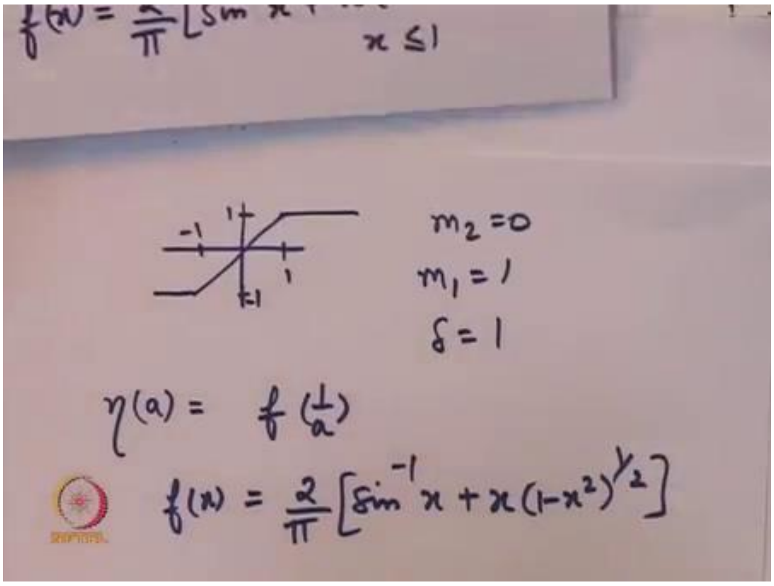

Let us go by the standard saturation non-linearity, where the slope is equal to 1 over the range minus delta to delta with delta also equal to 1, this formula that we have written here. Let us see what this evaluates to, eta of a becomes equal to just f of 1 by a plus 0 where f of x is equal to 2 over pie, sin inverse x plus x 1 minus x square 1 by 2. So, one can apply this formula for this special case. So, let us just draw a graph f of x itself looks like this.

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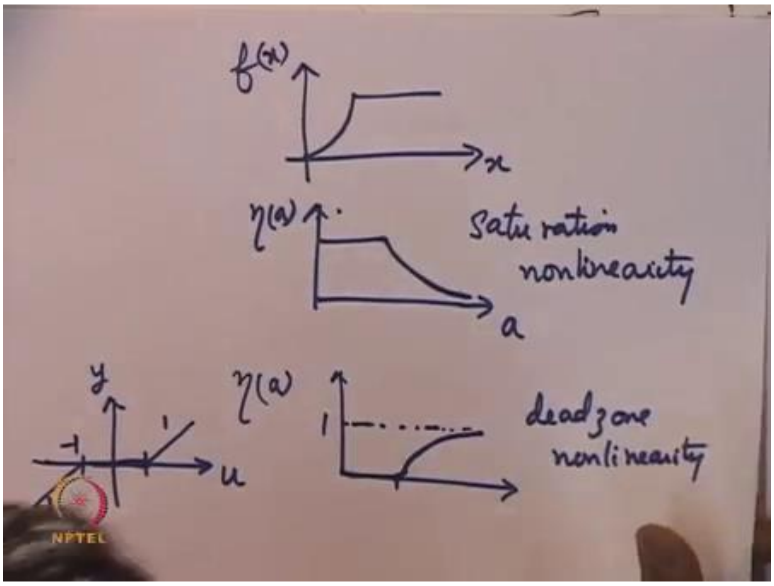

f of x has a function of x looks like this. So, what we have is just f of 1 by x, so f of 1 by x and that too for a larger than, we can get this as the formula for describing function for the saturation. One can plot this explicitly for example, on sy lab and check that this is indeed the case if time permits, we will plot this on sy lab and show it in this course. Now, one can also verify that the describing function for the so called dead zone non-linearity.

What is the dead zone non-linearity? A non-linearity whose input output looks like this, that is also again memory less time invariant and odd. It has some slope say m 2 or say equal to 1 for this range, but for certain range it is just dead. There is no output response seen, as long as the input is smaller than the range plus minus 1. So, here we expect that the gain is initially 0 and then it saturates to 1, and it saturates to the slope for very large amplitude, this zone over which it is dead is, very small fraction, because it is a very small fraction we will expect that it will eventually tend to 1.

So, this is how one can check by evaluating into that particular formula. So, the next thing that we will do is, we will take an example of a non-linearity which has some memory for example, the jump hysteresis and we will derive the formula for that particular purpose that is our first example where the describing function turns out to become a complex function, it has a imaginary part also.

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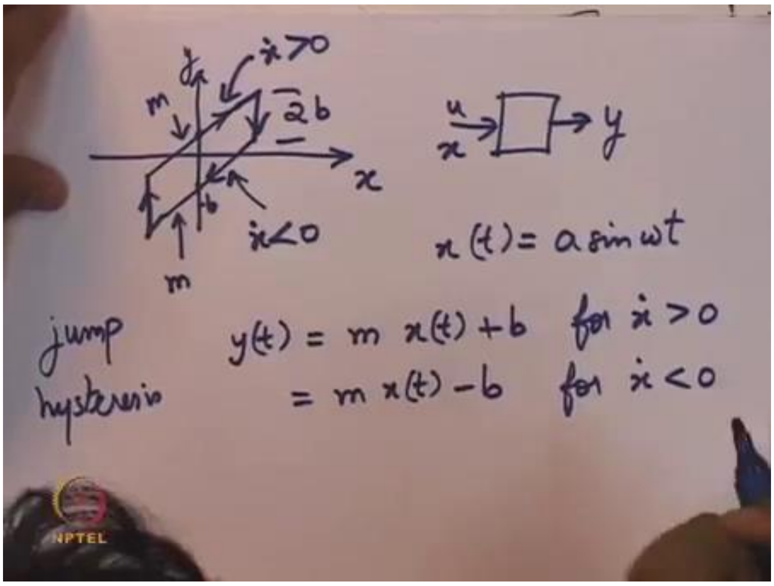

So, consider the jump so called jump hysteresis. So, what is the jump hysteresis? Suppose this is x, this is the system whose input is u, input is called x for this purpose, output is y. So, whether it is increasing or decreasing is what decides whether it is on this curve or this curve. So, this is the case when x dot is less than 0, this is the case when x dot is positive. We might say what happens when x dot is equal to 0. Of course, x dot for the reference signal x of t equal to a sin omega t x dot is equal to 0, but at just one point, so at that time it is jumping from there to here.

So, we will say output and what about the slopes, these slope is m this slope is also m, the jump amount is equal to b. So, we will say that so y of t is equal to m x of t plus b for x dot positive, this is equal to m x of t minus b for x dot less than 0. So, let us evaluate the describing function from by first principles for this particular hysteresis, this hysteresis is what we will call jump hysteresis, what is jump about it, when x dot has increased fully and it is decreasing, that time the output jumps that is the output y, the output jumps by amount 2 b jumps down by amount 2 b. And the other hand after x has decreased fully and x dot is negative and it has decreased and when x starts increasing again, that time the output jumps up by amount 2 b. So, for this particular non-linearity, we will derive the describing functions both the real part and the imaginary part. So, while we do this, we will note some properties of the describing function, what are the various properties?

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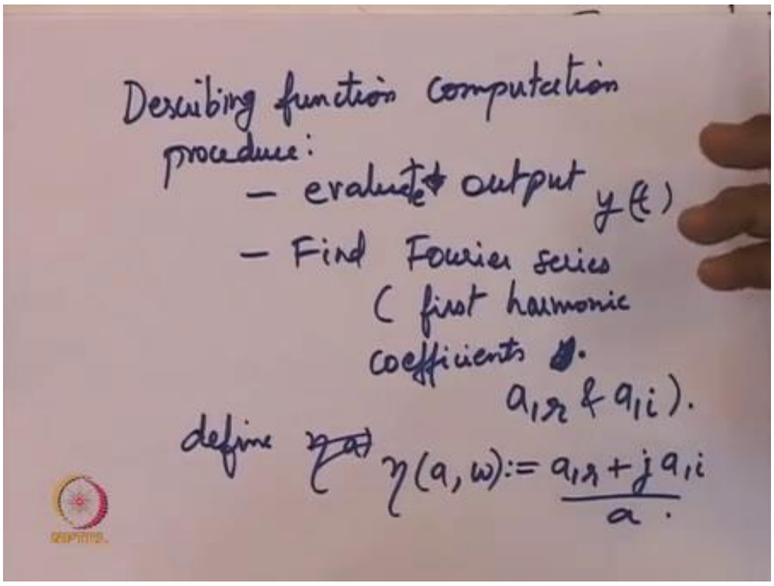

So, let us just recap the describing function calculating procedure that will give us some very important properties of the Describing Function Computation Procedure, evaluate output y of t find Fourier series, Fourier series in particular first harmonic coefficients, we are not interested in other harmonics, first harmonic coefficients y a 1 r and a 1 i; these are the two things that we require now. And then and then define describing function as in general can depend on both a and omega a 1 r plus j a 1 i by a.

So, now notice that if the output, if it turns out that the non-linearity's, if two non-linearity's are added by an amount added to each other then the outputs will just get added to y 1 plus y 2. Then the Fourier series of the sum of two signals is just a sum of the harmonics, the Fourier series extraction procedure is linear in its, in its arguments. So, that is what makes that the Fourier series expansion of two signals y 1 plus y 2, when you add them you have to just add the Fourier series coefficients.

So, this that is what makes this particular step in the procedure linear in the non-linearity also and now the non-linearity comes not just to the addition of two signals y 1 plus y 2. But also to the scaling of a signal by a constant by a static constant, if the if a non-linearity just gets scaled by a constant k then the Fourier series coefficients have to just be scaled. By an amount and hence the describing function has to also be just scaled by same amount k. So, what does, what does this particular property mean?

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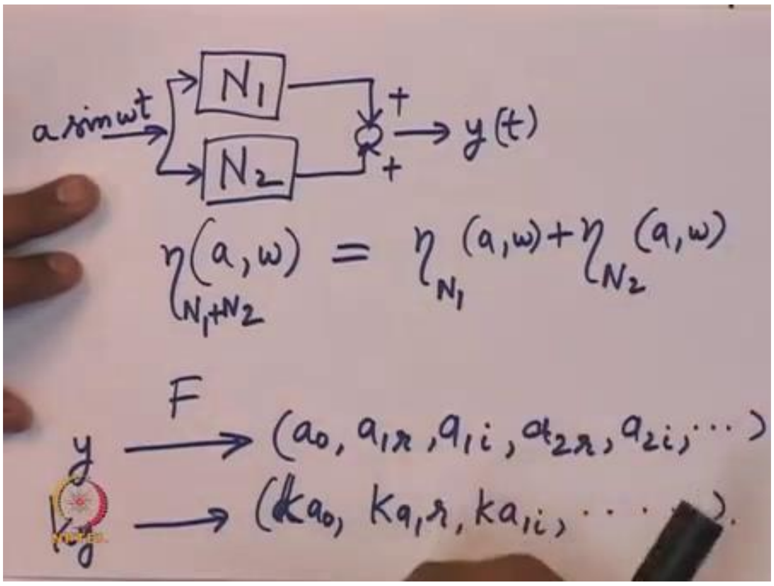

If you have non-linearity N 1 and non-linearity N 2, and if you have done lot of work to calculate the describing functions, and the describing functions where calculated and now you were suddenly told, that the output is the sum of the output of the two nonlinearities, then the describing function of this big block.

Let us call this describing function of N 1 plus N 2 of this block again is a function of a and omega will turn out to be nothing but describing function of N 1 a comma omega plus describing function of N 2 a comma omega. Why did we conclude this? That is because we said the output here is nothing but the sum of the two outputs, here and here. Now, if you are given with the Fourier series coefficients of these. Is it very difficult extract, the Fourier series coefficients of these? No, because the Fourier series let us call this map f that takes a signal y and gives you a 0, a 1 r, a 1 i, a 2 r, a 2 i and so on.

This map is linear in the signal y, because it is linear in the signal y if you multiply this y by a constant k, so k y will just go to k a naught k a 1 r to multiply this by a constant k means a every time instant the value of y of t is just scaled to k times y of t. So, this just gets scaled to k a 1 i and so on. Similarly, y 1 plus y 2 goes to just a 0 1 plus a 0 2, a 0 1 and a 0 2 are the first entry of y 1 and y 2.

So, to say that this map f that takes a signal y and gives you the takes a periodic signal y and takes and gives you a Fourier coefficients to say that it is linear in the signal means, that these Fourier coefficients can just get added. Of course, it is said that this space of periodic signals is at a vector space for you to introduce this sum etcetera. If you take two, you should take two signals, which are periodic in the same period capital T and you have to add them, again you get a signal that is periodic with that time period again.

And also you scale it by constant k, it will be periodic again with the same period. So, that is what allows us to say that, if one has done lot of work to calculate the describing function of nonlinearities n 1 and n 2, the describing function of the net non-linearity n 1 plus n 2 as defined in this block is just sum of the describing functions.

And that is coming because the procedure of calculating describing function goes through this Fourier series coefficients calculation. And this Fourier series coefficients extraction procedure happens to be linear, why is this linear? Because we have that integration operation etcetera and that integration operation is also linear in its argument in the signal y. So, how is this useful? So, we will see quickly how this is a very useful property.

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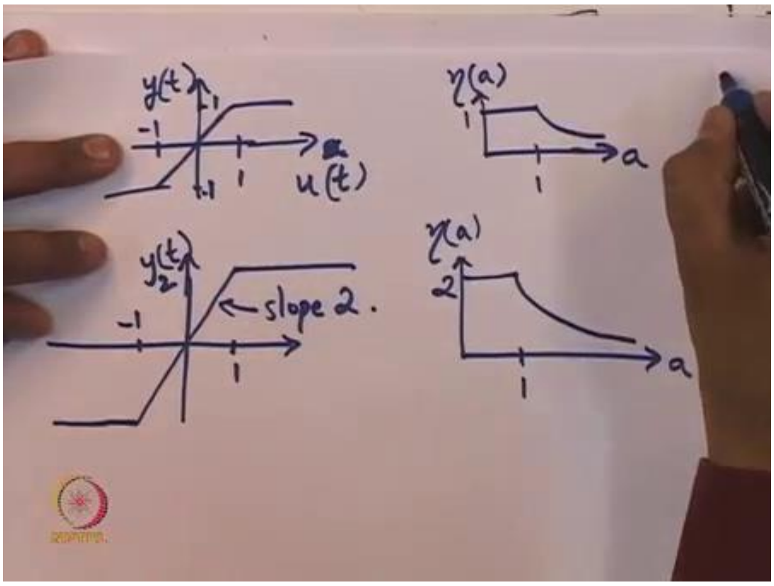

So, if we have calculated for a saturation non-linearity for example, and the output is y of t for the range 1 minus 1 and it is standard. And now, we say that no actually this is not what we wanted, but we in fact wanted scaling by constant k equal to 2 let us say what is that the this part 1 minus 1 cannot be changed by that, that change has to happen by a slightly more complicated procedure. This is y 2 in which slope of this is slope 2. So, notice that this particular non-linearity and this non-linearity are pretty related, this non-linearity has to just be scaled, the output has to be scaled by an amount k equal to 2 to get this non-linearity.

Because of this particular property, if the describing function of this has been calculated like we did before to a function like this, we have this value equal to 1, all we have to do is scale this 2, in which this range is equal to 1 again, but here it starts from 2. So, notice that this is just this one multiplied by 2. Let us take another example, in which we this is an example where it is just scaled. So, let us take another one where we add two non-linearity's.

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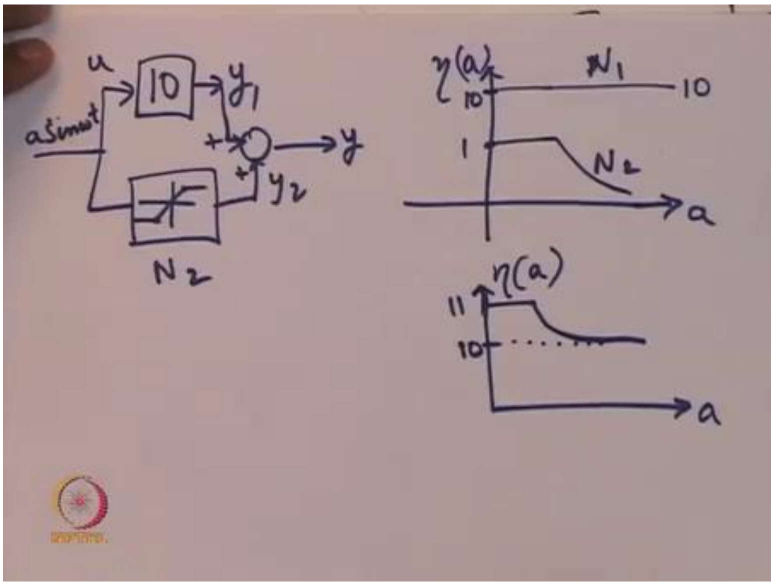

One non-linearity tends to be just multiplication by 10, another non-linearity happens to be the standard saturation non-linearity, this is the input which is a sin omega t and this is y 1 while this is y 2 and the two are added to give you the net output y. Let us plot both of these on a same graph, both are odd memory less time invariant non-linearity's.

This one has this and the other one has had this to this constant had value 10, and this is starts at 1 and comes down to 0, notice that this graph is not scale not up to, not to scale. So, this is for y for non-linearity n 1 of course, it is a linearity in this case and this is for non-linearity n 2. So, what about this net 1 there what is the describing function of the non-linear map here reference input to y, that is just the sum of these both, it starts at 11 and comes down to, and comes down to 10.

So, this net to say that we can just add these two describing functions as a function of a is what makes a describing function linear in its argument the non-linearity. So, we are going to use this very crucially to find out the describing function of the jump hysteresis of course, one can do it from first principle also, but we are going to do almost that, that will be also benefit by understanding this particular structure that describing, that the describing functions have between each other.

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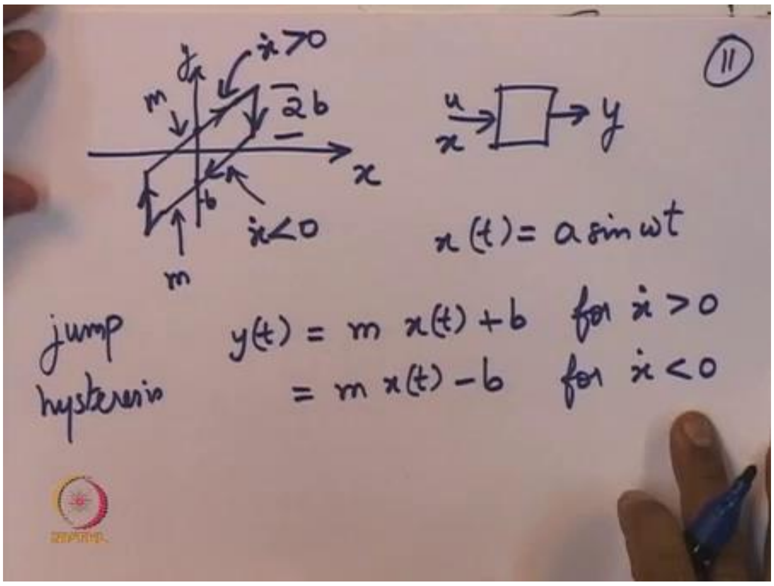

So, let us take the jump hysteresis graph again, so recall that this is our jump hysteresis. When the input is increasing, that time it follows this amplification by m except that it is shifted up by amount b, and when the input is decreasing that time the shift is to minus b and there is also scaling by constant m.

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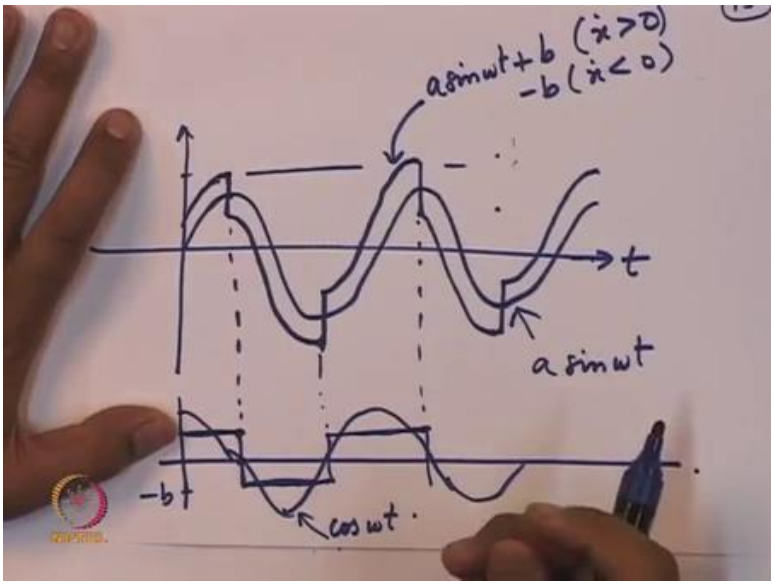

So, let us first consider the case that m is equal to 1, this is the input a sin omega t, the output has two parts, one is the input itself, but it has been shifted up. After the shift is what we are talking about, after it is gone up, there is a shift down by amount 2 b amount b will come back to the same graph and another amount b will bring it b lower. From here it shifts up, so this is how the graph looks. So, notice that this is the super imposition of two graphs which two that we will draw on another, on another page.

This is time axis one of them is just time omega t, this one is the output a sin omega t plus b for x dot greater than 0 minus b for x dot less than 0. So, this is the output. Notice that we have take amplification equal to 1, that is why we are able to see that it shifts by amount 2 b plus b on one side of this minus b on other side. So, we will write this as the sum of two things, that way it is better that we write on some here only. So, this jump is what we can say is equal to b here minus b, again b here.

So, when does this switch sign, it switches sign when sin omega t derivative of sin omega t changes sin, derivative of sin omega t is nothing but cos omega t. So, notice that this is this actually the signum function applied to cos omega t, the conclusion that I am trying to draw from here is that we have a signal a sin omega t, what we have added to that is b times the signum function applied to cos omega t, that is what we are trying to conclude form this figure. How did we conclude that we noted that this is our original signal a sin omega t, when this a sin omega t is increasing namely form here up till here, it has shifted up by amount b. Why is this shift by amount b?

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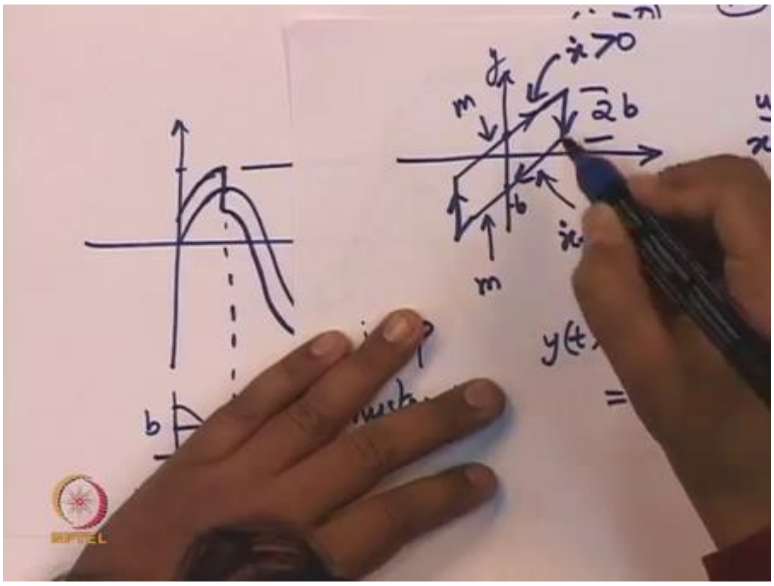

They call that this figure here, this was our jump hysteresis, when x dot was positive, it was scaled by m, m is equal to 1 for now and it is also shifted up by amount b. As soon as x dot becomes positive to negative, it jumps down by amount 2 b and follows this curve here, this is also again scaling by m, but it is amount minus b below the, below the line m, this is the line m with slope m and passing through the origin.

This one is amount b lower, this sign is amount b above, so that the shift is exactly 2 b and it is symmetric, it is symmetric about this point origin. Symmetric not really about the x axis or the y axis because there is this dependence on x dot. But when we say along this axis, there is amount 2 b here on this, there is amount b on this side, amount b on this side. In that sense it is symmetric.

So, what does this mean coming back to this figure, after sin omega t has reached its peak when sin omega t starts decreasing, that time the shift is down by 2 b amount, so that it comes by b amount lower than a sin omega t. But then notice that this sign omega t derivative has changed sin is nothing but to say that cos omega t function itself has changed its sign, that because it has cos omega t has change its sign means to say that sin omega t derivative has changes it sign.

So, this means that we are adding b where cos omega t is positive, we are adding minus b when cos omega t is negative, again we are adding b when cos omega t is positive. So, this is nothing but the signum function operated on cos omega t. So, what does this mean, what this means is that the describing function of the jump hysteresis can be very easily calculated by applying the signum non-linearity on the derivative of sin omega t. Derivative of sin omega t is nothing but cos omega t.

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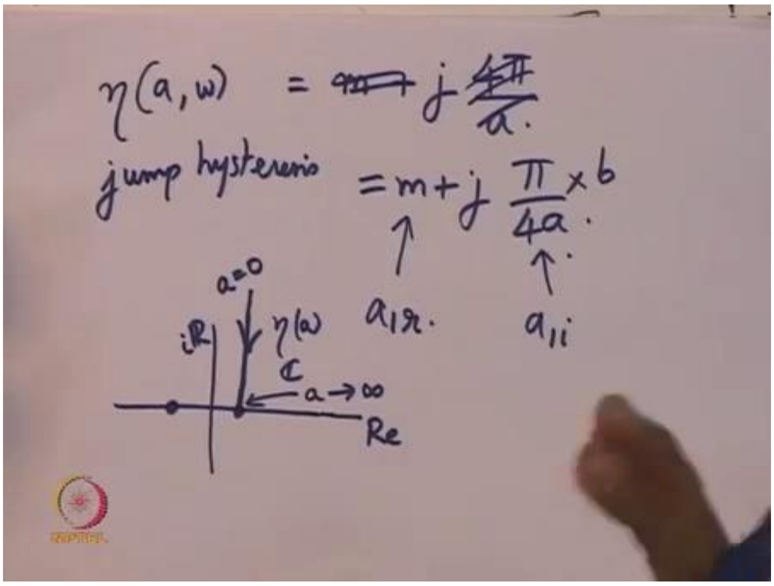

So, the describing functions of a, of the jump hysteresis equal to just a constant m applied to the scaling. Which we have taken equal to 1 plus j times j times what, the describing function of signum on-linearity, which we found was equal to 4 pi over a. I think let me verify, sorry I just now verified, it is not this, it is equal to m plus j times pi over 4 a, why did we bring this in, because we saw that the signum non-linearity was being applied to the cos signal, cos omega t signal. And the imaginary part comes precisely as the Fourier series coefficient, the first harmonic of cos omega t.

It is a coefficient of, this is nothing but a 1 I, while this is nothing but a 1 r, this 1 we noted was nothing but the describing function of the signum non-linearity, but that time it was odd. Hence, it applied to the real part only, but now it is coming with the cosine tern and hence we have multiplied with j here. So, this is how the jump hysteresis describing function looks. So, we are no longer able to plot the describing function as the function of a, but we have to plot it here.

So, I missed one thing, so notice that this one was scaled by amount b, why because the jump was not between plus and minus 1, but the jump was between plus b and minus b. Hence, we have multiplied this by amount b. So, this is describing function in the complex plain, so it can sort of that if m is positive, this is how for a equal to 0 is very large, and it comes down like this and this is for a tending to infinity.

As the amplitude of the signal tends to infinity, the jump amount is relatively very small and hence it amounts to just amplification by m, it turns out to come on the real axis, this is the imaginary axis in the complex plane, this is the real part. The significance of plotting the describing function on the complex plane, as the plot in the complex plane will become vector here very soon, when we use the describing function for finding periodic orbits.

So, because the describing function is complex here, it is no longer real like we did like we said it for odd memory less nonlinearities so far. Here the describing function is complex it depends on m a and b for the jump hysteresis, b was the amount by which it jumps 2 b was the amount by which it jumps, m was the slope for the case that x dot is positive or negative and a was amplitude of the input signal a sin omega t.

So, when we plot this m is some positive number is what I have taken here, hence we have plotted here and it comes from a very large for a equal to 0, it is some number with a very large imaginary part and it decreases, imaginary part is decreasing as a is increasing. And it finally, comes down to the real axis, for a tending to infinity. What is the reason that for a tending to infinity comes to the real axis, we have to go back to the plot of the jump hysteresis.

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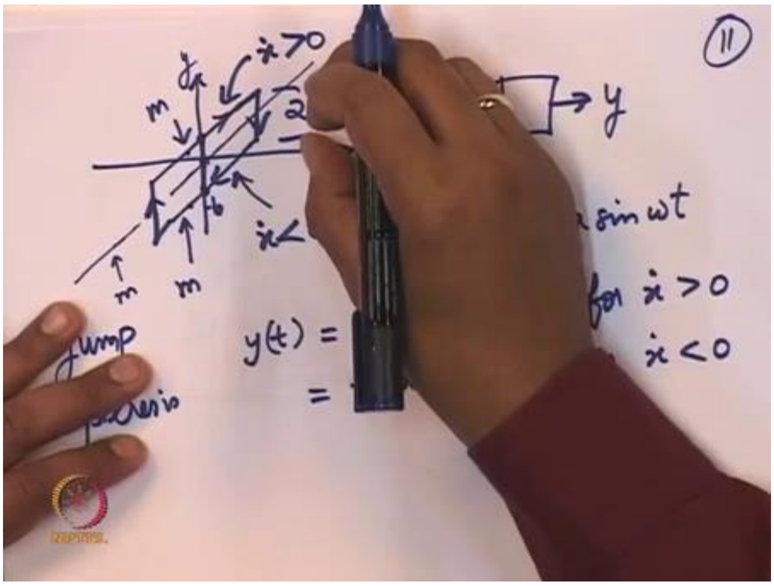

So, when the amplitude is very large, that time one can think that the jump amount 2 b is a very small fraction of the total signal, because it switches by amount 2 b no matter whether amplitude is 10 or 100 or 1000. So, the fraction of jump is a very small fraction of the total signal. Hence, the imaginary part is going to be very small, imaginary part itself is come because it is no longer memory less, it depends on whether x dot is positive or negative.

And also because it is not odd, it is not an odd memory less non-linearity. Now, we are going to see how this describing function is to be used for finding periodic orbits. That is the next important topic that has hysterically been the reason that describing function has been investigated so far for finding the amplitude and frequency of limit cycles, and as we noted in the very start of this lecture and also a few lectures ago, sustained, robust sustained oscillations can be implemented only by non-linear circuits. And we saw how the saturation non-linearity for example, can give us robust sustained oscillations. With this third order system in the feed forward path, third order linear system in the feed forward path. So, let us come back to that example.

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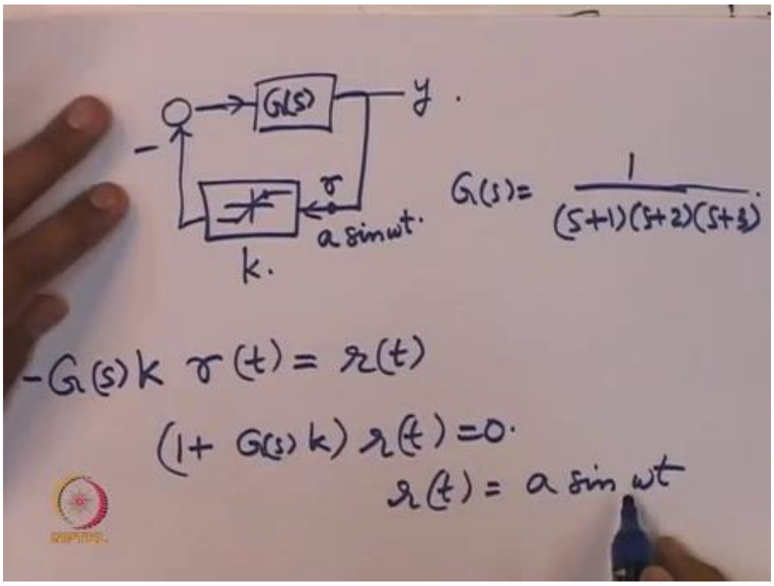

We have G of S, we have the saturation non-linearity, this is the output y, this G of S is equal to 1 over S plus 1, S plus 2, S plus 3, and we have some signal a sin omega t here. So, let us first take the case, that this non-linearity is just a pure constant, that can also be thought of to be the case, when amplitude is smaller than the range over which it is linear.

As long as the amplitude is smaller it is within that range the this system will be seen as the linear system one can think of it as just a constant k. So, when would we have periodic orbits in the close loop, we will have periodic orbits if assume that the external input is 0, so we have some signal here r, it gets amplified by k goes through this comes back there, and it is equal to r.

So, notice that r of t is here gets multiplied by k, then G S acts on it and there is also minus sign here because of course, minus sign is operated on k before G of G operated. And that gives you back and this r and this r are the same this, ignore this small different between the two, this is nothing but to say that 1 plus G S k r of t equal to 0. We will say some signal r of t equal to a sin omega t happens to be a periodic solution, if it satisfies this differential equation. What is the meaning that it satisfies the differential equation when we substitute a sin omega t into this, notice that we will get into this r of p that time.

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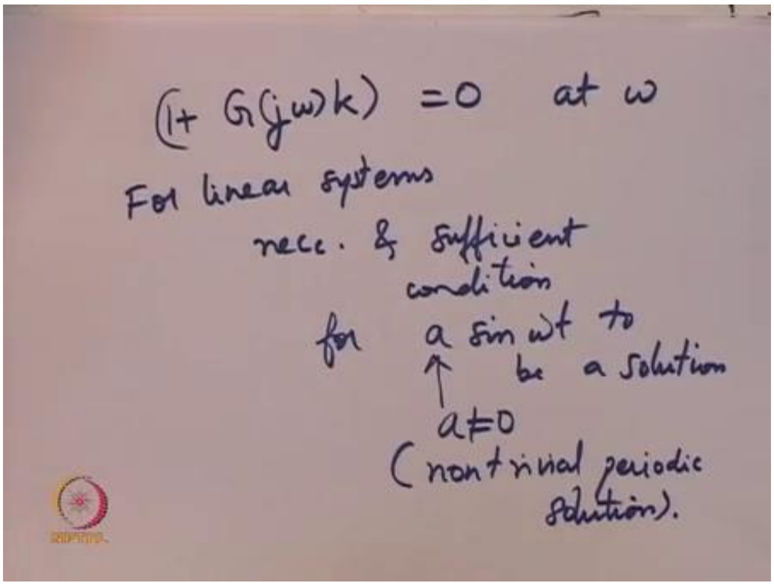

We get 1 plus G of j omega. Hence, k equal to 0 at omega. So, this is like this is for linear systems, for linear systems necessary and sufficient condition, for a sin omega t to be a solution. Of course, one might say that look a sin omega t will be a solution even if this is not equal to 0, because we could just take amplitude a equal to 0. So, a of course, we should say that this amplitude a is not equal to 0, that is to say that non trivial periodic solution. So, when do we have non periodic, non trivial periodic orbits in the system, we can have non periodic orbits if 1 plus product of the gains is equal to 0. It is extremely important equation, what does this equation mean?

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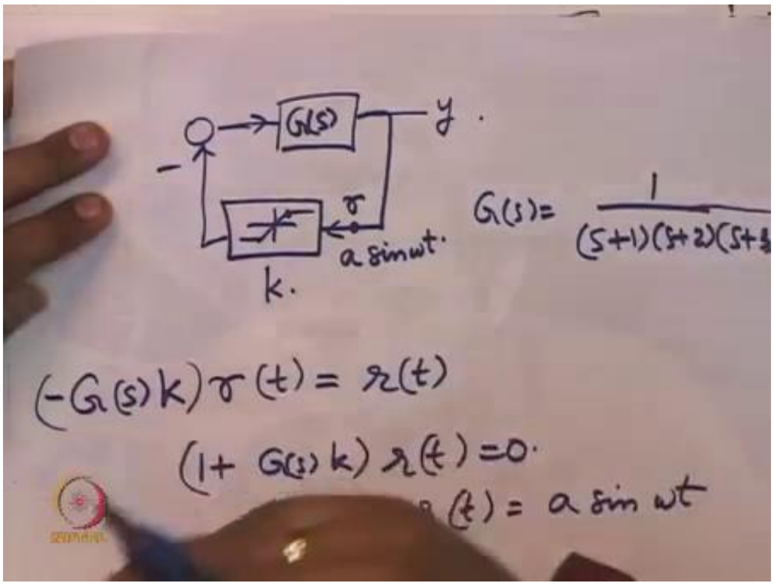

Look at this, consider the gain from here from minus 1, and the gain from here, the net gain should be equal to minus 1. If you start form r, this is the net gain that we got from here that gain should be equal to plus 1. It depends on in the definition loop gain whether you take this minus sign into account or not.

So, r of t goes through this and comes back here, but it is the same signal, this is the very hand waving way of understanding this, understanding this argument that you take a signal at some point here, it under goes again by amount k, it under goes by another gain, first by minus sign by minus 1 and then by G of j omega, why is G of j omega the gain, because that is the meaning of a transfer function, the transfer function is precisely the gain when you give exponential signal into in it and when you give sin omega t as a signal. Then the amplification is exactly G of j omega at steady state of course, this requires that all the transients have died. So, how this translates to the describing function, how for linear time invariant systems, the amplitude does not play a role is what we will see in the following lecture.