Nonlinear Dynamical Systems
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Lecture - 22
Non-linearities based on circle criterion

So, in the last class what we were talking about was a circle criterion. So, the general idea was that we know, if you have a passive system and you inter-connected, passive linear system and you have a non-linearity which is passive, that means something which lies in the 0 infinity sector. Then the new inter-connected to we get asymptotic stability. Now, one could think of class non-linearities which is not really the 0 infinity sector, but some other let us say k 1, k 2 sector.

So, in the last lecture we went through these various transformations that you can do. These transformations change the given non-linearity, which is in some k 1, k 2 sector, into a nonlinearity the 0 infinity sector. Now, when you have a linear plant inter-connected with this given non-linearity and we want to talk about the stability of this closed loop system.

We could change the non-linearity from the given k 1, k 2 sector to the 0 infinity sector, but you see that is on the feedback loop. So, then appropriate changes made to do on the linear plant and when we do the appropriate changes on the linear plant, then the inter-connection between this new linear plant and the new non-linearity is completely equivalent to the inter-connection between to the old linear plant and the old non-linearity.

The only difference in this whole process is that, now the new nonlinearity is in the 0 infinity sector which means it is passive. And because that is passive the corresponding new linear system if that is passive, then we know that in the new system with the new linearity and the new non-linearity that is asymptotically stable and because the two systems are equivalent. Therefore, the old linear system inter-connected with the old non-linearity stable.

So, this is the essential idea that was used. Now, transforming a given non-linearity in given k 1, k 2 sector to a non-linearity in the 0 infinity sector, we can do that using these loop transformations, but then if one does not want to lose this loop transformation, but we are given a linear plant we are given linear plant, the old linear plant and the non-linearity.

Then in a by loop transformation the linear plant is converted to some new linear plant which must be passive, but without doing this conversion of this linear plant. If one can predict whether the new linear plant along with becomes passive or not by looking of the nyquist plot of the whole linear plant, then that there is some advantage in this. The circle criterion is one thing which let us do that. So, perhaps I will just repeat a bit about what we have already discussed earlier. So, perhaps the situation where you are looking at non-linearity in the k 1 k 2 sector. We will see what the linear plant changes to and. So, suppose we consider a non-linearity.

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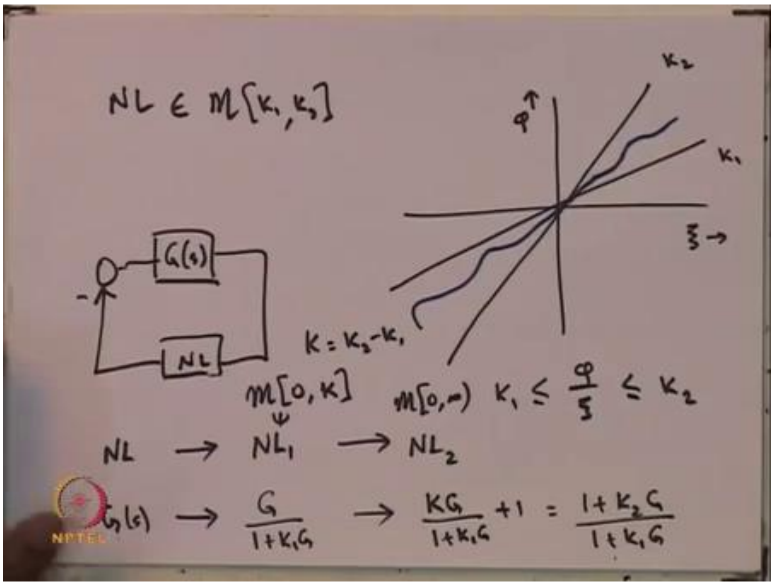

So, suppose we consider a non-linearity, which is in the k 1 k 2 sector. So, what we mean by that is if you think of psi as the input to the non-linearity and phi as the output to the non-linearity. Well there is this line with slope k 1 and there is this other line with slope k 2. What we saying is that the non-linearity is such that it lies in the k 1, k 2 sector. Of course, the other way to talk about this is that phi by psi is, this thing is greater than k 1 and it is less than k 2.

So, this is another way that you can that you write this, you know characterize this non-linearity. Now, if one is looking at a linear plant G of s and inter-connected with this non-linearity N L in this feedback form. So, suppose we have this particular situation then we want to talk about the asymptotic stability, I mean under what conditions of G s I mean what should be the characteristics of G s, the stable G s is inter-connected to the non-linearity in k 1 k 2 sector.

The resulting system is asymptotically stable and then what we have discussed with last class is that, this non-linearity in N L I mean this non-linearity N L. This can be converted into a non-linearity. So we can go through in to 2 steps. So, first you have this N L and first convert it into let me call it N L 1 which is something in the. So, N L 1 belongs to. So, this is nonlinearity in the 0 k sector where this k is k 2 minus k 1. So, you can do one transformation like this.

Then this can be followed by another transformation, the second transformation is when you convert something in the 0 k sector to this second non-linearity which is a passive non-linearity. That means in the 0 infinity sector. Now, when one does this, then the linear plant which we had here G or G of s, that also gets transformed in a certain way and we have talked about it earlier.

So, the way it gets transformed is when you take the non-linearity N L to N L 1, then the linearity gets transformed to G upon 1 plus k 1 G. So, this becomes new linear plant with this non-linearity. So, the inter-connection of these 2 equivalent to the original inter-section that we were interested in. Then this conversion from the 0 k sector to the 0 infinity sector makes a conversion here, which makes this k G upon plus 1 plus k 1 G plus 1, but then we saw that this is equivalent to 1 plus k 2 G upon 1 plus k 1 G.

Now, this non-linearity with this plant which is 1 k 2 G and 1 plus k 1 G. This inter-connection feedback the inter-connection between this linear plant and this non-linearity is exactly the same as inter-connection between this original plant and this non-linearity and because this non-linearity is the 0 infinity sector. We now can use the passivity theorem. So, if this resulting plant from G given G and use k 1 and k 2 and make this new plant.

This new plant is passive and stable or in other words the nyquist plot of this new thing lies in the right half plane and its stable. Then this inter-connection is asymptotically stable that translates to this original inter-connection being asymptotically stable, but one would like to check that, what do we want to check is the following.

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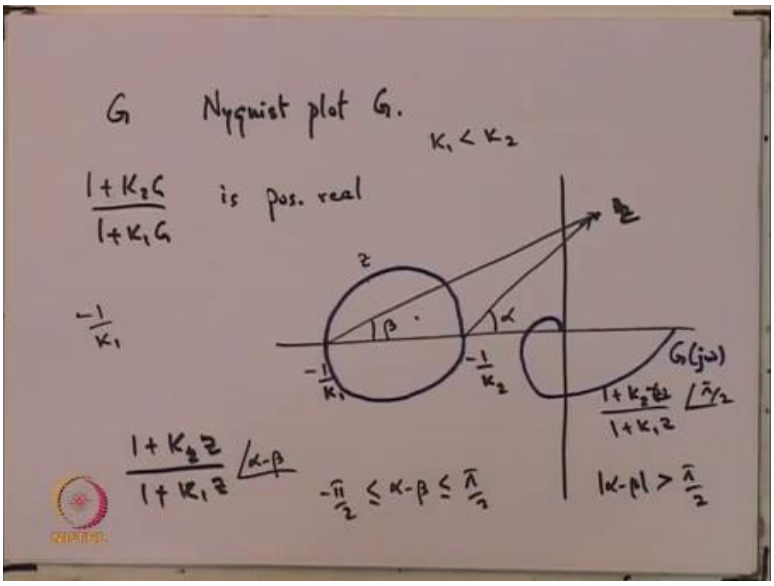

So, given G and therefore, the nyquist plot of G. We want to check whether this given 1 plus k 2 G upon 1 plus k 1 G, whether this is positive real. Then in the last class I sort of demonstrated how we do this checking. So, one thing you do is, you should look at the denominator. This gives I mean this gives that a pole of this system is like when G is equal to minus 1 by k 1. So, of course, k 1 is less than k 2 and since k 1 is less than k 2 therefore, minus 1 by k 1 that is this point and minus 1 by k 2 is this point, when you are trying to evaluate this transfer function given in G j omega.

So, we said that suppose you have any point z here and can we call it z. You want to evaluate 1 plus k 1 or k 2 z upon 1 minus 1 plus k 1 z, then the angle of this is essentially, you draw these vectors here to here and look at the these angles alpha and beta. The angle of this transfer function is going to be alpha minus beta. Now, asking for this transfer functions nyquist plot, lie in the right half plane is the same asking for alpha minus beta to be in this range between minus phi by 2 and phi by 2.

Then we made the main statement of the circle criterion, which was that you look at the circle. Now, inside this circle if you have any point z. I mean along the boundary of the circle is taken point z then 1 plus k 2 z upon 1 plus k 1 z. This as an angle, which is precisely phi by 2 and phi by 2. If it is there and phi by 2, if it is in the lower semi-circle.

If it is in the upper semi-circle is plus phi by 2 and in the lower semi-circle minus phi by 2. If the point z is outside then this alpha minus beta satisfies this condition and if the point z is inside the circle then alpha minus beta in fact turns out to be. I mean the modulus of the alpha minus beta turns out to be larger than y by 2. This is where we stopped last time. So, what does this mean this means.

So, suppose you have given this G and given this nyquist plot of G. The nyquist plot of G lies completely outside this circle. So, maybe this is the nyquist plot of the circle, this is the nyquist plot of the plant. So, G j omega, now this lies completely outside circle. So, for every point along the nyquist plot because of that argument we have given earlier alpha minus beta, this angle is going to be between pie by 2 and minus phi by 2.

So, what it means is for this particular plant G, if you calculate 1 plus k 2 G upon 1 plus k 1 G and plot the nyquist plot of this new transfer function. Then that nyquist plot is going to lie completely in the right half plane, but in the discussion that we had earlier, we had said that given up linear plant, if the nyquist plot lies in the right half plane. That mean is the real part of the every point on the Nyquist plot is positive that does not necessarily mean that transfer function given transfer function is positive real. I had mentioned that if one also insists that not just an imaginary access, but all of the right half plane maps into the right half plane. Then that transfer function is certainly positive real.

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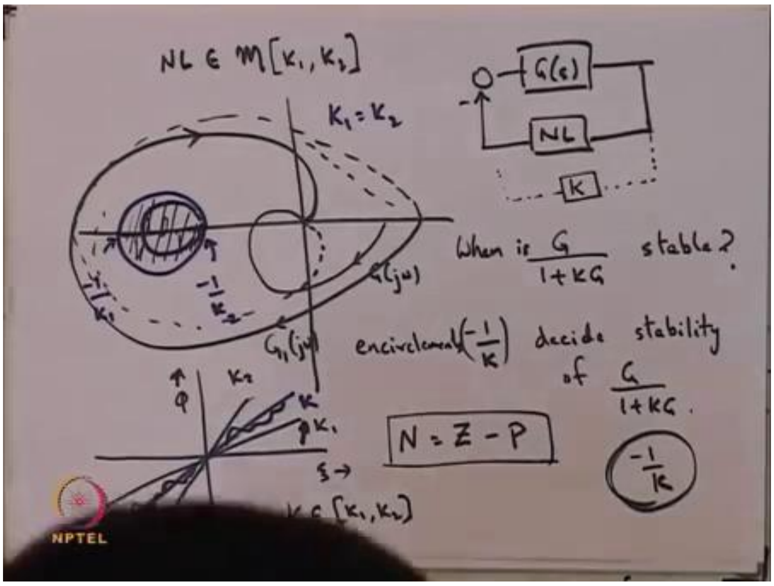

Now how does one guarantee that given such a situation where? So, what we were looking at earlier. So, let us say we have the circle here. Let us say this minus 1 by k 2 and this is this here is minus 1 by k 1. You have a nyquist plot and let me think of the nyquist plant like that. So, G j omega of course, this G j omega does not enter into the circle. Therefore, when you transform G j omega into 1 plus k 2 G upon 1 plus k 1 g. So, let me call it G 1.

So, if you draw G 1 of G omega then that will lie completely in the right half of the complex plane, but when you bring this G 1 into this may be. In this particular case I would not know what the nyquist plot of G 1 looks like, but let us suppose that it looks let say something like this. Now of course, this would be the other half of the nyquist plot. Now, whether the right half plane under this map G 1 maps inside or outside. How does one decide that because that will decide that resulting transfer function, that you have got apart from the positive real, it should also be stable.

Or apart from being or I mean apart from being positive real it should also be stable or another equivalent definition is that apart from the nyquist plot being on the right hand side all of the right half should map in to the interior or rather into the right half plane. So, how can we now check that, with respect to this original, you know the nyquist plot of the original plant which is G j omega. Now, it turns out that the way one does is very similar to the nyquist plot criterion that one uses for linear plants.

So, let me now try and motivate this interpretation. So, here we go, let me draw that thing once more. So, let us say this here is the nyquist plot we have. Let me suppose that this here is the circle that we had obtained earlier. So, this as minus 1 by 1 by k 2 and this is minus 1 by k 1. Now, we are in this particular situation, analyzing this closed loop system, which has the linear plant with a non-linearity in feedback loop. This non-linearity is a non-linearity that lies in the k 1 k 2 sector.

Now, what do we mean by this k 1 k 2 sector. Well this is also clear, this is line with k 1 this is a line with k 2 this will be input to the non-linearity this being output. So, what we mean when we say non-linearity is lying in the k 1 k 2 sector, is the non-linearity sector like this. Now, instead of thinking of non-linearity let us think of a linearity I mean let us think of a linear element that lies in the sector. So, let us say something like this. So, this has slope k where k is in the interval. So, k is in the interval from k 1 to k 2. So, the slope of this blue line is k 1.

So, instead of this non-linearity let us assume the feedback. Instead of the non-linearity is really a linear feedback with value k. So, let us assume that this is the portion, which is connected and not the non-linearity. Now, if k is connected instead of non-linearity then the resulting transfer function is going to be G of 1 plus k G. When can we say that is G upon 1 plus k G, when can we say when is G upon 1 plus k G stable. The way we decide when this transfer function is stable, is again by looking about nyquist plot.

What we should have is that the nyquist plot should not in circle the point minus 1 by k. So, encirclement of minus 1 by k decide stability of G upon 1 plus k G. How is that done, that is done by using the nyquist you know the nyquist criterion, which is that suppose the original transfer function G was stable. Then the number of encirclements of this point minus 1 by k must be 0. On the other hand G was unstable, if you recall that there was this theorem.

If 0, if the number of zeros in the right half plane of the of the transfer function given by z and the number of poles in the right half plane of G omega is given by the p. Then we had something like and is equal to z minus p this kind of formula in the nyquist criterion. What that translates to is depending upon the number of right half zeros or right half poles of G of s. One can specify that this G of G omega should encircle this particular point, minus 1 by k.

The appropriate number of the times in the clock wise the anti-clock wise direction depending upon, you know whether the number of zeros with larger number of 0 number of zeros. The number of zeros in right half plane larger or the number of poles in right half plane is larger. So, if one makes the assumption G is a stable plant for example, then in that case this G j omega should not encircle the point minus 1 by k. Notice that this minus 1 by k is going to be some point here because the slope is between k 1 k 2.

Now, if you look at all these linear plants which can lie between k 1 and k 2. Each time you will get some you know for stability. Suppose, you start with G of s, which stable then for the resulting closed loop system to be stable, you would say that G of j omega should not intersect some point minus 1 by k. This point minus 1 by k will vary here between the point minus 1 by k 2 which is what will have if you take this slope of the linear part to the k 2 or minus 1 by k 1, if you take the slope to be k 1.

So, as you vary the k you get all these points in the real part of inside the circle. It says that G j omega should not encircle any of them. That is of course, if you start off with G of s which is stable then this nyquist spot should not encircle. On the other hand, if you start off with some G s which is not stable. That means it has poles or zeros in the right half plane. Then what you would get is each of the times it should encircle the point minus 1 by k, appropriate number by times in the clock wise and anti clock wise.

So, you could have a very well G omega which looks like that. Then for each one of these points this guy might result in. So, let me call this G 1, G 1 G omega and this is such that for each of this points 1 by k, it encloses it an appropriate number of times. So, if I also draw its reflection, it should go something like that, which means that any point minus 1 by k gets enclosed once and twice in the clock wise direction.

So, twice in the clock wise direction means the original transfers functions. Suppose, the original transfer function had 2 poles in the right half plane then because you have these 2 encirclements therefore, the resulting transfer function is going to be stable. Now, that was for the nyquist criterion pole does. Now, that is when you have a linear feedback and for each 1 of this linear feedback what we are claiming is these points, but now we do not have a linear feedback, but we have the non-linearity.

This non-linearity you can think of as like we think of linear feedback with a linearity line in between the slopes k 1 and k 2 with some pertivation. So, you could think of this nonlinearity of something linear like this k, but with some pertivation. Now, one way to view this circle is that these pertivation from this k are captured here within the circle. So, the any non-linearity in this given k 1 k 2 sector can be thought of as a linearity with pertivations. That linearity with pertivations when further linear parts you get this thing and the pertivation is the rest of the circle.

So, if you avoid any point in this rest of the circle, and you have G of G omega which avoids that point, but then the original G of s. Suppose, it is stable then in fact this whole circle should not be encircled, but if G of s had unstable 0, unstable poles then the whole thing should be encircled with the appropriate number of times. Now, here is a very interesting way to think of this. Suppose, you think of this k 1 going up that means the that lies between that k 1 and k 2 k 1 is allowed to go up. Now, as k 1 is allowed to go up therefore, the value of k 1 is changes.

Therefore, this becomes. So, k 2 is kept constant therefore, this becomes another circle a smaller circle and then as it is allowed to go up and up. Finally, let us say this k 1 is made larger and larger until finally, k 1 is equal to k 2. Then what would have happened if this circle would have shrunk until it becomes just this point minus 1 by k 2. Now, if this interval is shrunk from k 1 k 2 k 2 k 2; that means k 1 as become k 2, then there cannot be a non-linearity.

The Only the feedback that you have is in fact the linear feedback with linearity being k 2, but what that would have meant is that this circle has shrunk down to this one point minus 1 by k 2 and by the nyquist criterion for linear plants, we know that the number of encirclements of that minus 1 by k 2 by this G j omega, would depend upon the open loop G j omegas. I mean the open loop plant G s whether it is stable or not.

If it is stable for example, we should not have any encirclements of minus 1 by k 2. If it is not, if it is unstable it has poles in the right half plane, then it should be an appropriate number of encirclements means of the point 1 minus 1 by k 2. So, in some sense all this non-linearity lying between k 1 and k 2 is captured by this circle. That circle shrinks down to a point when you shrink this interval down to making it a linear gain. Conversely if you start from a linear gain and expand it out, then as you expand it out the uncertainty comes out in the form of this circle, expands out into that circle with the appropriate size.

If the transfer function does the correct number of encirclements for that circle, then the resulting system is asymptotically stable. So, in a sense it is the generalization of the nyquist criterion one uses for the linear plants. Now, you could have the various circle criterion for various different non-linearities. So, let us now look at what happens as you change the non-linearity or the sector in which the non-linearity is present.

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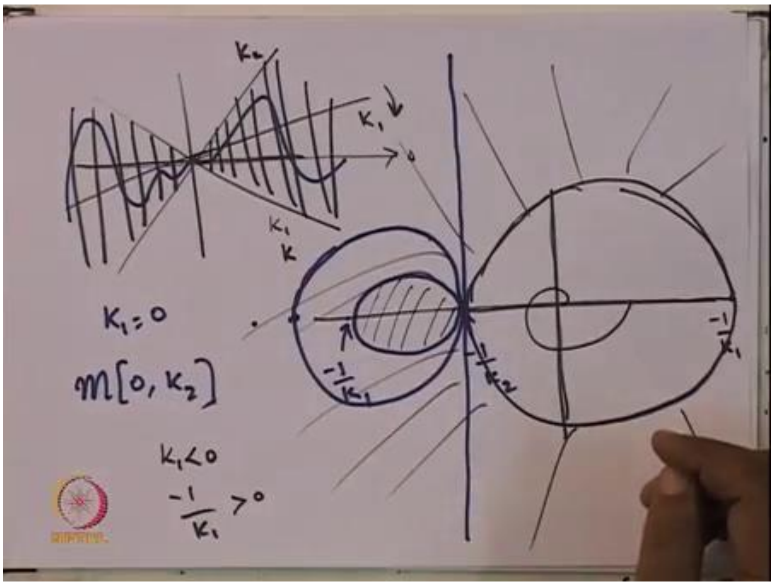

So, suppose you take this non-linearity and let us suppose this is slope k 2. This is slope k 1 now as a result of this what you are going to get here, the circle that you are going to get. Well the circle you are going to get is something like that like so. Well that might not look like a circle, but just assume that this is a circle. So, this is minus 1 by k 2 and this is minus 1 by k 1 and it is in the k 1 sector and this is the circle criterion. That means the nyquist plot should not enter the circle.

So, in some sense the forbidden region. So, the inside of the circle is the forbidden region. So, long as the nyquist plot lies outside you find the transformed nyquist plot lie on the right half. Now, let us do one thing let this k 1, so the non-linearity lying in the k 1 k 2 sector. Let us move this k 1 downwards that means the k 1 k 2 the lower limit is made even lower. So, this is 1 by k 1. So, as k 1 is made smaller minus 1 by k 1 becomes a larger thing. Therefore, the resulting circle is larger. So, as you have made it smaller. So, suppose you made it this small, then we will end up with a circle which is larger, this might not look like a circle, but you have to imagine this a circle.

As you keep lowering k 1 further and further this circle become larger and larger until this k 1 so much that k 1 became equal to 0. That means now we are thinking of the non-linearity in the sector 0 k 2. Now, what is going to happen here this circle that point minus k 1 by k minus from larger and larger until when k 1 becomes 0, this value minus 1 by minus 1 upon k 1 become minus upon 0 that is infinity.

So, it is gone real far off and then this circle criterion essentially tells you that particular circle is everything to the left which means, the nyquist plot in this particular case. If you are looking at k 1 0 at k 2; that means, if you are looking non-linearity like this. Then the nyquist plot should lie to right. Now, instead of pulling this k 1 down if you keep the k constant and push k 2 up. Then this is the original circle, when you push k 2 up this gets extended until when you hit infinity it becomes the circle with minus 1 by k 1 here and 0 here.

So, some circle like that, if you expand k 1 if you bring k 1 down to 0 at the same time take k 2 up to infinity and make it as a 0 infinity sector, well as you are taking k 2 up this point keeps expanding until you get a circle like that. As you keep expanding k 1 it goes up to infinity. So, finally, you have this imaginary axis everything to the left of this is the forbidden area. So, your nyquist plot should lie completely in the right half plane which is got essentially the, result about positive reality is all about.

So, this result is in fact more general kind of a result of which the positive real condition is a special case, but now interestingly we can do more things. For example, keep k 2 like this k 1 could be extended to such an extent k 1 in fact becomes negative, so one is looking at a non-linearity which can lie in this whole sector. Where, so this is the 0 slope in this whole sector we could think of non-linearity like this.

Now, what would happen in this particular case. So, if you go back here, we keeping k 2 constant and k 1 your extending it keeps going until it reaches infinity and therefore, this whole region is the forbidden region. After that when k 1 becomes negative 1 upon k 1 minus 1 upon k 1 is in fact a positive quantity, which is you know close to plus infinity. So, then what is going to happen is when you are looking at k 1 k 2 with k 1 less than 0 therefore, minus 1 by k 1 is greater than 0. So, the minus 1 and k 1 is probably some point here minus 1 by k 1 minus 1 by k 2 is here.

So, you can these 2 points and think of this circle here. So, you get a circle like this, but there is a catch, it turns out that now the back portion is the outside of the circle. In other words the nyquist plot should lie completely within this region and anywhere outside is the forbidden region. So, earlier we had a circle and inside of the circle is forbidden and anywhere outside is allowed, but now when this k 1 as become negative it turns out that you again becomes a circle, but it is the outside which is forbidden and the nyquist plot as to lie inside.

Now, one way to think of this is in the following way. So, suppose you have the complex plane. So, what you can do is you have this complex plane and all points which are the infinity points, you can think of holding the complex plane up and all points which are infinity think of them together as 1 point. Therefore, now this complex plane has become as like a sphere. Now, this circle that we drew on the complex plane, if you now translate it onto this sphere you would be getting a circle on this sphere, on this sphere on the surface of this sphere somewhere your drawn this circle.

Now, if you draw a straight line on the complex plane, then think about the straight line. The straight line if you translate onto this sphere, you will mark all the points in the in the complex plane on the corresponding and fear, but you see that all infinity you collected up and you have the special point. So, if you thinking about this sphere think about the north pole of this sphere as the special point which is all the infinity is collected together.

So, when you looking the straight line this straight line goes to plus infinity and minus infinity, which means when you translate it into a curve on this sphere it will touch the north pole. So, straight lines essentially translate into circles on the sphere which pass through the north pole. So, if the circles pass through the north pole now this is the good part. You had you see in this diagram you have a circle and the circle kept expanding that means you have the circle in the left half plane which is contain the forbidden region and it kept expanding.

Now, it kept expanding until it became a straight line that is when the k 1 became a 0 slope. So, that gave you a circle, which pass through the north pole. So, you see you had small circle on the surface of the sphere and the circle kept expanding and the inside of this, I mean on the sphere you are drawing the circle and inside of the circle is the bad region. The outside region of the circle is the good region as far as the surface of this sphere is concerned. You translate that in do paper this is what you get?

Now, as the circle keeps growing finally, when it becomes this straight line that means the slope k 1 is equal to 0. Then the circle has grown in such a way that it now passes through the north pole. Now, when the slope is further reduced from k 1 equal to 0 then what happens, is that the circle which pass through the north pole has got larger and the infinite point is a forbidden point. Now, on this sphere if you have a circle that circle will translate either in to a straight line, if it passes through the north pole or into a circle.

Now, the point of at infinity will correspond to the infinite region I mean the point you know, the outer region when you translate even to the map, it translates to the outer region. Now, when the circle expanded. So, that became larger this point was the forbidden, I mean the infinite point I mean the north pole was a forbidden point.

So, that is precisely what happens here, when this keeps expanding out and comes to the other side then the pointed infinity is a forbidden point. So, the forbidden point is the outer part. This part is the nice part. Now, if you keep this k 1 constant here now start moving k 2. So, k 1 has a negative slope and now you start moving k 2. As you bring k 2 down what is going to happen is this nice region, this k 2 down means is this is going to go that way.

So, this nice region is something which is going to keep expanding because this minus 1 by 1 k 1 is constant and minus 1 by k 2 keeps going further and further. So, it keeps expanding that way. So, the nice region keeps expanding, but the outside region is the bad region until when k 2 hits 0. So, when k 2 hits 0 this minus 1 by k 2 is gone off to infinity, which means you have a straight line here. The good portion is this side and the bad portion is onto the other side.

Then suppose k 2 becomes negative then minus 1 by k 1 is here and minus 1 by k 2 will be further to the right. Then you have would have a circle there and the interior of that circle has to be avoided, whereas the exterior is the good portion. So, maybe I would just draw a series of pictures with the circle and I will also draw which is the good region and which is the bad region. So, suppose you have a non-linearity like this.

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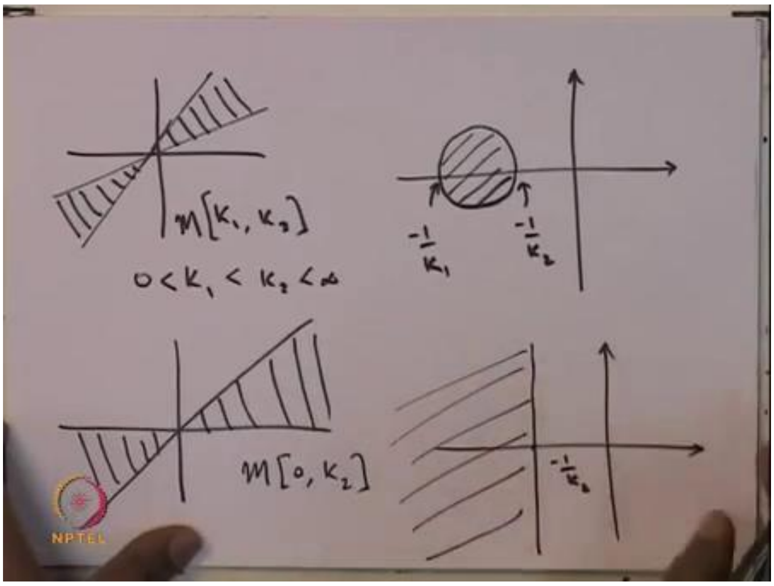

So, the non-linearity is in this shaded region. So, the non-linearity is between k 1 k 2 where k 1 is positive less than k 2 less than infinity. What this translates to is a circle, this is the point minus 1 by k 2, this is the point minus 1 by k 1. The forbidden region is the shaded part. This is what we first showed. Now, if the non-linearity is expanded such that the non-linearity is in this sector like this. So, the non-linearity now is in the 0 k 2 sector then what this translates to here.

So, here you have this outside is the good part. Now, here what you will have minus 1 by that minus 1 by k 2 is still there, let me probably draw the arrows for the original axis. So, minus 1 by k 2, but minus 1 by 0 is infinity. So, what you have is this line is like this. This thing that I am shading is the forbidden region this is a nice region. So, if you had a non-linearity like this this circle is there and this circle is the forbidden region. If you had non-linearity lying between 0 k 2 then it is this thing, this whole a half plane in some sense is the forbidden region and the G j omega has to lie there. I mean the nyquist plot has to lie to the right of this this particular thing.

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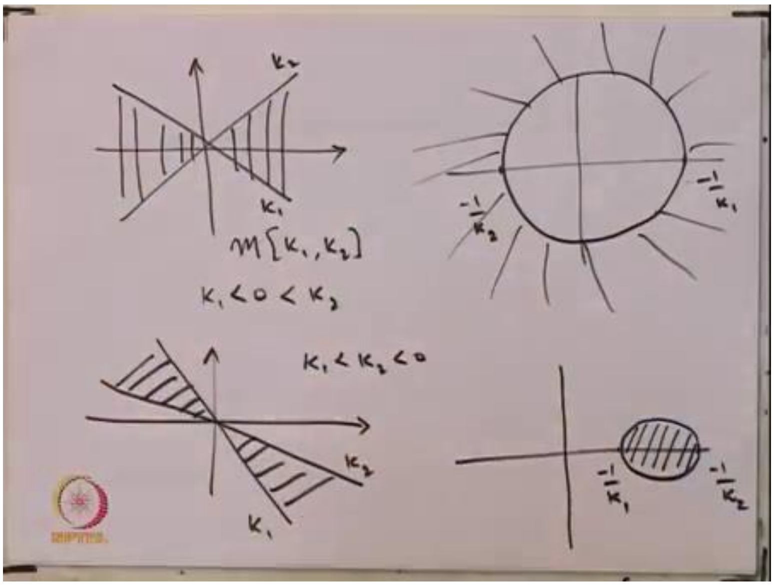

Now, further if you have a non-linearity. So, let me draw the axis and you have a non-linearity which lies in this area. So, that non-linearity is in the k 1 k 2 sector. So, this is k 1 this is k 2, where k 1 is less than 0 is less then k 2. Then what the circle is criterion really tells us is because this is less the k 1 is negative. Therefore, minus 1 by k 1 is up here and here somewhere is minus 1 by k 2.

So, you will have a circle and forbidden region is the outside of the circle. So, unlike the earlier case now the forbidden region is the outside of the circle. Then if you push further and you have non-linearity which lies in a sector like that. So, this here is k 1 here is k 2. So, what we have is k 1 is less than k 2 which in turn is less than 0. Now, in this case what you have is so you have minus 1 by k 1 and here you have minus 1 by k 2 and here you have a circle.

Now, it turns out as forbidden region is again the inside of the circle. So, there are all these various interpretations that happen. So, one quick way to see is, if you have a non-linearity in the range between k 1 and k 2, and therefore you can plot the points minus 1 by k 1 and minus 1 by k 2. You can draw the circle which connects these 2 just like here or here or in the earlier 2 cases. Like here or when one of the slopes was 0 that is infinity.

So, these two, now if the sign of k 1 and k 2 are both the same, like in this case or in this case. Then what ever the circle you have got the interior of the circle is the bad part, but if the sign of 2 are different like in this case, then it is exterior that is the bad part. So, this sort of sums up the various situations that can happen. Of course, k 2 could be made equal to 0 or k 1 made equal to 0 and you have the special cases.

K 2 could go off to infinity, which means that in the original axis it will hit 0. The circle will hit 0 and the special case of 0 infinity sector being the you know the left half is the forbidden part and the right half is a nice part. So, that gives the complete sort of interpretation for the various aspects of the circle criterion. Now, other thing is that of course, we have been till now asking about global asymptotic stability, but sometimes using circle criterion, we can talk about local stability and we cannot talk about the global stability.

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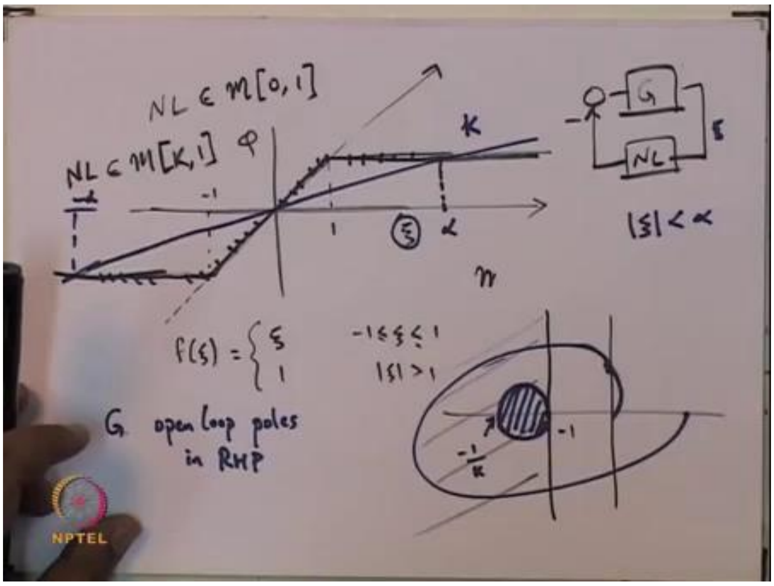

Now what I mean by that is the following. So, it might. So, happen that the non-linearity has some characteristics, which may be look like this so let us say, it has some characteristics like this. So, I mean one way that you could write out this characteristics is that the non-linearity is such that f of psi that is a non-linearity is equal to psi, when a when let us say psi is between plus 1 and minus 1.

So, minus 1 by psi less then plus 1 may be I will make it equal and this is equal to 1. So, this like a saturation. So, it is like a linear and then it saturates is equal to 1 and when mod is psi is greater than 1. So, you have some linearity like this. If you have a non-linearity like this, now what is the sector under which this non-linearity lies. Well clearly this slope and the other slope being the 0 slope. So, we could think of it as lying in that sector.

So, you can think of this non-linearity as a non-linearity, this non-linearity as a linearity that lies in the 0 1 sector because this 1 0. Of course, this non-linearity lies completely within this particular sector. Now, if you use circle criterion to translate this, what that means is the 0 and there is 1. So, what that means is if you looking at the nyquist plot. So, corresponding to 1 there is minus 1. So, everything to the left of this is forbidden and you can have a nyquist plot lying on the right and then such a plant so such a G when inter-connected with this non-linearity will give us, will give us global asymptotic stability, but now the following could happen.

May be you had a G and that G had Nyquist plot, which looked something like that. Let us say that this G was such that it had open looped poles in the right half plane. Now, by circle criterion it might be, that there is the circle here with the inside being forbidden and this nyquist plot is such that for this circle, the resulting closed loop stability can be predicted, but now if this is the case.

So, here this might be some minus 1 by k and this minus 1 by k might correspond to a slope like k here. Now, if you look at the original non-linearity, the original non-linearity is this one. The original non-linearity of course, lies in the infinity sector, but if you have to think of a non-linearity in the k 1 sector. So, if you have to think this original non-linearity, as lying in the k 1 sector, then as for as this non-linearity is concerned it is only up to here up to this value.

Let me call this value alpha and here minus alpha. So, it is between minus alpha and alpha that means for the input of the non-linearity lying between minus alpha and alpha. Can this non-linearity that originally drawn this non-linearity here. We could think of this non-linearity as lying in the k 1 sector, if you restrict yourself to minus alpha and alpha. Now, this psi that means input to the non-linearity is essentially the value of the signal of the branch here.

If this branch value is restricted to psi not psi less than alpha then the non-linearity we are considering, this non-linearity has characteristics which lie in the k 1 sector and because of it lies in the k 1 sector and the G j omega does not intersect this particular circle we can say for that. So, long as this psi is restricted to something less then alpha, this given system is asymptotically stable.

So, it is not globally asymptotically stable, but it is asymptotically stable. So, long as you look at only that portion of the face phase, where psi is a less than the modules psi is less than alpha, then for that restricted region of the phase. Of course, this includes the situation when psi is equal to 0 and psi is equal to 0 the output is also equal to 0 and that in fact is the equilibrium point that we want to get things into. So, you have the face plane you have the original face plane and for psi less then alpha, you have an area surrounding it.

What we can say from this is nyquist plot criterion is that. So, long as this psi is less than alpha and you start somewhere for this system with this is psi value being less than alpha you guarantees. So, this like local asymptotic stability as into global asymptotic stability. So, there are various things that you can do with this circle criterion depending upon how the original nyquist plot looks like. So, anyway so that I guess I am out of time for this lecture. So, let me stop here now and I will continue with the new topic in the next class.