Nonlinear Dynamical Systems
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Lecture - 19
Definition for Positive Realness
And Kalman- Yakubovitch – Popov Theorem

In the last lecture, I spent the whole lecture, showing the proof of positive real lemma, but throughout the proof I kept talking about the definition of positive realness and the definition of the matrix situation for positive realness and so on. So, I would in this lecture initially try to spend time and give a sort of a better idea about what this definition of positive reality especially for matrices are… Now as far as the positive, as far the definition for positive realness for matrixes are concern, I will be following what is given in the book by Klein. But as I was saying earlier, for the scalar case there is no real agreement about what exactly is the definition of positive realness.

So, what I would do is, I would just revisit what I had said about positive realness, and I would start with this scalar case. And then I would give you the definition for the, for the matrix case, and motivate you know what are the advantages and disadvantages of the equivalent definitions, which are therefore for positive realness. So, let me start by revisiting what the definition of positive realness is.

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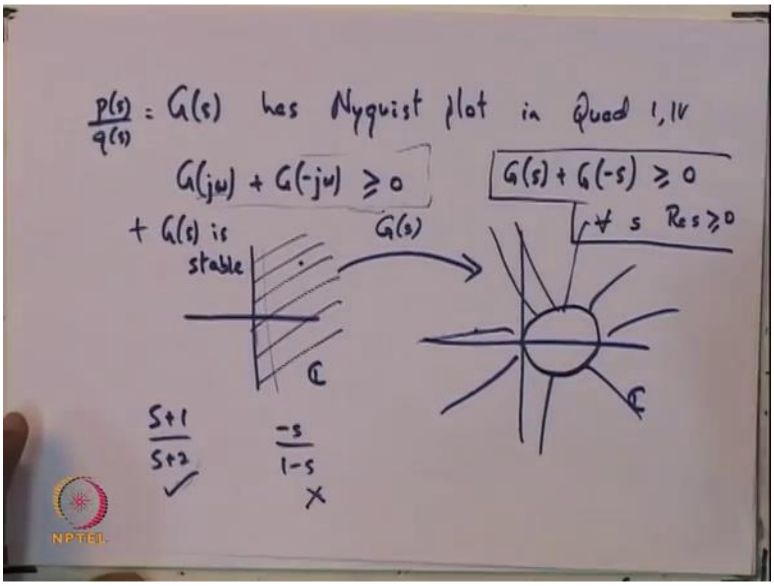

So, given a transfer function G of s, one definition for positive realness is, you look at the Nyquist plot. And if the Nyquist plot lies in the first and the forth quadrant, so if you have a Nyquist plot which looks like that for example, then that transfer function can be called positive real, this is one particular definition of positive realness that you could use. So, if the Nyquist plot is in the first and the forth quadrant then you call it positive real. And sometime back I had given examples of transfer function, so this is single input single output case, so it is some polynomial divided by some other polynomial.

Now, if you look at, if you look at the theory of Nyquist criterion then you know that, if the degree, if the relative degree that means, if p is a degree 3 polynomial, and q is a degree 4 polynomial. Then, the relative degree is 1 or minus 1 depending upon you are the way you want to look at it, basically, only if the relative degree is 1 or 0. Can you expect to have the Nyguist plot restricted to this half?

The reason for that is because, if it is more than, in the relative degree is more than 1 then it turns out the angles that is so if the for example, if the denominator is 2 degrees higher than the numerator then what would happen is finally, it would I mean, the Nyguist plot would enter this quadrant. So, one observation that you have, for single input single output case is a Nyquist plot is restricted to this half of the complex plain, if and only if the relative degree is 1.

Now, even if the relative degree is 1, we still not guaranteed what we want as for as the positive real lemma is concerned. And the reason for that is several so let us take this definition that, G s has Nyquist plot in quadrants 1 and 4. And this is a same as saying G of j omega plus G of minus j omega is greater than equal to 0, mind you in this particular case I am thinking of G s as a single input single output case.

So, it is some polynomial divided by some other polynomial, and so this one is greater than equal to 0. So, that means, when you evaluate the transfer function along the imaginary axis, that imaginary axis, the image of the imaginary axis lies in the right half, lies in the right half of the complex plain. Now, of course, we have this map from the complex plane to the complex plane, and there is a, what we are saying is that the image of the imaginary axis is something that lies in the right half of the complex plain.

The image, I mean this map here is G s so any point s here goes to G s there. Now, there are several examples of such things. So, for example, if we use S plus 1 by S plus 2, this transfer function when you look at this map and you look at where the imaginary axis maps to, that will like completely in the right half. If you look at minus s upon 1 minus s, this was also an example that I used earlier, and if you look at the map, if you take G s to be minus s upon 1 minus s, and you look at the map then again the imaginary axis fill map to something here.

So, for example, in this particular case, it will map to some, something like that. Now, if positive realness definition is just taken to be this then both of these will turn out to be positive real. Now, if you take these states space representation of this, state space representation of this, and try to use the positive real lemma, the positive real lemma will be applicable to this only because, it is a the denominator has it is roots in the left half plane.

Whereas, in this case the denominator has it is roots, not in the left half plane, but in the right half plane and so you cannot apply the positive real lemma. Earlier I had also talked about idea of dissipativity and idea of storage function so in both these cases you will get a storage function. The only difference between these two is that, in this case you will, because the denominator has all it is I mean, roots in the left half plain that means, this is stable transfer function.

Therefore, in this particular case, the storage function that you get is positive. On the other hand, in this case, also you can construct a storage function, but this storage function is not going to be positive, it is going to be negative. Now, in physical systems, if we talking about storage function, it is a, it is some function which stores energy. And if you going look at a function that stores energy, and that function is negative that does not make sense I mean, what does, what does it mean to say that it the amount of energy stored in the system is negative. As a result it does not make sense in this particular case, though you can still find a storage function, where is it make sense in this case, and you can find the storage function.

So, in both cases so as soon as this equation is satisfied, you can find the storage function, but it is only when the, when the transfer functions is hurvitz, that you can find the storage function which is positive. Where is, if it is not Hurwitz then you can find the storage function, but that will not be positive. Now, if you look at this further, that means instead of just looking at it this way, if you put the condition that G s plus G of minus s is greater than equal to 0, for all s such that the real part of s is greater than equal to 0.

That means, instead of just looking at the imaginary axis, you look at where this whole half plane maps to under G s. And this whole half plain should map, such that it falls in this right half. If you put this added condition then because the imaginary part is in the boundary of this thing that means, if the condition is this condition is satisfied then this condition is automatically satisfied, but there is something more satisfied that means, whatever is here now gets map to the right half plain then in that case, this function, for this transfer function it would be all right, for this transfer function it will not be all right.

So, if you take this transfer function, and see where this right half maps to, what you will get is this whole area outside the curve. Of course, what that means is some of these points get map to points here, which is not in the right half, but in the left half, whereas, if you had taken s plus 1 by s plus 2 then the imaginary axis would have map to a curve, and the right half would have map to the inside of the curve, which means every point here on the right half is getting going to get map to the right half plane.

And so this equation captures the fact that this is satisfied plus, the storage function is positive. And therefore, in some sense this should be the real definition of positive realness. But for historical reasons this is usually given as the definition for positive realness, and there are some places where this definition for positive realness is used. But if you give this definition for positive realness then such functions also permissible, and to disallow these functions, the additional condition that is given is, this condition holds plus G of s is stable. And as a turns out, these two conditions together is equivalent to this condition.

So, the various books that you go through might have some mixture of this definition as a definition for positive realness, and this is for the scalar single input and single output case. And I mean depending upon your taste you can adopt anyone of them as what you would believe positive realness to be, but they all roughly say the same thing. But there are these certainties that need to be handled. So, let me now give you the definition for a, for positive realness as far as matrices are concerned.

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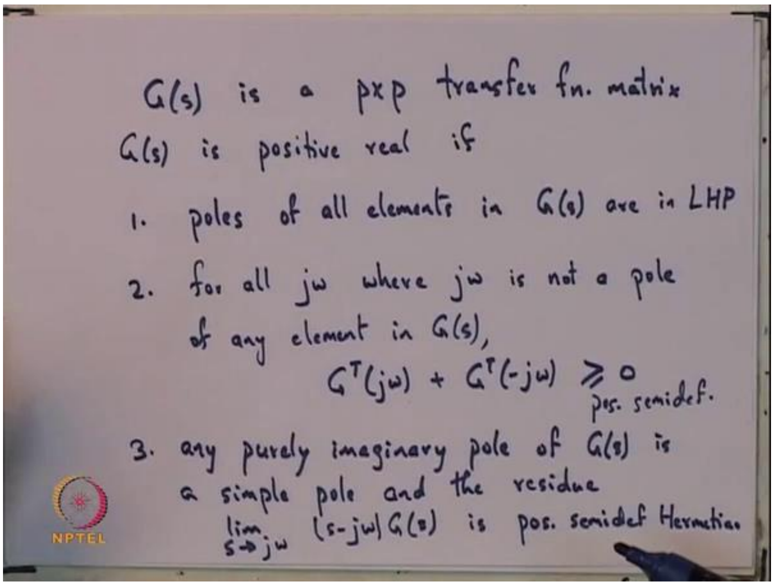

So, let us assume G of s is p cross p transfer function matrix. Now, of course, in all these cases this matrix, the transfer function matrix has to be a square matrix because a number of inputs have to be equal to the number outputs. So, G of s is the p cross p transfer function matrix then the following conditions have to be satisfied for G s to be, to be declared as a positive real transfer function.

So, G of s is positive real and this particular definition that I am using is the definition that is given in a the book by Kalim, non-linear systems by Kalim so just like what I said in the 1 D case is, here also in the multiple input multiple output, there could be other opinions, but I am just sticking to this for now, is positive real, if number 1, the first condition is poles of all elements in G of s, are in the left half plain that means, every elements so every entry of G of s is a transfer function and, everyone of those transfer functions is hurvitz, is stable.

And the second condition is for all j omega, I mean imaginary, purely imaginary value where, j omega is not a pole so where j omega is not a pole of any element in G s, G transpose of j omega plus G transpose of minus j omega, is a positive semi definite matrix. So, this condition is precisely like the Nyquist plot condition in the single input single output case.

So, what we are saying is for all j omega where, j omega is not a pole. If j omega was a pole for some entry in the transfer function matrix of course, then this thing will not be very well define, and this there are problems. So, you remove all those omega as which might, on the imaginary axis, which might be a pole of any one entry of G of s, and for all the others you will have this. But of course, remember these are matrices so the sum of these two matrices, one is claiming is a positive semi definite matrix, a positive semi definite matrix. And there is one more condition, the third condition is that any purely imaginary.

So, any purely imaginary root, or rather pole, purely imaginary pole of G of s is a simple pole, is a simple pole. It is a simple pole that means, it does not have multiplicity high, greater than one, and the residue, and the residue. So, the way you obtain the residue is a limit s tending to j omega of s minus j omega times G s is positive semi definite Hermatian. So, there are these three conditions, so the third condition is that for any purely imaginary pole of G s, so any purely imaginary pole of G s is a simple pole, and the residue limit as you tend towards a pole of this particular thing, that residue of course, here is a, is a matrix so it is in fact the positive semi definite Hermatian matrix.

So, the definition for positive realness is this. So, now if you just specialize, you take G as to be a one cross transfer function, well the poles are all in the left half plane. Well as we have send it before, and this is a Nyquist condition, a Nyquist criterion conditions so the Nyquist plot in the first and the forth quadrant, and this one is the stability so these two conditions of what. This is an additional condition that appears in the matrix case, in the scalar case, this it is clearly true, but in the matrix case it is a bit more involved, and therefore, this condition makes it is appearance.

So, this is the definition for positive realness as far as matrices are concerned, and so now if you take a G cross G, p cross p transfer function matrix G of s. And you want to talk about it being positive real you put these things these criterion in, and you can check whether it is positive real. And in the earlier lecture when I talked about the positive real lemma, well there G s could just be taken to be positive real with this definition, and as for as the realization is concerned whether it is single input single output or multiple input, multiple output the realization would be in terms of those matrices in so that matrix condition would remain unchanged.

So, now since, we have also already done the positive real lemma, there are variations of positive real lemma. If you remember the positive real lemma, ultimately in the statement about the matrices, there is this specific matrix p, which is a positive definite matrix, positive semi definite matrix. Now, this positive semi definite matrix during the, proof of the positive real lemma or in fact when I showed that the existence, I mean the existence of those equation are equivalent to the system being passive. I had made use of the fact, that this matrix p defines the storage function.

Now, this if p is a positive definite matrix then the storage function is positive definite. If p is the positive semi definite function, then the storage is positive semi definite. Now, between the situation when the storage function is positive definite, and the storage function is positive semi definite, there is a slight problem and this problem is very much similar to the kind of problem that you would get, when you use systems with inputs, no outputs and you are using lyapunov theory.

Now, in lyapunov theory when you take a, when you take a function, which is positive definite then that actually guaranties whatever is the conclusions that you can draw, from using lyapunov theory whereas, if you take something which is positive semi definite, you cannot utilize it to the full power of the lyapunov theory. So, you cannot, as a lyapunov function candidate you have to always take something which is positive definite, and you hope that it is derivatives is negative definite. But if the derivative is negative semi definite, you cannot draw that strong conclusion, I mean your conclusions that, you can draw is weaker and so on so forth.

So, in the same way as in exactly the same way, as far as storage functions are concerned, when you have positive definite storage functions it is good, and when you have positive semi definite storage function, it is not that good. And the statement of the positive real lemma only guaranties that this storage function is a positive semi definite, not positive definite. Now, in order to guaranty the positive definiteness of storage function, one brings in this additional thing.

So, there we had shown that positive semi definite storage function, I mean positive semi definite p, and those other matrix conditions are equivalent to the transfer function being positive real. Now, one can give an additional thing which is additional definition kind of thing, which guaranties that, the storage function is strictly positive definite. Now, the storage function being strictly positive definite, is equivalent to the transfer function being strictly proper real.

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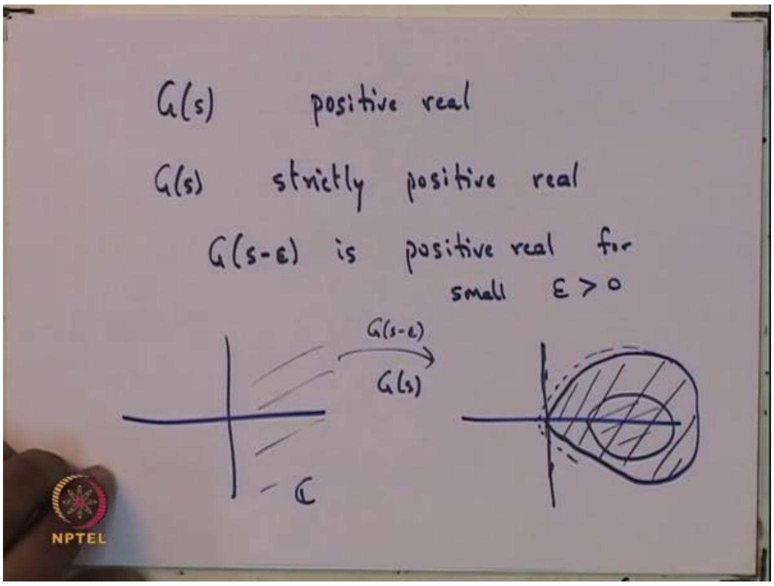

So, we have already given some definition for G s being positive real. So, there is Nyquist condition and so on so forth now, we say G of s, is strictly positive real, when G of s minus epsilon is positive real. For epsilon greater than 0, I mean epsilon greater than 0 small, for small epsilon greater than 0.

What we mean by this is, you see earlier when I was talking about positive realness. I had checked that of course, this is a map G of s so any point s goes to the corresponding point here, and the imaginary axis map into something, and then the right half mapping to the inside. Of course, this is a good thing that can happen and that is the definition of positive real. Now, in this particular situation where, which, I have drawn, where the imaginary axis maps to this, and the right half maps to the inside.

Clearly if instead of G of s we take the map G of s minus epsilon, that also about map to, you know some neighborhood of this. And so in fact this particular, this particular situation G of s is strictly positive real. It might happen, that you have some plots which looks like that, the imaginary axis maps to something like that, and then the right half plane perhaps maps to something like that.

And now, if you look at G of s minus epsilon then the pertivation might be such that, this gets move like that. And because, it is gets moves like that, G of s minus epsilon does not satisfy the positive realness condition and so this will not be strictly positive real, and this touching in the imaginary axis of this area. This region, it maps to some other region here and, that region how it touches the imaginary axis that in some sense defines the positive definiteness and positive semi definiteness, of the storage function.

And so when you do this G of s minus epsilon that means, you prepare by epsilon in some sense what you doing is you have moving this imaginary axis. And so if there are these places where it touches the imaginary axis, this image then when you shift the imaginary axis then those things get map to the left half. And a G of s minus epsilon does not, does not remain positive real.

So, such things which are on the boundary, they are not strictly positive real, anything else is strictly positive real. Now, of course, just like the positive real lemma, there is also a famous lemma which instead of talking about the equivalents of positive semi definite p and G of s being, G of s being positive real, it is talks about the equivalent strictly positive real.

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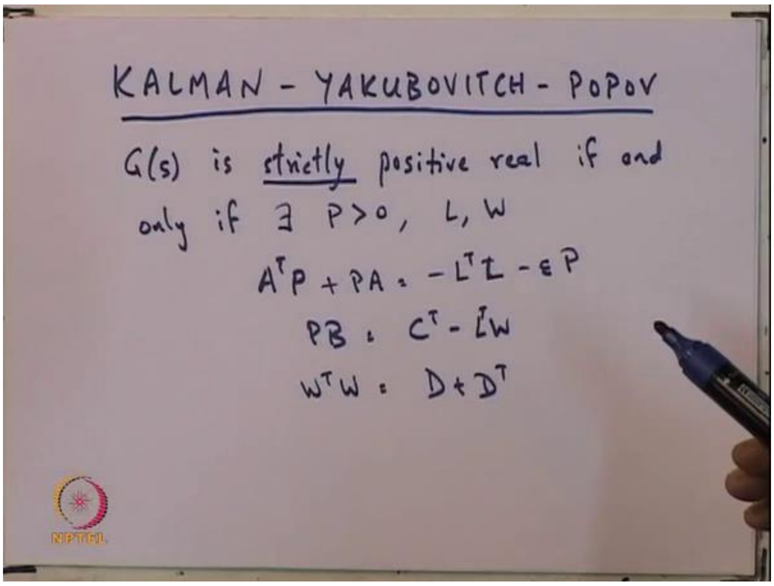

And this particular theorem is attributed to three famous people Kalman - Yakubovich and Popov so this lemma is attributed to all three of them, and the lemma is exactly same as the positive real lemma. The only difference is that G s is strictly positive real so instead of positive real earlier you had only positive real now you have the additional strictly if and only if.

And so now, you have the equations which are A transpose if and only if there exist, p positive definite and other two matrices L and W such that, A transpose P plus PA is equal to minus L transpose L, L transpose L minus epsilon times P. And PB is equal to C transpose minus L transpose W. And W transpose W is equal to D plus D transpose. Of course, here just like in the earlier strictly in the positive real lemma, here also all the assumptions are that this A, B, C, and D come from the minimal state representation of G s.

So, as you see between the positive real lemma and this lemma, the only difference is that G s on one side we are saying as strictly positive real, and the other side in this lyapunov equation, instead of A transpose A, P plus PA been equal to minus L transpose L, there is this additional epsilon P. And this sort of guaranties the strictly positive real situation, and it guaranties the positive storage function. So, let me now revisit and look at, what we have been talking about earlier and why we started looking at this positive real transfer function and so on.

So, the reason why we started looking at this positive real transfer functions is first of all, there was aizerman conjecture. And from the aizerman conjecture a certain guess was taken, that you know if you have a nonlinearity in some certain sector, and you have a feedback connection of that non-linearity with the linear system. Then if that linear system with those particular gains gives you a stable closed loop system, then the linear system with the nonlinearity in that loop, will give a stable system. And then we also show that counter examples were given so aizerman conjecture is not correct.

Now, after that we came into this passive systems and what the passive systems are and, we have a lot of results with respect to passive system. Now, the important thing about passive system is, that if you inter connect, two passive systems I mean, if you have a feedback connection of two passive systems, then the resulting system is also passive. And this makes things very good, because what was I really saying is, if you start of with some system, which is passive and you have another system which is passive, and you interconnect the two. The new system that you get which is the interconnection of the two systems is also passive.

And this I mean, especially if you think about this passive it in terms of the energy that means, the passive system is something where the total amount of energy supplied is either dissipated or it goes to increase the stored energy. Then, this seems very natural, but what we will now do is, we will formally show that when you interconnect to passive systems, the resulting system turns out to be passive. As a result it turns out that this concept of passivity is something that goes a long way in answering the question raise by aizerman, and in fact providing an answer which is similar to what Aizerman guessed.

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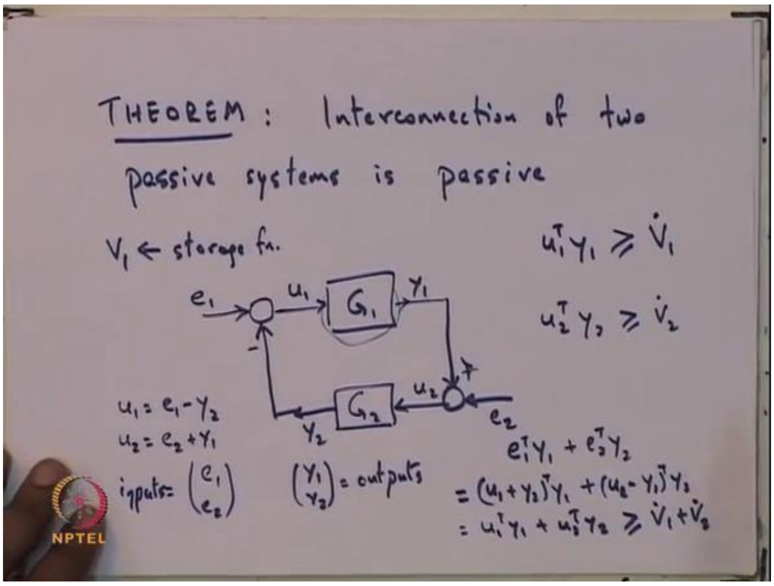

So, let me begin by first talking about the lemma or the theorem ok might has well call it a theorem, interconnection of two passive systems is passive. So, what do I mean by this interconnection so let me assume this is system one, let me call it G 1 so let me call the input u 1 and the output by y 1. And let me have a second system G 2 so input you see when you are talking about input and output, one needs to probably draw an arrow so that, it is clear what is the input and what is the output so u 1 is the input, and y 1 is the output.

And let us have another system G 2, and this system has u 2 as the input and y 2 as the output. Now, G 1 is passive what does it mean to say G 1 is passive well, one thing that from all the discussion that we had about passivity is that, u 1 y 1, let me say u 1 transpose, so rather than thinking of it as single input single output, I could think of it as multi input multi output.

So, I am saying u 1 transpose y 1 is greater than equal to v 1 dot, where this v 1 is the storage function of the first system. We had said that, for the passive systems, the product of the input and the output is greater than equal to the rate of change of the storage functions. So, v 1 is like the amount of energy stored in G 1 roughly and so u 1 transpose y 1 is like the amount of energy supplied, and the energy supplied is greater than equal to the rate of change, where the power supplied this greater than equal to the rate of change of the stored energy in the first system.

Now, this one being passive essentially, you have a similar statement u transpose y 2, must be greater than equal to v 2 dot. And this v 2 dot is the storage function of the second transfer function. Now, let us look at what we mean by interconnections so let us interconnect it in the following way. So, I put in this thing, and I will assume that there is some input v 1 coming into the net system, and maybe I subtract this so what this essentially tell me is that u 1 equal to e 1 minus y 2. And I do a same kind of thing here so there is some input here which let me call it e 2.

Now, of this system, this is the interconnected system and in this interconnected system, I can think of the vector e 1, e 2, the vector e 1, e 2 as being the set of input. And I can continue to think of y 1 and y 2 as a set of output. So, then input times output is essentially e 1 y 1 plus e 2 y 2. Now, for e 1 if I substitute u 1 plus y 2 so I get u 1 plus y 2 times y 1 plus now, I have not written the equation for this here this I will continue to call it positive so what I have is u 2 is equal to e 2 plus y 1.

So, this e 1 y 1 plus e 2 y 2 is equal to u 1 plus y 2 times y 1 and for e 2, I can substitute e 2 is u 2 minus y 1 so u 2 minus y 1 times y 2 of course, there are these transposes, but that really does not matter. Now, you see you have the y 2 transpose y 1 and you have y 1 transpose y 2, but with negative signs of they are sort of cancel. So, what you left with is u 1 transpose y 1 plus u 2 transpose y 2 which from these two inequalities, you know is greater than is equal to v 1 dot plus v 2 dot. Now, what does it is mean? This means that if you take this e 1 and e 2 as the inputs for the interconnecting system and the y 1 and y 2 as the outputs.

So, for this interconnected system, when you look at the inputs multiplying the outputs this is greater than equal to, the rate of change of a storage function which is in fact the some of the storage function of the first one and the second one. So, in physical systems if this was the physical system and it had some elements which stored energy.

On this another system which has some elements which is stored which is storing energy, then the complete storage function is the sum of this storage function plus this storage function. So now, this is an extremely powerful sort of a result and therefore, I mean what we can say is, if we have two systems, which are passive and interconnect it, then the inter-connected system continues to be passive.

Now, if I mean how this theorem becomes really powerful is by the following means. You see suppose, you think of this G 1, this system G 1 as a linear system which is passive. G 2 is some system, which is, let us say a non-linear system, but you can sort of by some means show that this is passive. Then if you interconnect this two then the interconnected system is also passive. So, if this was a non-linear system and you manage to find some storage function for this non-linear system then in some sense you have found storage function for the complete non-linear system.

Now, if you are talking about for example, lyapunov theory and so you do not think of these inputs. Earlier we had discussed how given a general non-linear equation, you can split it up into a linear part and the non-linear part, and now if you can show that this linear part is passive and this non-linear part is passive independently, and for this non-linear part you can find some storage function. For the linear part of course, we already have the positive real lemma and the Kalman - Yakubovich and Popov lemma by which you can find storage function then the some of these two storage functions act like the storage function for the net system. But that 0 input that some of the two storage function would act like the lyapunov function.

And therefore, this is in why in fact the a two construct the lyapunov function for that particular system. Now, how this connection with aizerman idea is what I will now try to explain, and for that first let me consider nonlinearities, which are, which are memory less. So, what I mean by a memory less nonlinearities are following, if you give a certain input to the nonlinearity you will get an output, but this output is not dependent upon what happened in the system earlier in time or later in time whatever. It is an instantaneous map.

So, what I am trying to say is that whatever is the input the output is completely determine by what the input is at this instance, the output at this instance completely determined by the input at this instance, such maps they would call a nonlinearity. If we have a nonlinearity, I mean such a map we would call memory less nonlinearity.

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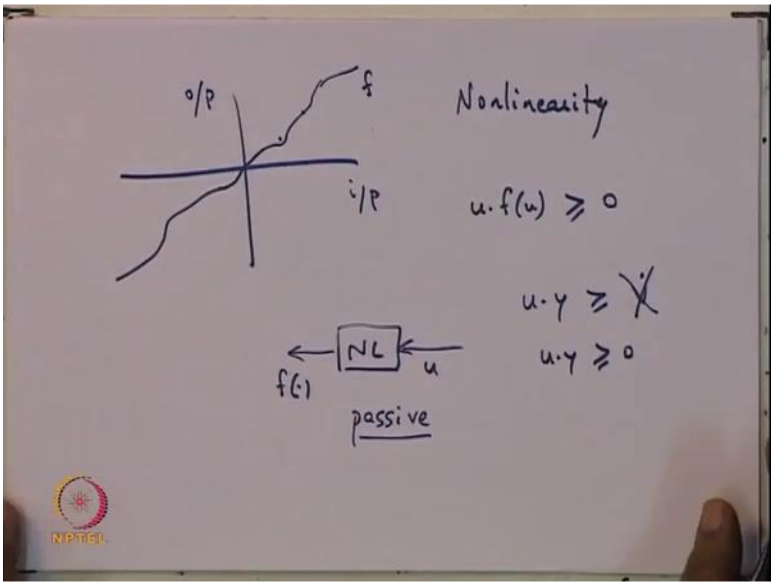

So, if we have a memory less nonlinearity then one way you can characterize that nonlinearity is by this map that you have the input and here you draw the outputs. So, for any given input, there is a particular output, for this input there is some particular output. And so you can connect all those dots and you get a curve. Of course, this curve was a straight line passing through the origin then the non-linear system is not really non-linear, but is linear, but if you have this situation is a non-linear, so this is a nonlinearity.

So, for any given so this is like a lookup table if you want given any input, you go through the graph and you will know what exactly the output is. Now, if you have a nonlinearity such that, this curve so let me call this nonlinearity f then this nonlinearity is such that, if the input is u then u times f of u is greater than 0, greater than equal to 0 then as for as the non-linear system is concern.

So, here is the non-linear system suppose, this is the input going into the nonlinearity and what you get here is f of that. And what you saying is the input multiplied by output is always greater than 0 then from whatever we have been discussing earlier, we could call this passive. And now if you call this passive, this thing is memory less that means, in the sense it just depends on what the instantaneous input if one could think of this storage function.

So, the definition of passivity is u dot y input multiplying output is greater than equal to the rate of change of the storage function. Suppose, you take the storage function to be the 0 storage function then you get this that means u dot y is greater than equal to 0. And so assuming that this storage function is 0 storage function is one way that this equation is satisfied. And so you have a passive system this non-linear system is passive, and it has a storage function which is a 0 storage function, and here is the beauty of the whole thing.

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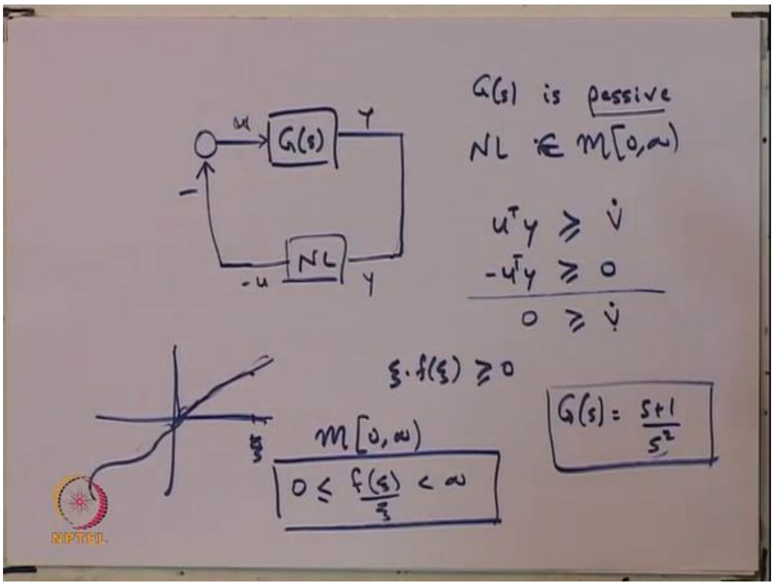

Suppose, you take a matrix, a linear system G of s and you take a nonlinearity, and this nonlinearity is in the first and the third quadrant. Incidentally nonlinearities which are in the first and the third quadrant are often denoted by this. This is the nonlinearity lying in the 0 infinity sector that means, they lie here or here, the slop or rather if you take any u, the f of u. In fact I had given this kind of definition earlier, if you call the input psi, f of psi by psi is less than infinity.

Such nonlinearity is 0 infinity nonlinearity, and if you have any nonlinearity like this then of course, this is true psi times f of psi is greater than equal to 0, and from what I said in last line one could view this as passivity. So, if you have a nonlinearity which belongs to this class, and you have a feedback structure which looks like this then if this G s I mean in this linear part, if G s is passive, and the nonlinearity is belongs to, is a nonlinearity like this then G s is passive, the nonlinearity is passive.

So, from what we had talked about earlier, the interconnection of these two part, the interconnection of these two systems also passive. Now, this is passive, what we mean by that is, if this input is u and the output is y then what we saying is u transpose y is greater than equal to v dot where v is the storage function as far as this guy is concerned. And the nonlinearities passive, well it is the same y here, and out here what you have, what would be the negative of this u? So, let me call it minus u, but this minus u and y it obeys the rules of the, of the nonlinearity therefore, minus u transpose y is greater than equal to 0. This is for the nonlinearity and because 0 infinity sector this must be true, this G s is passive so this must be true. I add these two I get 0 greater than equal to v dot.

So, this is an autonomous system, in which I have v provided G s is passive, I can find the storage function to the positive definite function, and I have therefore, a positive definite function whose derivative is less than 0, therefore, the resulting system is asymptotically stable. So, if you remember aizerman conjecture said that if you had some trajectory, which lied in the sector perhaps something like this. And his conjecture was that you if had a G of s such that, on the feedback put any gain between 0 and infinity, and it gives you a stable system.

Then, if put a nonlinearity then the resulting closed loop system is asymptotically stable. And that was a proven to be false, but what we have got is a very similar result, what we are saying is if we take any nonlinearity in the 0 infinity sector, and out here you were not going to take any transfer function such that, you know you put any gain between 0 and infinite and the resulting feedback system is asymptotically stable. That is not what you going to do, what you are going to do instead is instead of that condition you are going to take a G s, which is passive, and if you do that then the resulting system is such that its asymptotically stable.

So, if you recall earlier a few lectures back I had shown that, in aizerman conjecture there is some sort of counter example are shown, and if you remember in that counter example, the G s that I took was s plus one upon s squared. Now, if you look at this particular transfer function, and you look at its Nyquist plot, it will be clear that, this is not a passive transfer function. And as a result one cannot expect this interconnected system to be, to be passive, and therefore, asymptotically stable.

So, the interconnection of passive systems being passive that result, in fact is a solution to the aizerman conjecture, in the sense it is a positive, it is a positive reply to aizerman conjecture, in sense that if you have a nonlinearity in the 0 infinity sector. Then, that is like interpreting the non-linearity as a passive nonlinearity. And therefore, you interconnected with the passive linear system, and the resulting system is passive and because the resulting system is passive, what you have is asymptotic stability. So, I amount of time for this lecture and so we would stop here today.