Introduction to Robotics
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Lecture No 2.2
Homogeneous Transformation Matrix

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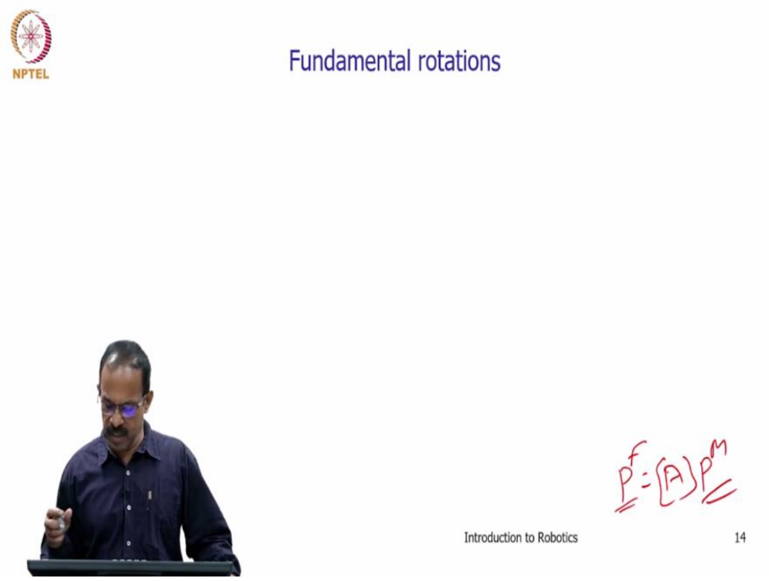

Good morning. Welcome back to the discussion on Robot Kinematics. In the last class we discussed about Coordinate Transformation Matrix. That is if you have a mobile coordinate frame and a reference coordinate frame, how do we map these two coordinate frames using Coordinate Transformation Matrix was the discussion we had in the last class. And we found that the transformation can actually be represented using a 3 by 3 matrix.

And the point in the mobile frame and the point, point in the mobile frame can be mapped to the point in the reference frame using a relationship where we use this $[P]^F = A[P]^M$, where P^m is the point in the mobile frame and then P is the P^f is that same point represented with respect to the fixed frame. And this A is known as the Coordinate Transformation Matrix. And we saw this Transformation Matrix for a fundamental rotation motion and we call that as a Fundamental Rotation Matrix.

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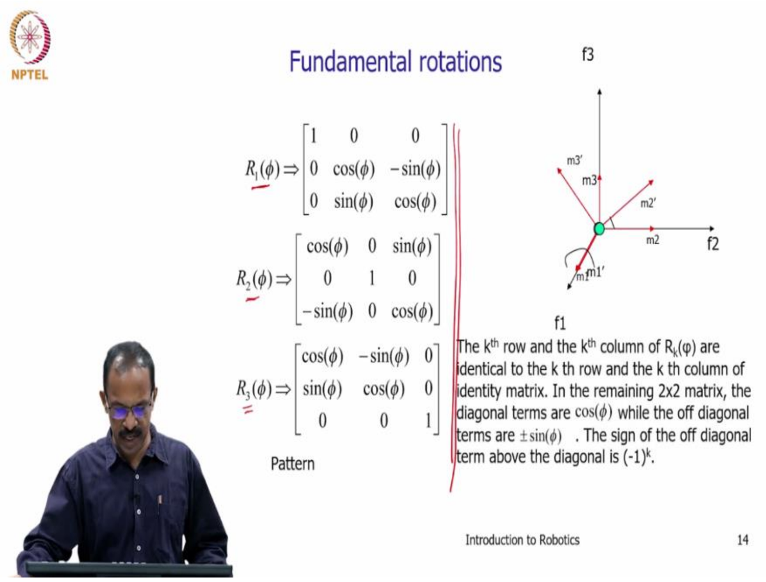

And if there are two coordinate frames and if they are rotated with respect to a axis, then you will be able to get the rotation matrix like this, this 3 rotation Fundamental Rotation Matrix. The rotation with respect to the first axis or the second axis or the third axis, you will be able to get this rotation matrix like this. So, this is the Fundamental Rotation Matrix. Then we saw that suppose we have multiple rotations happening in the coordinate frames, then we can actually use a Composite Rotation Matrix to get the complete Transformation Matrix.

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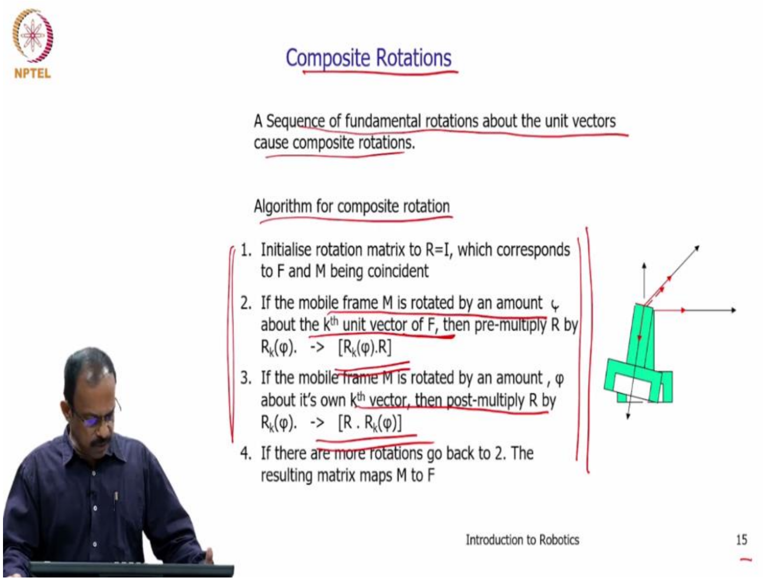

So, that is the Composite Rotation Matrix. Here, if you have a sequence of fundamental rotations about unit vectors of either a fixed frame or the mobile frame, you will be able to get the Composite Rotation Matrix using an algorithm. And this algorithm we saw that if the rotation is with respect to the axis of the fixed frame, you have one procedure if it is with respect to the mobile frame or the rotation of the mobile frame is with respect to its own axis, then you will be having a different algorithm to choose.

I mean different way of computing the Composite Rotation Matrix. So, if the mobile frame is rotated about the kth unit vector of F that is with respect to fixed frame, then you do a pre multiplication or if it is rotated with respect to the mobile frame kth vector on its own frame or its own axis then it is post multiplication. By using this pre or post multiplication, you will be able to get the Composite Rotation Matrix irrespective of the number of rotations you make. So, you can have any number of rotations, you will be able to get the final rotation matrix using this algorithm. So, this was what we discussed in the last class.

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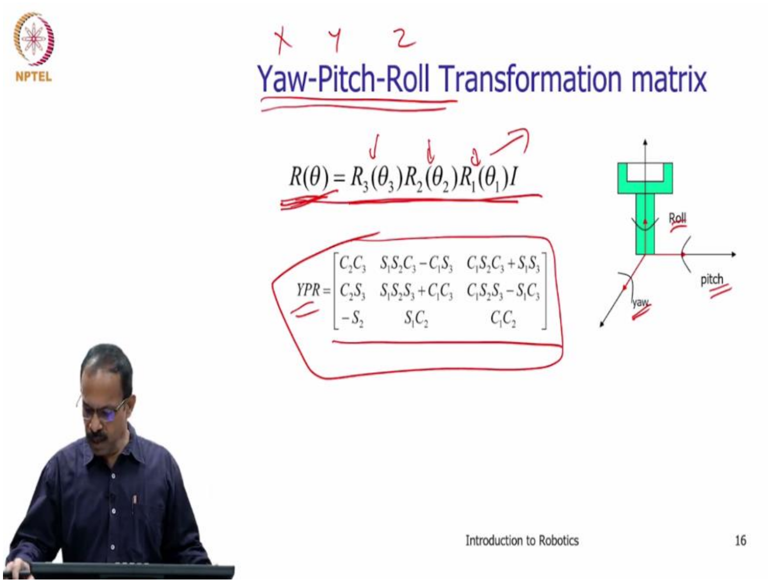

So, let us see how this is actually used for something called a Yaw-Pitch-Roll Transformation. So, Yaw-Pitch-Roll Transformation is one thing something which is commonly used in the mobile, Sorry in the manipulator Kinematics, because the rotation of the tool point or the list of the manipulator is represented using the Yaw-Pitch and Roll motions. So, it is rotation with respect to X, Y and Z axis.

So, X, Y and Z axis is the Yaw-Pitch-Roll that is you have a wrist like this. So, this is the Z axis, this is the X axis and this is the Y axis so, this is how it is defined. So, you have a Yaw motion with respect to the X axis, you have a Pitch motion with respect to the Y axis and you have a Roll motion with respect to the Z axis.

So, that is known as Yaw-Pitch-Roll transformation. Suppose, you make a Yaw-Pitch-Roll rotation for the wrist, you want to know what will be the transmission of that tool point with respect to the reference frame. So, that is the Yaw-Pitch-Roll Transformation Matrix. So, if you have this one, so, this is the tool, so you have this Yaw axis, Pitch axis and Roll axis.

$$R(\theta) = R_3(\theta_3)R_2(\theta_2)R_1(\theta_1)I$$

So, if you make a Yaw motion, then a Pitch motion, then a Roll motion all done with respect to the fixed frame, then we get this as R3, R2, R1 because R1 is with respect to the X axis, this is with respect to the Y axis and this is with respect to the Z axis. So, you will be getting it as pre multiplication and R theta is R3, R2, R1. So, this is the Composite Rotation that you can get.

Now, we know what is R1 because we saw this R1 is the fundamental rotation matrix with respect to first axis that is R1 that is with respect to X axis. Rotation Matrix R theta as

$YPR = \begin{bmatrix} C_2C_3 & S_1S_2C_3 - C_1S_3 & C_1S_2C_3 + S_1S_3 \\ C_2S_3 & S_1S_2S_3 + C_1C_3 & C_1S_2S_3 - S_1C_3 \\ -S_2 & S_1C_2 & C_1C_2 \end{bmatrix}$ So, this can see this would be the Rotation matrix or the transformation matrix that you can get once you have these rotations R3, R2 and R1.

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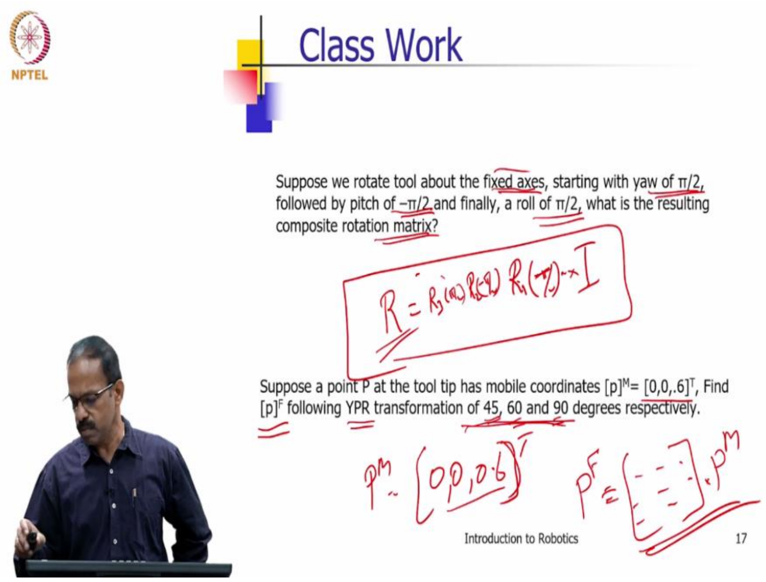

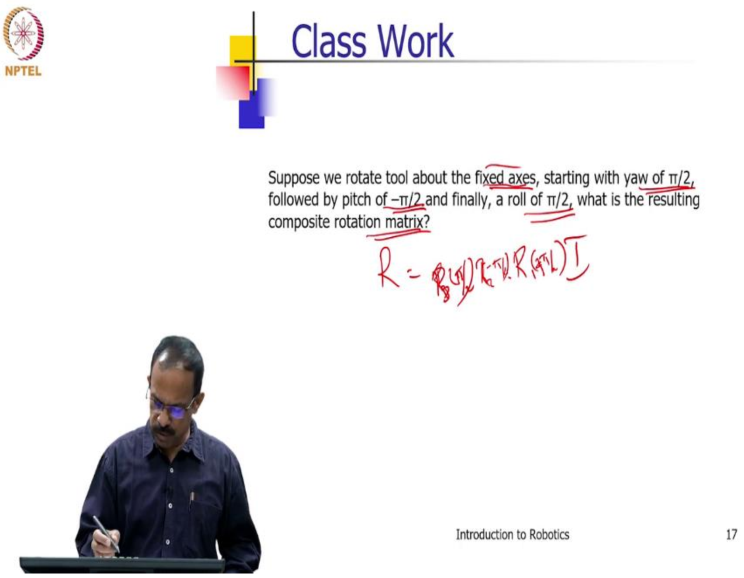

Now, you can take an example for a this Transformation Matrix before we move forward to translation we will see the will take one example. Suppose we rotate the tool about the fixed axis. So, if the rotation is all about with fixed axis, starting with the yaw off π/2 followed by a pitch of -π/2 and finally, a roll of π/2. What is the resulting composite rotation matrix?

Okay, you can actually try this, what will be composite rotation matrix. So, what we do we initially said $R(\theta) = R_3(\mathrm{pi}/2)R_2(-\mathrm{pi}/2)R_1(\mathrm{pi}/2)I$

Let me do this again so, I then you have this R1 pi by 2, R2 minus pi by 2, R3 pi by 2. So, that is the R that you can get. So, this would be the transformation matrix that you can get. Now, you substitute the value of pi by 2 in this R1 and similarly minus pi by 2 in R2 and pi by 2 in R3, you will be able to get the rotation matrix.

Now, suppose there is a point P with respect to mentioned suppose the point P at the tool tip has a mobile coordinate of 0,0,.6. Now, assume that $[P]^m = [0{,}0{,}0.6]^T$ and then we want to find out PF following this YPR transformation. So, YPR is the Yaw, Pitch and Roll that we saw and then these angles are given as 45, 60 and 90.

So, we need to find out PF using this relationship $[P]^F = A[P]^M$ so, apply the values of 45, 60 and 90 in the YPR transformation matrix that is theta 1, theta 2, theta 3 and then you get this matrix and multiply this with PM, you will be getting it a the new point P^m will be obtain P^f will be obtained by this relationship. You can try this and find out how the point 0,0,.6 will be represented with respect to P with respect to the fixed frame after the transformation of Yaw Pitch, Roll. Can I give this to as an exercise for you, you can try it.

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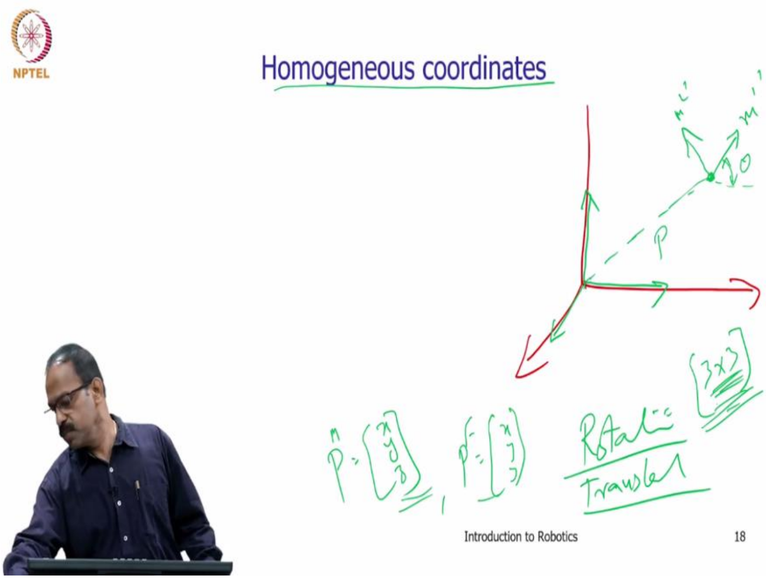

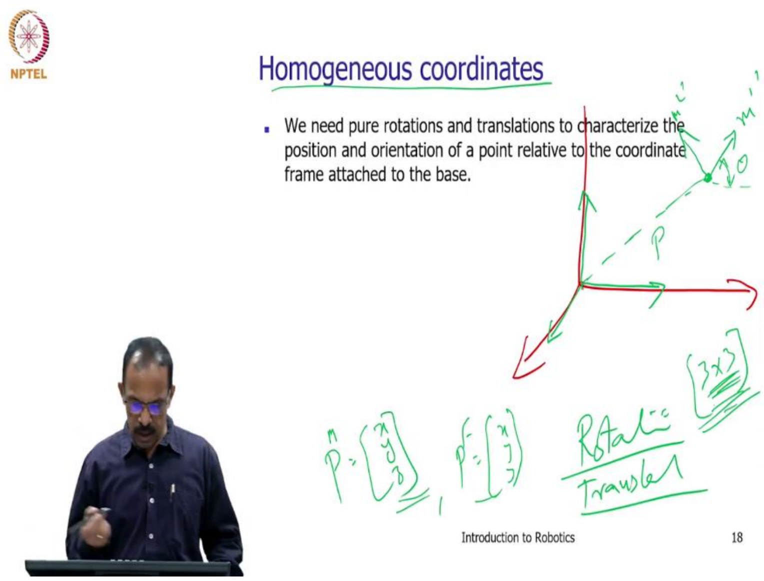

Alright. So, so far, we discussed about the rotation of coordinate frame with respect to the axis of one of the either the reference frame or it's on a frame. But as we discussed any moment of the coordinate frame may not be always a rotation. So, if you have this is the fixed frame or the reference frame and this as your mobile frame, this mobile frame can actually rotate with respect to one of the axis, and it can actually translate also, it can actually translate to here and then can have a rotation also.

So, this is the rotation angle and this is the translation. So, you can actually translate by P and it can rotate. So, that would be the new m_1, m_2. Call this m_1', m_2'. So, now suppose we want to represent the both the rotation and translation. So, you have a rotation and you have a translation also of the frame, because this frame has rotated and translated. And we know that the rotation can be represented in the three-dimensional space, the rotation can be represented using a 3 by 3 matrix.

Now, suppose we have a Translation also, then how can we represent this translation using this 3 by 3 matrix it may not be possible to represent the rotation and translation using a 3 by 3 matrix though we are in the three dimensional space, we may require to have a higher dimension space to represent the rotation and translation.

But all of our points are normally represented using 3 dimensional coordinates. So, you represent the P, suppose you have a point P then we will say the coordinates are X Y Z. So, and if you have to represent this as PM, and then the PF also will be 3 dimensional. So, PF also will be 3 dimensional with an X Y Z.

$$P^m = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \ P^f = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

So, the rotation transformation can be represented using a 3 by 3 matrix so, we have no issue in multiplying this and getting this 3 by 3 matrix and multiplying. But if you want to have represent it represent the translation and rotation then we are not it is possible to do with the three-dimensional space.

So, we need to go for a higher dimensional space to represent both translation and rotation of the frame to represent the points in 3D space, if the coordinate frames are translating and rotating in 3D space, then a three dimensional space is not sufficient to represent the points we need to go for a higher dimensional space and this higher dimensional space is known as the Homogeneous coordinates. That is, so if you had to represent characterize the position and orientation or the point related to the coordinate frame attached to the base, so but both rotation and translation are needed.

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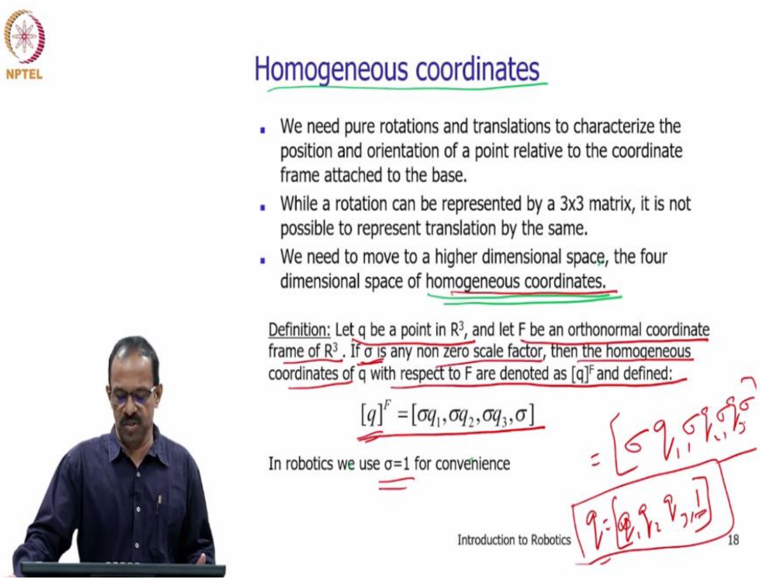

Now, if you have to represent this, well rotation can be represented by a 3 by 3 matrix. So, the rotation can be represented by a 3 by 3 matrix It is not possible represent Translation by the same. Therefore, we need to go a higher dimensional space and which we call it as the Homogeneous coordinates.

So, we call this as the Homogeneous coordinates. So, this is known as the Homogeneous coordinates. What is how is the homogeneous coordinate defined? So, what we do we will define a point in space in a 3D space, a point can be defined using a 4 dimensional vector, so that is the Homogeneous coordinate.

So, let q be a point in three-dimensional space and F be an orthonormal coordinate frame of R3. So, we have a point q in three-dimensional space and we have an F in orthonormal coordinate frame R3. Then if sigma is a non 0 scale factor, then the homogeneous coordinates of q with respect to F are denoted as $[q]^f = [\sigma q_1, \sigma q_2, \sigma q_3, \sigma]$. So, this is the way how we define a Homogeneous coordinate the point p, point q can be represented as sigma q1, sigma q2, sigma q3 and sigma. So, this is the way how you can represent a point in four-dimensional space.

And we take sigma is equal to 1 for convenience. So, the point p point q= $[q_1, q_2, q_3, 1]$. So, this is known as the Homogeneous coordinate of a point. So, we are not making any major changes, we are simply saying that we can represent a point in space at in a three-dimensional space, a point can be represented using a four-dimensional vector.

And the first three is the coordinates of the point in three-dimensional space and the last one is unique that is the one is the last element in the vector. So, this way, we are saying that any point in three-dimensional space with respect to a fixed frame can be represented using a four dimensional vector. So, that is known as the Homogeneous coordinate of a point q.

So, once we have defined this as a four dimensional. Now, we can actually represent the transformation of the translation and rotation using a 4 by 4 Matrix. So, now we can actually go for a 4 by 4 Matrix because your q is a 4 by 1 vector. And therefore, we can use a 4 by 4 matrix for transformation, because earlier it was a three-dimensional vector. Now, we have a four dimension one, so we will go for a 4 by 4 matrix to represent that represent the rotation and translation of a coordinate frame with respect to fixed axis or a mobile axis.

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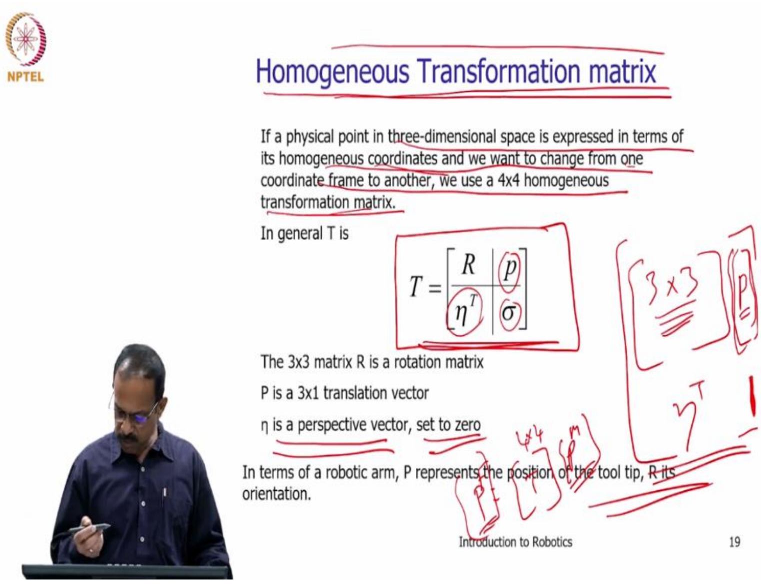

So, the homogeneous transformation matrix now, we have a homogeneous coordinate frame, homogeneous coordinates therefore, we can actually define a homogeneous transformation matrix. And if a physical point in three-dimensional space is expressed in terms of homogeneous coordinates and we want to change from one coordinate frame to another, we use a 4 by 4 transformation matrix

That is, you can have a 4 by 4 transformation matrix to represent the coordinate transformation from one frame to other frames. So, the if the space is expressed in terms of homogeneous coordinates and one coordinate frame the other coordinate frame transformation can be expressed using a 4 by 4 matrix and that matrix is known as Homogeneous transformation matrix. So, previous one what we saw was a three-dimensional rotation matrix. Now, when we convert that into a four-dimensional space, we call this a Homogeneous transformation matrix.

So, in general, if T =$\begin{bmatrix} R & p \\ \eta^T & \sigma \end{bmatrix}$ is given by this a rotation matrix, a position vector P and sigma and eta transpose. So, the general structure of the homogeneous transformation matrix will be like this, you have a 3 by 3 rotation matrix. So, you will be having a 3 by 3 rotation matrix, which represents the rotation of the coordinate frame, then you have a position vector, a vector which represents the translation of the coordinate frame and then you have one which represents the sigma of the homogeneous coordinate and then here something called an Eta a vector.

Eta transpose here, then this eta is known as a perspective vector and normally set to 0. So, this perspective vector will be set to 0 and this will be sigma and therefore, you will be having this as the 4 by 4 homogeneous transformation matrix. And this transformation matrix can be used for calculate finding out the position of coordinate the point in three-dimensional space, because now P^f= [T]P^m, because P^m is again homogeneous coordinate, P^f is homogeneous coordinate and this is a 4 by 4 matrix.

So, this way the transformation of coordinate frame when there is a rotation and translation will be able to represent using this 4 by 4 matrix and the first 3 by 3 part of the T represent that rotation. This vector represents the translation and this is the sigma and this is a eta transpose which is a eta is a perspective vector normally set to 0. So, this is known as the homogeneous transformation matrix.

I hope you understood. So, what we are trying to do is to represent the point in homogeneous coordinates and then get a 4 by 4 matrix to represent the rotation and translation of coordinate frame. So, now the, the translation of the frame can be represented using this vector rotation can be represented using this matrix, and therefore, you have the 4 by 4 matrix.

So, in terms of robotic arm, P represents the position of the tool tip, with respect to R. R its orientation. So, in terms of robotic arm, you can say, this is the orientation of the tool and this is the position of the tool with respect to the reference frame. So, that is the way how we look at the homogeneous transformation matrix.

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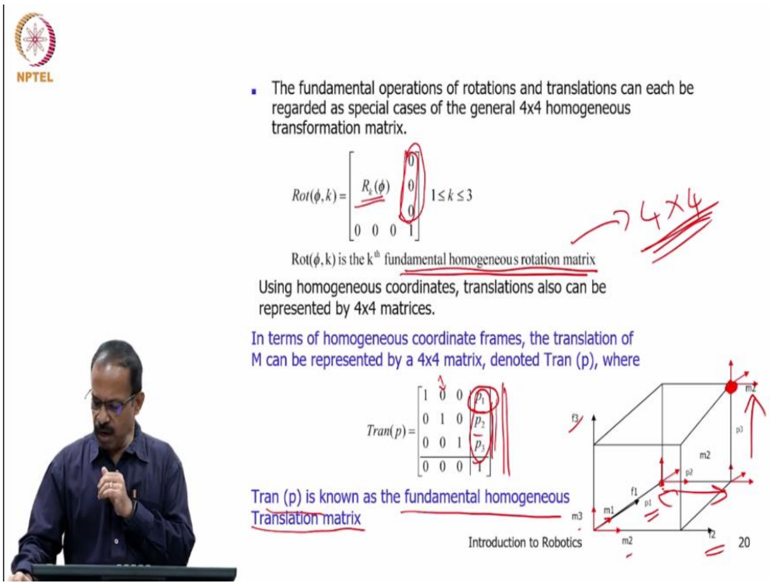

Now, since we have a 4 by 4 Matrix to represent the transformation, we can actually convert the rotation matrix, the 3 by 3 rotation matrix can be represented using the homogeneous coordinates, and then we call it is homogeneous rotation matrix. Or we can see that the fundamental rotation matrix can be represented as a homogeneous rotation matrix by assuming that the translation is 0.

So, $R(\emptyset, k) = \begin{bmatrix} & R_k(\emptyset) & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $1 \le k \le 3$,

this is your fundamental rotation matrix and this is the translation which is 0, that does not translation the pure rotation. So, the fundamental rotation matrix can be represented as a fundamental homogeneous transformation rotation matrix by setting this to 0. So, now your fundamental rotation matrix or fundamental homogeneous rotation matrix is a 4 by 4 matrix so, hereafter we will be using only 4 by 4 matrix, and therefore, if it is only pure rotation, we will make it as a 4 by 4 homogeneous rotation matrix.

Similarly, you can get a fundamental homogeneous translation matrix also which can be represented like so, you have a sorry this is the translation if there is only translation is there, then we have P1, P2, P3 as the translation along X Y Z axis. And this will be a unit this will be identity matrix, because that is there is no rotation. So, this will be identity matrix and you have a translation vector.

$$Tran(p) = \begin{bmatrix} 1 & 0 & 0 & p_1 \\ 0 & 1 & 0 & p_2 \\ 0 & 0 & 1 & p_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, P1, P2, P3 would be translation and translation P is known as the fundamental homogeneous translation matrix okay. So, this is the fundamental homogeneous translation matrix. Now, if you can see that how it is represented, so if initially you have the fixed frame f1 f2 f3 then you have the mobile frame M1, M2, M3. Now, if this is translating P1, if the mobile frame is translating P1 along f1 then you will get just a new mobile frame.

That is P1 along f1 then you have P2 along f2 and P3 along f3, so this would be the final position of the coordinate frame, if there is no rotation, this would be the final position of the coordinate frame and this transformation of the coordinate frame along f1 f2 and f3 can be represented using this fundamental homogeneous translation matrix. So, this is the fundamental homogeneous translation matrix.

So, now the rotation and translation can be represented using a 4 by 4 homogeneous matrices. So, you can have fundamental homogeneous rotation matrix and fundamental homogeneous translation matrix and if you have composite rotation, we need to follow the composite rotation composite transformations, we follow the composite transformation principles and then get the composite transformation matrix. So, again you will be getting the homogeneous composite transformation matrix.

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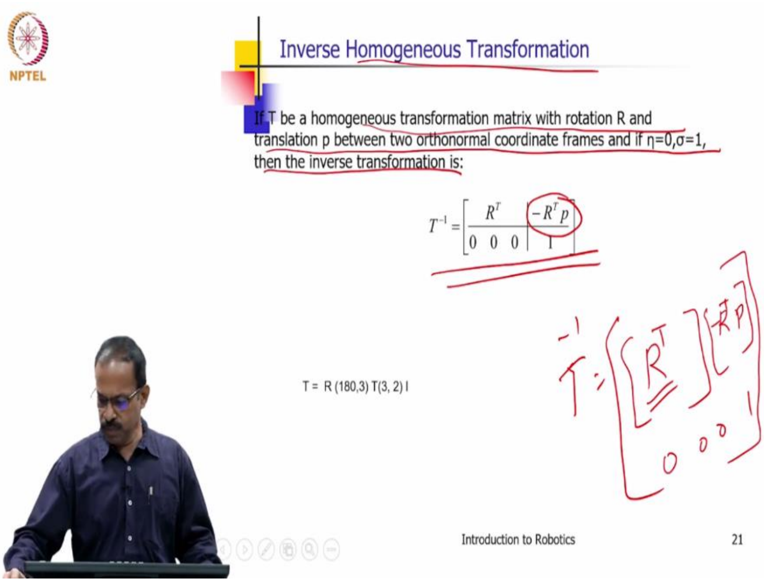

So, we will see how to get that one so, if you have a okay before that we go to the composite let us talk about the inverse homogeneous transformation matrix also. So, if T be homogeneous transformation matrix with rotation R and translation p between two orthonormal coordinate frames, and if eta is equal to 0 and sigma is equal to one, then the inverse of the transformation is given as T inverse is R transpose minus R transpose p.

$$T^{-1} = \begin{bmatrix} & R^T & & -R^T p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, you do not need to really go for finding out the inverse using the normal matrix inversion principles, what you need to do is if you have I mean if these conditions are satisfied, T is a homogeneous transformation matrix and they are orthonormal coordinate frames, then we can write T as T inverse a so, T inverse will be so first you take the R matrix and get the transpose of R because T will be R and p and then you take this minus R transpose p.

So, take minus R transpose and multiply it p be a vector and then you have 0,0,0,1. So, this will be the inverse of T. So, the inverse of a homogeneous transformation matrix can be directly obtained by using this principle R transpose and minus R transpose p. So, that is the inverse transformation.

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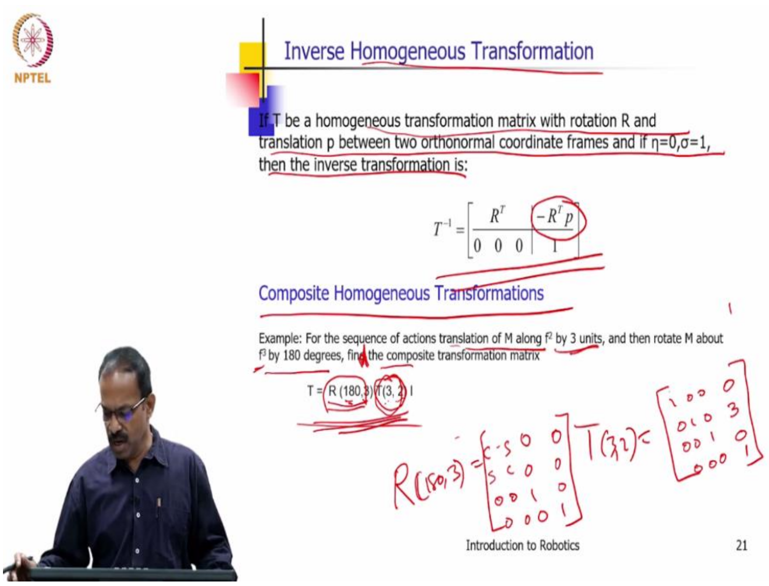

Now, if you have a suppose we will take a simple example for the transformation matrix and then see how do we get the transformation matrix suppose, in this the composite homogeneous transformation matrix, so, as I told you when you have multiple transformations, rotation translation again translation rotation etc. Then you will be able to get the Composite Rotation Matrix by the principle of Composite Transformation Matrix that we already discussed.

$$T = R(180,3)T(3,2)I$$

So, for the sequence of actions translation of M along f2 by 3 units that is there is a translation of M along f2 second axis by 3 units, and then rotate M about f3 by 180 degrees and find the okay find the Composite Rotation Matrix? Find the composite rotation transformation matrix from this one?

So, you have one translation and one rotation, you have to find out the composite transformation and rotations are all about fixed axis. So, it is both f2 and f3. So, you can find out what is the composite rotation. So, what we do? The principle is that you assume it as I this identity matrix first and then the translation along of f2 by 3 units. So, this T 3, 2 represents the transformation matrix or the fundamental homogeneous translation matrix with respect to second axis by 3 units.

That is T 3,2 and then pre multiply that with the rotation matrix, fundamental homogeneous rotation matrix about the third axis of fixed frame by an angle 180 degree. And if you find out these individually these two and then multiply you will be getting the composite homogeneous transformation matrix. So, if you write this T 3,2 so T 3, 2 will be will be equal to, so you can actually write it as so 1 0 0, 0 1 0, 0 0 1 because there is no rotation it is a fundamental translation and it is transition with respect to the second axis.

$$T(3,2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, you have 0 3 0 then 0 0 0 1 so, that is the fundamental translation, homogeneous translation matrix. Similarly, you can get R180 third axis. So, this will be we can write it as, so the translation is not there so, it will be 0, 0, 0 and this will be 0, 0, 1 and then the rotation with respect to the third axis, so you will be getting it as 0 0 1 0 0.

$$R(180,3) = \begin{bmatrix} C & -S & 0 & 0 \\ S & C & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, this will be cos 180 minus sin 180, sin 180 cos 180. So, this will minus. So, that is the way how you will be getting the translation I mean the matrices for rotation and translation. Then you multiply these two and get the Composite Rotation Matrix. So, this is how we get the Composite Rotation Matrix.

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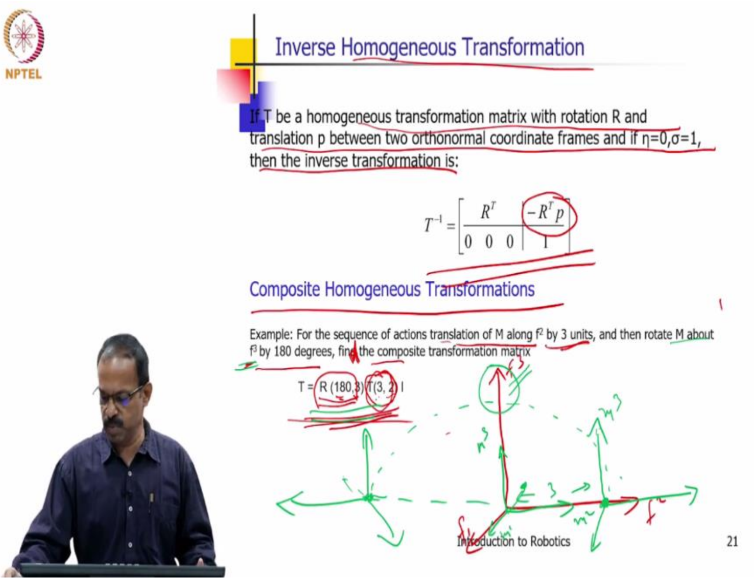

So, let us look at physically how it is actually happening. So, now to get an idea what is happening it what is happening. So, let us assume that you have a fixed frame. So, this is f1 f2 f3, f1, f2, f3. Now, I consult this as initially they are aligned M1 M2 M3. So, this is M1 M2 M3. So, the sequence of action is translation of M along f2 by 3 units. So, this M is translated along f2 by 3 units.

So, assume that it is reached here. So, it has reached here now, so this 3 units and this is the new position of the frame. Now, what happens this then rotate about M, rotate M about f3 by 180 degree. So, now this coordinate frame is rotated about f3 so, this is f3 and it is rotated 180 degrees. So, what is happening, so, it is not rotating with respect to its own axis. If the mobile frame was rotated with respect to its own axis, then it will be rotating with respect to this point only.

So, only thing this will actually come to the side, but since the rotation with respect to f3 this will be rotating all the way like this and coming up to here 180 degree. So, this will be the position and orientation this will be the position of the coordinate frame after this because it is rotated by 180 degree with respect to this axis. So, it can be actually rotated. So, the coordinate frame is somewhere here. So, it should be rotated like this and it will be coming this side. So, that is the way how it moves because the rotation with respect to this axis, so you will be getting a rotation. So, this is the way how the coordinate frame will be moving.

Now, when you do this kind of a transformation and therefore, suppose there was a point p here. Now, this point p will be moving here and then that point p will be reaching somewhere here. So, the point pf now you calculate pf, pm will be here some assume that it is 1 0 0 initially now it we actually moved out the way this set so, we will be getting a completely different coordinate point for the p with respect to the fixed frame. So, this is what actually happens when you do the coordinate transformation.

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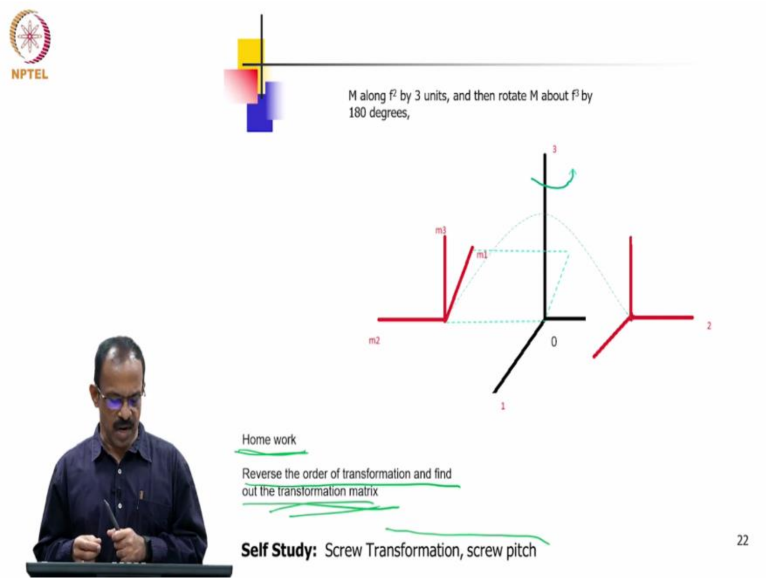

Okay, so, it is shown here the same thing to actually moved all the way up to here, so we will be getting it as M2, M3, M1 like this. So, this is what actually happens when we do the transformation okay. So, assume this is the work for you, now reverse the order of transformation and find out the transformation matrix. Suppose, the rotation was done first and then translation was done, what will be the Coordinate Transformation Matrix? Are they going to be the same or there will be difference?

You can check what will happen when you reverse the transformation reverse the order of transformation, then you can find out the composite transformation matrix at the end of these transformations. I hope you got the point what we are trying to say that whenever you have a moment of the frame, the mobile frame with respect to the reference axis there will be a, always there will be a matrix associated with the this transformation which can be used for calculating the position and orientation of the new coordinate the coordinate frame with respect to the reference frame.

And if you are to represent both rotation and translation in a translate in the matrix, then we have to go for a higher dimension space. So, we use a 4 by 4 matrix to represent the transformation and this 4 by 4 matrix is known as the Homogeneous Coordinate Transformation Matrix. And there are many other transformations also we talked only about this homogeneous transformation. So, there is something called a Screw Transformation, Screw Pitch etc. So, I leave this as a self study topic for you if you are interested you can actually refer to some standard textbooks and understand, what is Screw Transformation?

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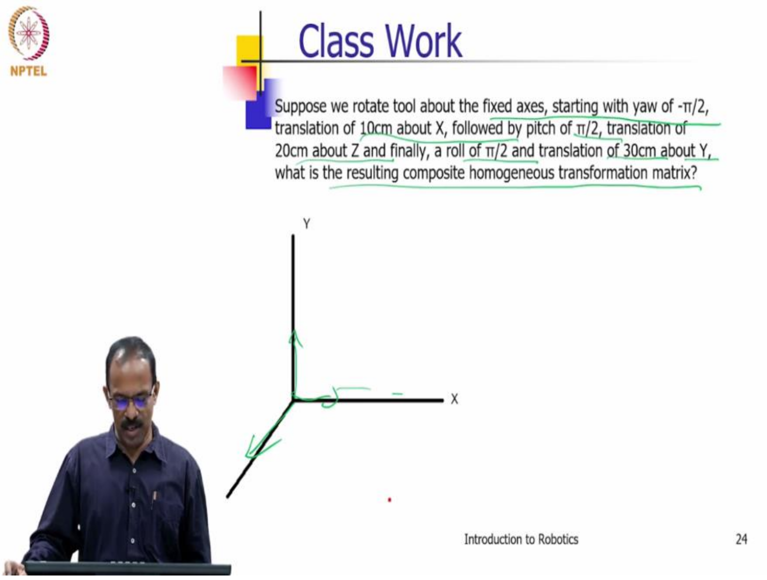

So, this is a class work or probably I will give this as an assignment later, suppose, we rotate tool about the fixed axis starting with the Yaw of $-\pi/2$ and translation of 10 about X followed by a pitch of $\pi/2$ and a translation of 20 centimeter about Z and finally, a roll of $\pi/2$ and the translation of 30 centimeter about Y. What is the resulting composite homogeneous transformation matrix?

So, there are a lot of transformation, there are many transformations taking place. So, you need to follow the same principle of Composite Rotation Matrix calculation. So, follow the principle whether it is rotation, I mean, the moment is about the fixed axis or the mobile axis. And then accordingly decide whether it is a pre multiplication or a post multiplication. And keep doing this till you look at the last transformation, and then find out the Coordinate Transformation Matrix.

And finally, to verify that you can actually plot the movement, so you can use the same method I explained the previous slide. So, create the mobile frame here. Initially they are aligned and then see whether what will happen when it is each one each transformation takes place. How is it moving? How is it rotating? How is it translating? And then see whether it actually matches with what you are actually seeing in the Coordinate Transformation Matrix. So, take any unit vector and then see whether you are able to get the correct result. So, that is the homework that I will be giving you later.

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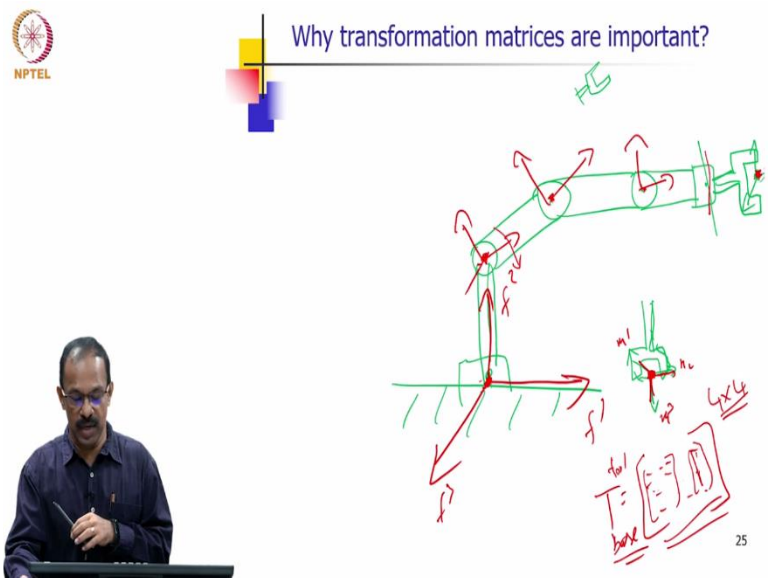

Okay, I am just skipping this for the time being okay. Now, let us see why are we learning all these transformation matrices? What is the importance of having this transformation matrix in about kinematics? So, as I mentioned earlier in one of the classes, so of course you have a robot like this you have a joint here and you have a joint here and then you have another joint.

So, this is the assume that these are the joints for the robot is another joint. Now, we know that our interest is to see what is this happening to the tooltip? So, because this is the one which actually we will be using for manipulating objects picking and placing an object and all. So, we are interested to know what is the coordinate of this particular point?

So, what we will do we assume that there is a coordinate frame attached to this. So, we have a coordinate frame attached to this. Now, as these joints move, this point will be moving so, after some time this may be coming like this or it may be coming like this because of the movement of these joints, so we can actually this can actually take place you can actually move around and reach many places within the work this space.

And this coordinate frame will be something like this. Now, here it will be something like this. So, it will be having this coordinate frame here. Now, I am interested to know what is happening to the, what is happening to this point? This point. what is happening to this point as these joints move? And how do I get this? I need to have a reference frame. So, I will have a reference frame here, I will say that, this is my reference frame.

So, this is my reference frame, I will call this as a f1, f2, f3. And then I will say, this is my mobile frame, I will call this as M3, M2, M1 etc. So, I have a frame here. Now, I am interested to know what is the position and orientation of this frame with respect to this as these joints move. So, we have a now we have a transformation problem, how this coordinate frame is getting transferred or transformed because of the various motion and then how can I represent the position and orientation of this tool with respect to the base frame. So, the robot kinematics is basically looking at the position orientation and velocities of this coordinate frame with respect to this frame.

And for that, we need to represent the relationship between relationship between this frame and this frame using a transformation matrix. So, we need to have a transformation matrix which maps the tool point and the base. So, we call this as the tool to base transformation. So, what is the transformation from this tool to this base is of interest to us. And then we represent that as a 4 by 4 matrix.

So, we represent this as a 4 by 4 Matrix the transformation from the tool frame to the base frame. And how is it moving because if we want to get this transformation matrix, I need to know how much it is rotated and how much it is translated. So, that is my interest. So, we got this can actually have can actually rotate and then translate with respect to the base frame and if I know the rotation and translation then only, I will be able to get this.

So, the relation between this and this I can get if I know the how much it is translated and how much it is rotated. And this translation and rotation take place not because of one joint, because this translation rotation can take place because of rotation with respect to this joint or this joint or this joint, this joint or this joint or a combination of these joints also, because all these joints can actually move and whenever the joint moves, there is a movement of the coordinate frame here.

Because any joint movement will affect the tooltip and it starts moving either linear or a translation or rotation it can happen. So, we need to know how these are related suppose it rotates if this one is rotating, then there will be a rotation and translation of this. So, I need to know how much it is rotated and translated so, that I can get this rotation transformation matrix. Similarly, this also.

And to do that, we cannot directly get it. So, we need to look at each joint Okay, how much this has rotated. So, how much this has rotated with respect to this or what is the relationship between this rotation and this moment or this rotation and this moment we need to know. So, all these can actually be represented using again using transformation matrices. So, what I will do I will assign a coordinate frame here and then find out what is the relation between this coordinate frame and this coordinate frame.

Similarly, I say another coordinate frame here and find out what is the relation between this and this when there is a rotation about this. So, now I know there is only rotation between these two, these two can only rotate, rotate. So, I will find out what is the relation between this coordinate and this coordinate frame and it is rotate. Similarly, I will do this and for all these, all the joints I will try to find the relationship and finally using all these transformations.

I get the final transformation between the tool and base. So, I will be getting a transformation matrix connecting this point to this point using the Coordinate Transformation Matrix. So, that is the way how we will be using the Coordinate Transformation in the Kinematic analysis of the Manipulators. So, to know this we need to know how the robots are being constructed and what are the ways in which these joints are arranged?

And then only we will know how to actually get this transformation matrix or how the transformation matrix can be generated for each of these joints. And to know that, we need to have some basic understanding of manipulators their physical construction and what are the major design parameters that will affect the transformation matrix. And unless, we know that it is difficult for us to understand the transformation taking place between coordinate frames. So, what we are trying to do is the next one or two classes, I will go through the basic mechanical features of the Manipulator.

Try to find out what kind of joints are used and what kind of actuators can be used and how this can actually leads to different kinds of construction of robots and the different robot classification happens because of the joint and then the positioning and then how this affects the overall workspace of the manipulator and some basic fundamental understanding of the Manipulator we will have in the next few classes. And then we will move forward with the kinematics using the Transformation Matrix. So, that is what we are going to do in the next few classes.

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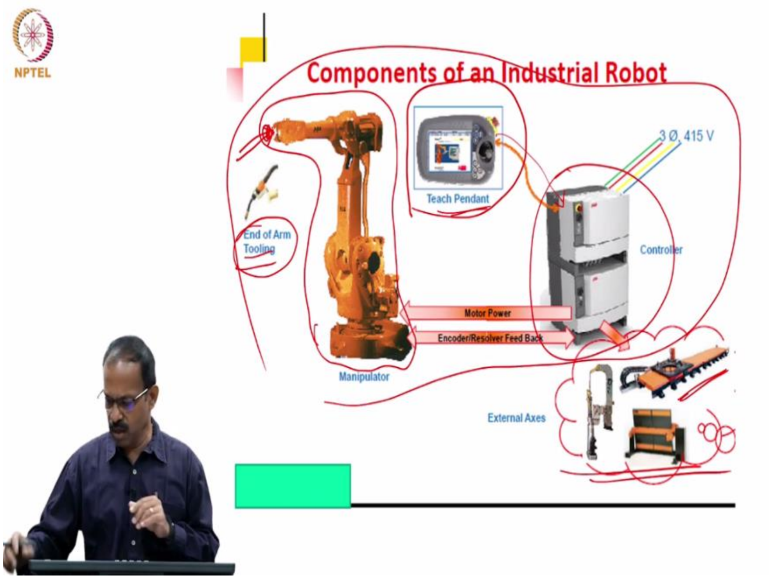

So, probably I will give you some basic introduction today and then we will continue in the next class. So, if you look at the industrial robots, to the whatever we discussed was more on the mathematical foundation for understanding the kinematics. So now, let us look at the components of an industrial robot. As you can see, this is known as the manipulator. So, the industrial robot is normally known as a manipulator. A manipulator is the one which actually manipulate physical objects in the 3D space. And that is why it is known as a manipulator industrial manipulator. So, that is the, the main element of the robotic system and industrial robot.

And anything that can be attached to the tip of these robots, and this is the tip of the last joint or the wrist point whatever you call, and that is known as the end of arm tooling. So, you can have a grasper for grasping object or you can have a welding tool or you can have a paint gun. So those, those things can be attached to here and that is known as an end of arm tooling.

And this will be having a motor and links to control it. I mean to move the joints and the other important element is the controller of the robots. So, the controller is the one which actually give the necessary commands to the joints to move depending on what you program you can actually make the program and store the program and depending on the instruction, it will actually give the commands to the joints.

And there is something called a teach pendant. A teach pendant is something which can be used for teaching the robot about the position and orientation of in the 3D space or you can use it for simple programming of the robot or effectively teach pendant can be used to control the robot and to move it move its joints and the way the operator wants.

And that is connected to the controller so the how the instruction comes through the controller, controller will process the information and then give the necessary commands. So, these are the major elements of a industrial robots. So, you have the manipulator, the end of arm tooling, controller and a teach pendant.

And we can have some external axis. So, it is not always used, but we can actually assemble this robot onto an axis like this, so that you can actually move along that axis. So, you can actually some kind of a mobility for the robot by adding some additional motion capabilities that is that external access. So, you can add something and this robot can actually be placed onto this platform and then there can be another mobility for it. So, the robot can actually move along this in this platform. So, it getting more work space capabilities. So that is the, these are the basic elements of an industrial robots.

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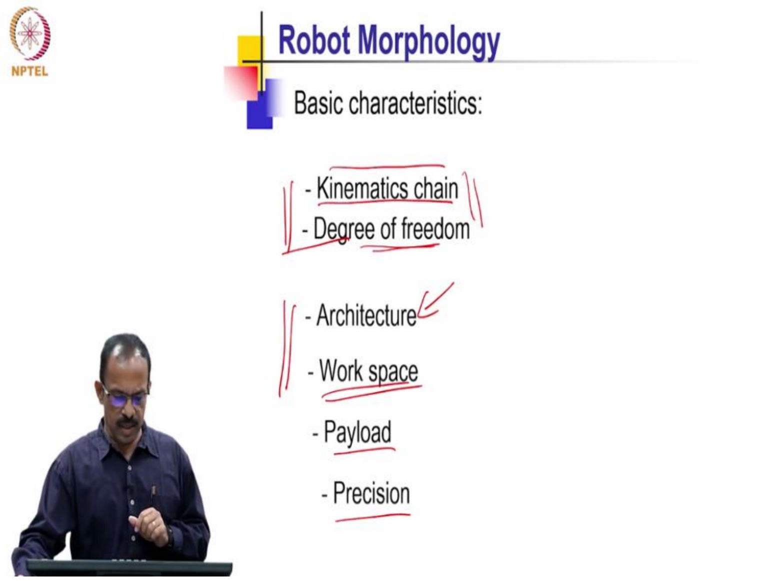

Now, the basic robot morphology, we talk about few things, there is the Kinematic Chain, and Degree of freedom. And this actually leads to the kinematic chain and the degrees of freedom lead to different architectures for the robots. And depending on the architecture, you have different workspace for the robots also.

So, we will see some basic features here, what are what are the different kinematic chain that you can have and how the degrees of freedom for the robot can be defined and basically how this actually leads to different architectures and workspace. And payload and precision are the again the features of mechanical the features of the robots.

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So, as I mentioned, the industrial manipulator is basically a kinematic chain, when I say kinematic chain, so you can see that there will be a base and to which link will be attached, there will be a base and a link will be attached. And then there will be a joint then there will be a link and joint, so like this it will be attached. So, this is known as a Kinematic Chain.

The chain of links and joints is known as the Kinematic Chain. Now, when you have it as a serial connection of this link and joints. So, this is the link join, link join like that. So, there is a serial connection, then we call it a Serial Manipulator. So, if the manipulator is obtained by serially connecting the links and joints, then we call it as a Serial Manipulator or normally it will be an open Kinematic Chain.

So, the chain will be their link joint, joint link, joint link like that and open Kinematic Chain. So, that is known as a Serial Manipulator. So, most of the industrial robots are serial manipulators. Okay sometimes we call this Anthropomorphic robot, like a human hand like now that is why it is known as Anthropomorphic. So, you can see that the all these robots are serial robots because you have a join here then a link here then join link, join like this an open kinematic chain. So that is the Serial Manipulator.

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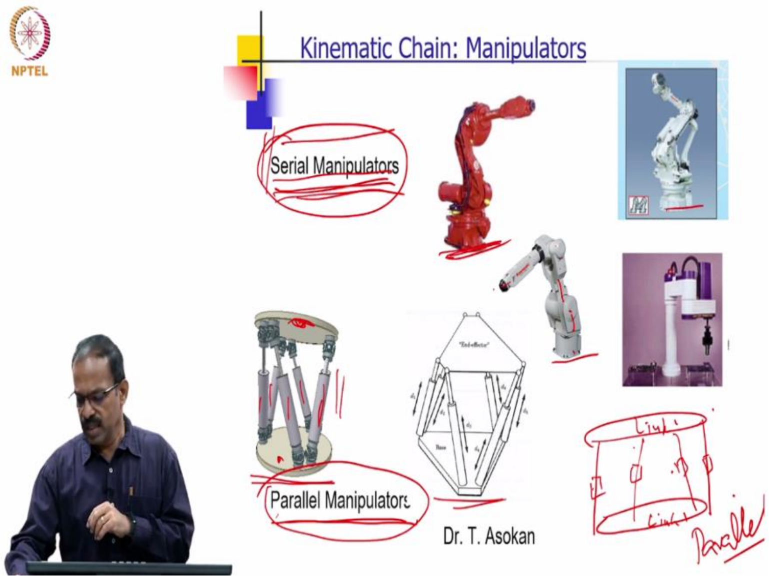

And another category that is available in the industry is known as a Parallel Manipulator. So, when these are not connected serially and we connect them parallely then we get a Parallel Kinematic Chain. So, parallel kinematic chain is that you have a link here, you have a link here and they are connected parallely using different joints. So, this is one link this is another link, so link one and two are connected through joints then this is known as a Parallel Linkage.

So, this parallel linkage leads to Parallel Manipulators. So, these are some example for the Parallel Manipulators. So, you have one base link and then their movable link and they are connected through these joints. Now, this link can actually move It can actually have all the six degrees of freedom are all the three-dimensional space motion can be possible and this is the Parallel Manipulator

The first type of industrial robots where this type Serial and most of the 90 percent of the or 95 percent of the industrial robots are Serial Manipulators. But you can actually see parallel manipulators also in the industry nowadays, they are having some specific applications and when that can actually have a lot of weight carrying capacity etc. So, there are so finding some applications in the industry.

So, these are the two major categories of industrial manipulators that is Parallel and Serial based on the Kinematic Chain. So, based on the kinematic chain, we can classify them as Serial Manipulators and Parallel Manipulators. And most of the literature that you see in the I mean whatever is published, you will see most of them related to Serial manipulators. But nowadays in the last few years, we had a lot of people doing work in the area of Parallel Manipulators also.

So, that is about the, the Kinematic Chain. Now, as you can see here, there will be there will be a lot of joints and links, but then how many links and joints we should have. So, what is the criteria for deciding that number of joints and links? Because we can I can have three links and three joints or I can have 5 links and 5 joints. So, what should be the optimal number or what should be the way we decide these numbers that actually is decided based on something called the Degrees of Freedom.

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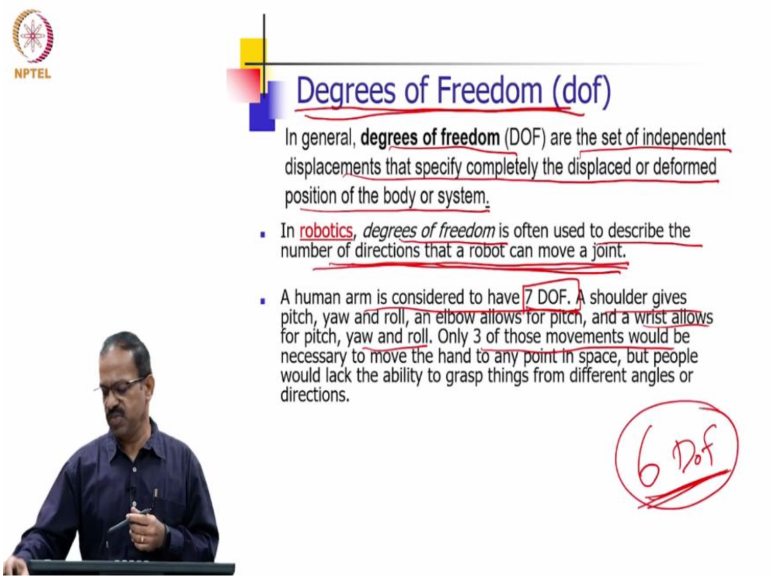

So, every robot will be having a degrees of a degree of freedom specified based on its kinematic characteristic, characteristics. So, in general, the degrees of freedom are the set of independent displacements that specify completely the decide or default position or body of the system that is a general definition for degree of freedom. So, we normally say that any object in space, so if you take any object in space, it has got some degrees of freedom.

So, I hope all of you know how many degrees of freedom this object has. If you take any object in 3D space, you can see that it has got many degrees of freedom. But in three-dimensional space, we say that he does what 6 degrees of freedom. So, any object in 3D space has got 6 degrees of freedom.

And there are 3 motions X, Y, Z directions, and then three rotation with respect to the X, Y, Z axis. So totally, we have 6 degrees of freedom for the object. Now, in the case of suppose, now if you want to manipulate this object in space, so if I have to manipulate this object in space, I need to move it to X direction, Y direction, Z direction, and then rotate, I need to have minimum 6 degrees of freedom for my hand also, because if I my hand cannot have, if my hand is not having six degrees of freedom, I would not be able to manipulate this object

And therefore, we say that in general, a robot also need to have 6 degrees of freedom, because then only it can actually manipulate objects in 3D space. And therefore, we define the degrees of freedom for robotics as. So, in robotics degrees of freedom is often used to describe the number of directions that a robot can move a joint. So, the degrees of freedom defined for in robotics is defined as the number of directions that the robot can move a joint.

So, if I can, if a robot has got a joint, it can actually move in one direction, then we call it as one degree of freedom, it can actually move in with respect to two axis, then we will call it as two degrees of freedom like that, that is the way how it is defined for robotics. And therefore, we have, a human arm is considered to have 7 degree of freedom.

That is, you have 3 degrees of freedom here and another 3 degrees of freedom here. So, we have three degrees of freedom in at the wrist and three degrees of freedom here and we have, another degree of freedom here. So, we have 7 degree of freedom for the human arm. And the shoulder gives you three degrees and the wrist allows another 3 degrees of freedom.

So, 3 of first 3, or the any 3 of 3 of these moments allow us to place the wrist in one location in the 3D space, I can actually position it wherever I want in 3D space. And then this allows me to have the orientation also so this way, the position and orientation. So, I can have, I can position this object anywhere in the 3d space using these 3 joints. And then I can orient it whatever way I want using the other three degrees of freedom. So, I have 3 plus 3 6 degrees of freedom.

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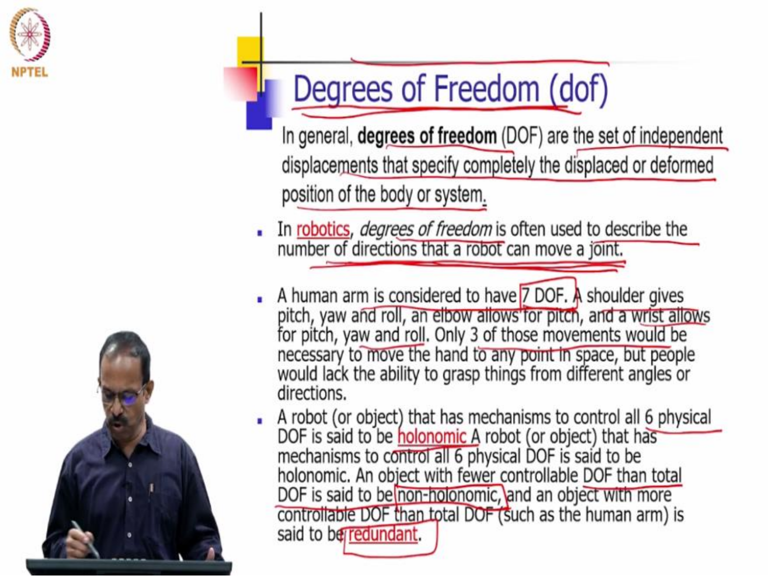

And since we have the objects are having six degrees of freedom, and we need to have six degrees of freedom for the robot to manipulate objects, all the industrial robots need to have minimum 6 joints, guess each joint assuming each joint is 1 degree of freedom.

We need to have a minimum 6 joint for the robot to have 6 degrees of freedom so that it can manipulate physical objects and a robot that has mechanism to control all 6 degrees of freedom. So, a robots can have all 6 physical degrees of freedom he said to be holonomic that is, we need to have minimum 6 and if it has got all the 6 degrees of freedom, then we call it as a holonomic robot.

And robot that has mechanism to control physical degrees is said to be holonomic. An object with fewer controllable degrees of freedom than total degree of freedom is said to be non-holonomic. So, whenever it has got, less degrees of freedom than the controllable I mean it has less controllable degree of freedom, then the actual degree of freedom that object has, then we call it as a non-holonomic object.

And whenever that is more degree of freedom, then we call it as a redundant object. So, we can have an holonomic robot or a non-holonomic robot or a redundant robot depending on the degrees of freedom. So, whenever the robot has got enough degrees controllable degrees of freedom as required for the motion of the object in the 3D space, then we call it as a holonomic robots.

Whenever it has got more degrees of freedom, it is redundant. Whenever it has less degrees of freedom, we call it as non-holonomic robot, so thats about the degrees of freedom. So, most of the industrial robot has got 6 degrees of freedom because we need 6 degrees of freedom to manipulate physical objects in 3D space.

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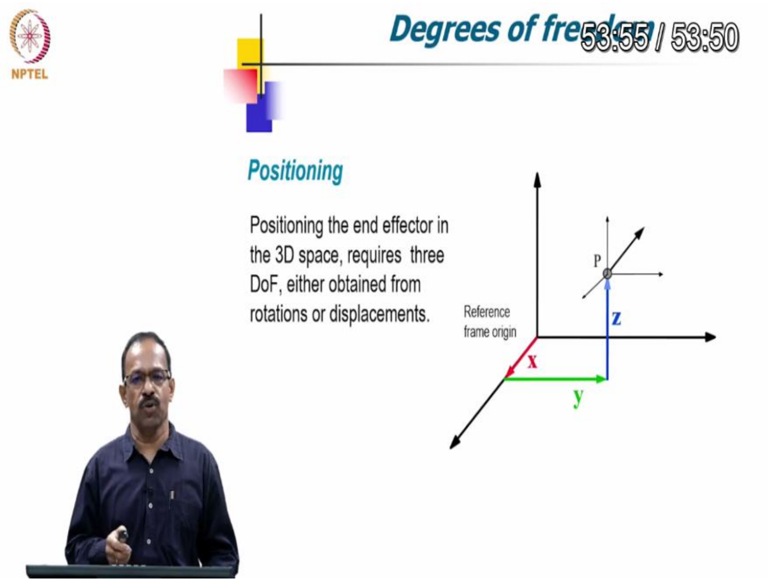

So, I will stop here. We will continue this discussion in the next class, how these degrees of freedom are used for positioning and orienting the objects in 3D space, so let me stop here. Thank you very much.