Introduction to Robotics
Professor Balaram Ravindiran
Department of Computer Science
Indian Institute of Technology, Madras
Lecture - 7.3
Recursive State Estimation: Bayes Filter Illustration

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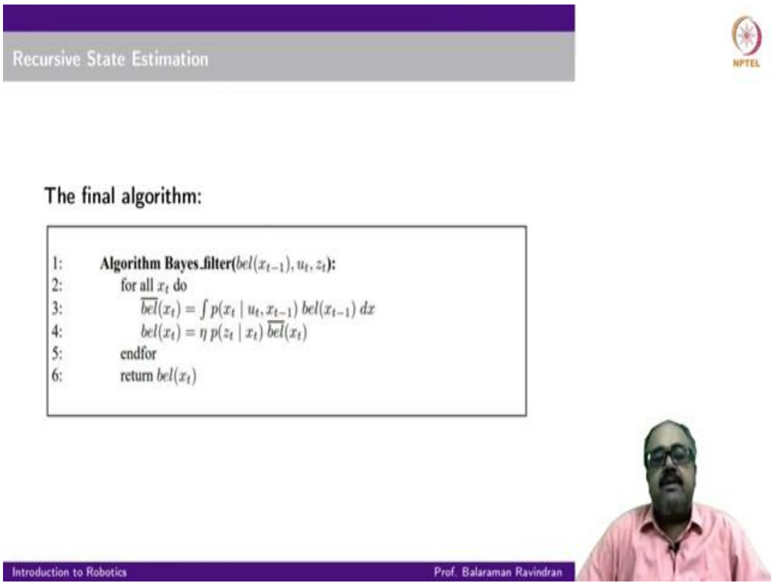

Hello, everyone, and in this module, we will start looking at exactly how the Bayes filter algorithm will operate. So as most of you would remember, the Bayes filter algorithm is going to have a prediction step. And followed by a measurement update or a correction update.

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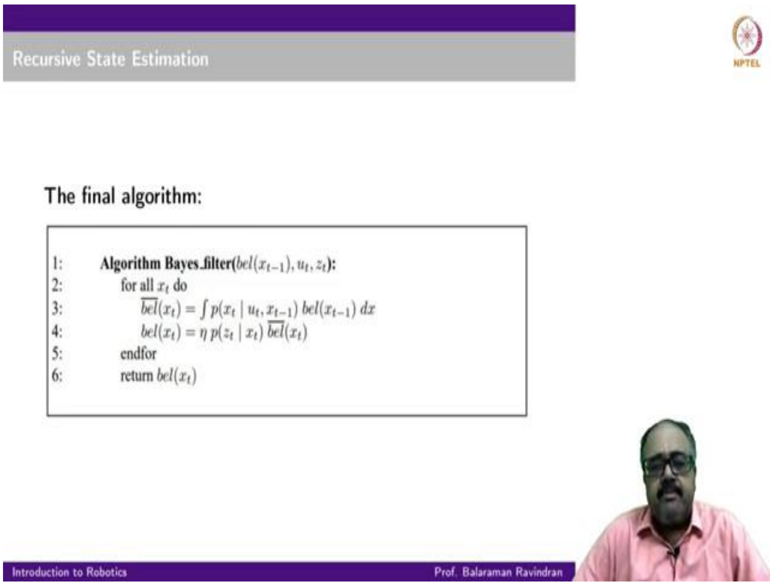

So before we get into the working of the Bayes filter algorithm, so let us just look at a simple example where we will illustrate this. So we are going to call this the door world, this example is taken directly from the textbook. So you have a robot and so it has a camera and it is standing in front of a door. So there is a door and the door could be in one of two states, door could be either open, or it could be closed.

So remember that our state signal could be much more complicated than just this one door being open or closed just for illustration purposes, so that the math and the computation that you would see on the next few slides is easy. So we did assume that the state is described by exactly one variable, which is the door, and the values this variable can take is that it is open or it is closed.

So my xt consists of just 1 variable, and that is not a vector. And at each time t, x could be either open or x could be closed. Now, what is my z? My z is using the camera the robot can sense whether the door is open or closed. My z is also a single variable and it is what I sense the door to be, the door could be closed or the door could be open. But it does not mean that the door is really open or closed, the camera could make a mistake.

For example, that could be a reflection on the door, which makes me think that there is a light coming from the other side and therefore I could think that the door is open while the door is actually closed. Sometimes the door could be open, but I might actually, it might be looking at a darker room and I might think that the door is actually closed. But so my camera has some amount of noise, it is going to tell me whether the door is open or whether the door is closed, but I do not know for sure.

So my x, at every time instant, is going to be either open or closed and my z also at every time instant is going to be open or close whether I sense the door to be open or whether a sense it to be closed. Now the next thing I need to do is look at the actions, so I need to have a set of actions here.

And so I am going to assume that the robot now to make things simple, the robot does not move, it has only 1 action, that it can open the door. It can push the door and try to open it. So again, there could be some noise, it might not push it hard enough. So you might have to push a little harder to get it to open and there is some noise. And of course, as always, the robot can choose to do nothing.

So really, the robot has 2 action choices, one whether it is going to push, or whether is going to do nothing, robot can just stand in front of the door, and then keep looking at it without doing anything to the door. So these are the 2 possible actions. So 2 possible states open or closed, 2 possible observations open or closed. And 2 possible actions, push or do nothing. So, so let us look at how this world is going to look like for us.

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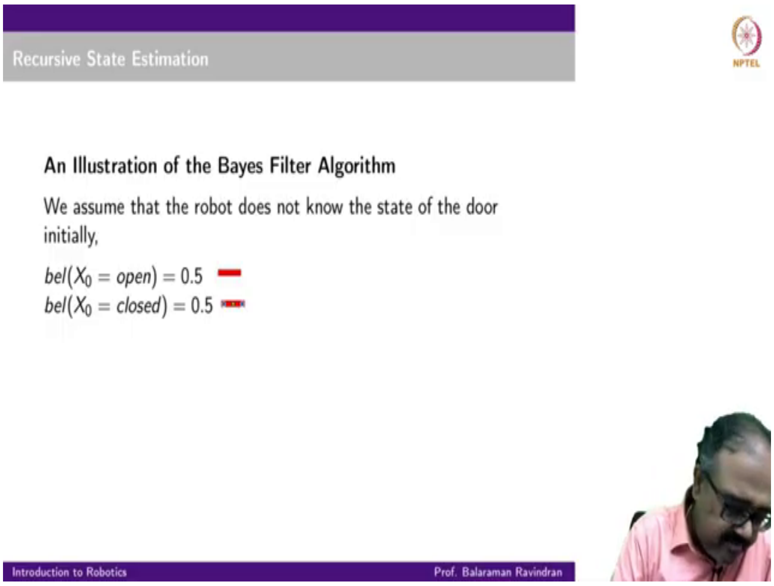

So when we start, so remember my x is open or close. So when I start, I do not know anything about the state of the door, because I have not sensed anything. Remember, we always said that the x naught is your initial state. If you have any prior knowledge, you could put it there. But otherwise, you have not done any actions you have not made any sensing or any measurements. Therefore, I start off with thinking my belief on whether the door is open or close is basically half and half. So I encode it like this.

So, the belief that the door is open that is I think the door is open with probability half. And I think the door is closed with probability half. And this is true before I make any sensing actions. Is that clear? Before I make any sensing actions, I think the door is open with probability half, the door is closed with probability half. Now notice that this does not mean that there is a half-half chance of the door being open or close, it just means that the robot thinks the door is open with a half chance, or the door is closed with the half chance. The door is either open or closed, so that is something to keep in mind.

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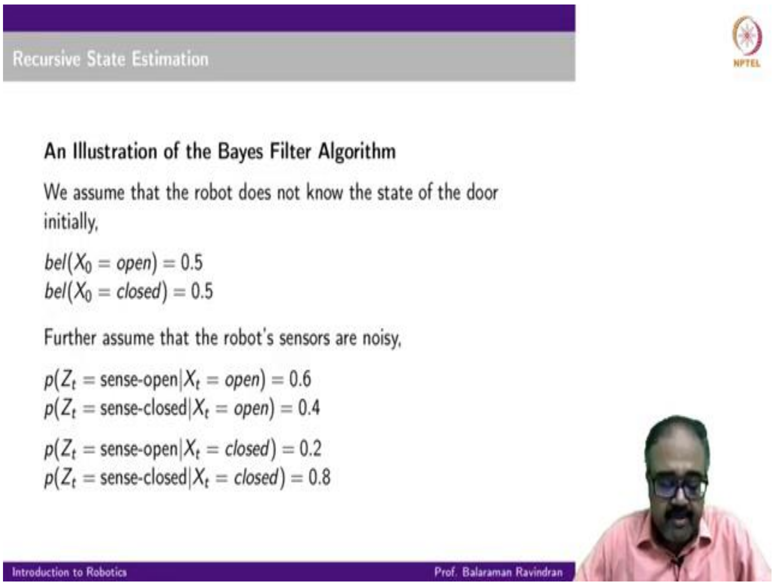

Next. So now this is the initial belief that we start off with. Now I am going to look at the dynamics of the world. So I need to know the sensor model, then I also need to know the motion model, if you remember. So the sensor model, I am going to assume that the camera is a little noisy. So even if the door is open, there is a small chance that I may sense the door as been closed. So how do I encode that? So my z, do you remember is sense open. So it is going to think of the observation as the door being open, and the door is actually open.

So if the door is actually open, the robot will think, or robot will sense it as being open with only a probability of 0.6. If the door is open, but the robot could sense it as being closed with a probability of 0.4. It could be a variety of reasons, the robot could think the door is actually closed when the door is open, with some probability 0.4. There could be some kind of occlusion on the other side of the door, or there could be something blocking your view of the door itself.

And therefore, you think that even if the door is open, the probability that you sense that it is closed is 0.4. So your reliability if the door is open, that you will sense it that it is open it just greater than tossing a coin, so it is 0.6, 0.4, it is a pretty noisy sensor. So the other way, when the door is closed, I have to look at how the sensor will behave when the door is closed as well. So when the door is closed, I am going to sense it as being open with a probability of 0.2 and I am going to sense it closed, with the probability of 0.8, because that is a fairly more accurate sensing.

If the door is closed, I will sense it as closed with a very high probability, which is 0.8. And if the door is closed, I will sense it as open with a probability 0.2 I mean these are just numbers that we have for illustration purposes. So you should not really be questioning this too closely as to whether these are realistic numbers or not, but they are not too bad. So is that clear? So we now we have the sensor model, the sensor or the measurement model that gives you what is the probability of zt given xt.

So xt could take 2 values, zt could take 2 values, therefore, you have to specify four probabilities here. So when xt is open, probability of sensing, it is open is 0.6, when xt is open, the probability that you are going to sense it is close is 0.4. Likewise, closed sense open 0.2, closed sense closed, 0.8. So at least the sensors do not completely mislead you. So the right sensory value has a higher probability.

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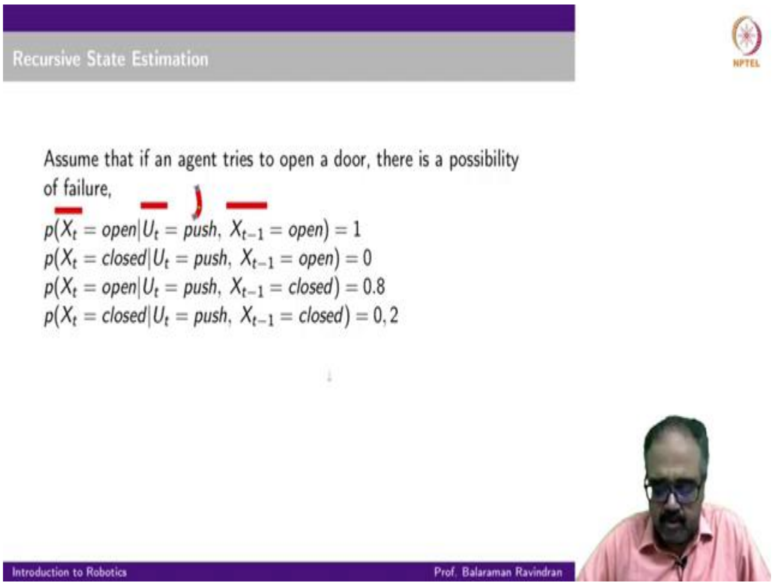

So, going to the next setup. And now we have to look at the motion model. So what is the motion model here for us or the transition model here for us? So I think of a particular state. So I am in a particular state xt minus 1, I do an action ut and then I land in a state xt. So I start in xt minus 1, I do ut, I land in xt. so that is our model. This is a Markov assumption. So we say we are not worried about the history. So I know what is the state I am in at t minus 1. I know what action I performed at time t. So what is the state, I am going to land up in time t.

So let us look at this again. Again, we have to look at multiple cases. So now for the first set of values I am showing on this slide, I am looking at the action push. So what happens when I push? So what happens when I push. So if the door was open to begin with, and then I push, what do you think is a likely outcome. It is going to be open, by pushing it I am not going to close the door, I have to pull the door to close it, I do not have a pull action, I only have push action.

So if the door is already open, and I push, it will continue to stay as open. So that happens with probability 1, that means it is a certain event. If the door is open, and I push, it will continue to stay open, and therefore the converse event when the door is open, and I push the door will close has a probability 0, so it will never happen. So that is what it means.

Now look at the other case, the door originally is closed, and I push and the door opens. I have a probability of 0.8, a fairly reliable action. But the door is closed and I push, there is a small chance of 0.2 for this, a small chance of 0.2 that the door could be closed, could remain closed, that could be, the door could be stuck, you are not pushing with enough pressure, there could be some other source of noise that the door remains closed even when I actually try to push the door open.

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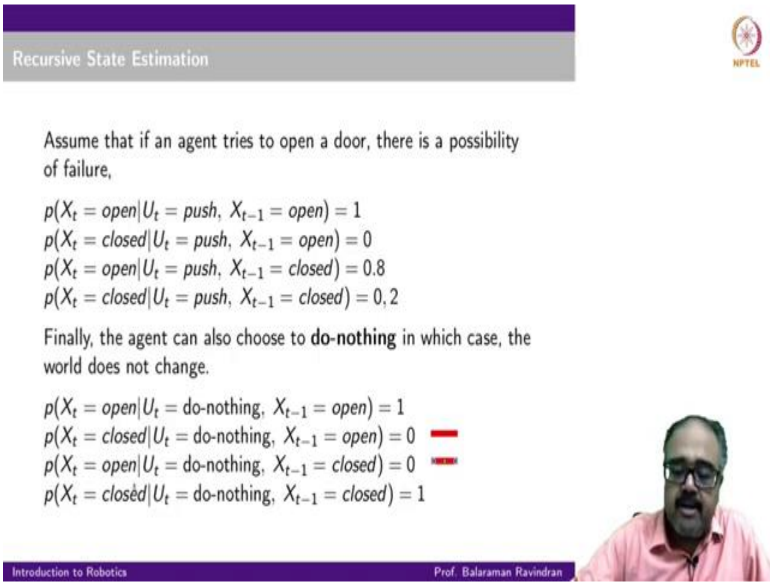

And finally, for the motion, so I could do nothing, that is other action. So, the first action was push and the second action is basically do nothing. And so, do nothing basically does not cause the world to change. So, if the door was open, and I do nothing, the door continues to remain open, if the door was closed, I do nothing the door continues to remain closed. So, this is basically the 0 probability events or when this state changes, the door was closed, I do nothing, and the door now becomes open, never happens probability is 0.

The door was open, I do nothing, the door now becomes closed, never happens the probability is 0. So both these events have a probability of 0. And so, if the door is open, it continues to remain open, if the door is closed, it continues to remain closed, if I do nothing. So all of this clear now. So I have my motion model, I have my measurement model and I also have my initial beliefs. Now we are ready to start computing our updated beliefs.

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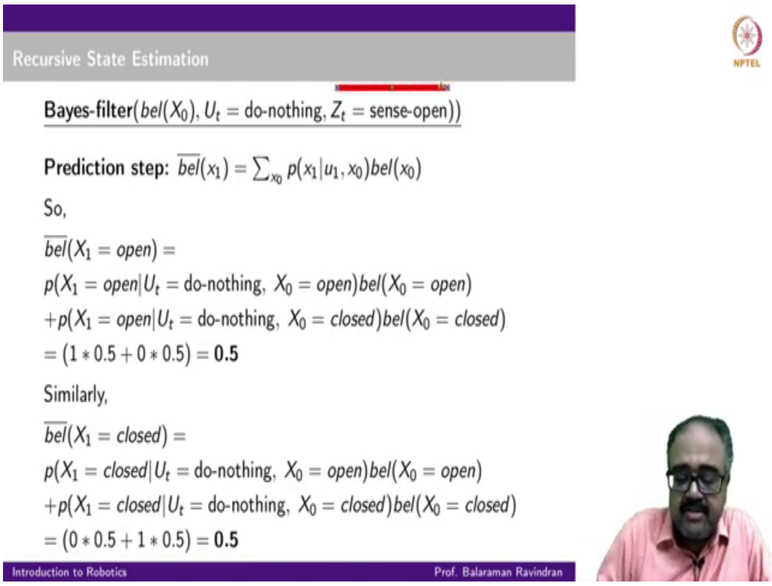

So let us assume that I am starting off with the initial beliefs that we had half and half and then action that I do is do nothing, I am not doing anything to change the state of the world. And then the sensory information I am getting is open. So, basically, I did nothing I get a sensory information that is open. Now, the way the Bayes filter algorithm works is I start off by first making the prediction step. So I first compute bel bar and I am going to compute bel bar of x1 the bel bar of x1 is essentially looking at all possible values that x naught can take and looking at the probability that x naught takes that value, then I know what u1 is, u1 has been given to me, which is do nothing.

So what is the probability of x1, given do nothing and that particular x naught, so I have to do this over all x naught. So, this basically gives me the bel bar x1. Remember in the algorithm that we wrote this x naught was actually written as the, summation was actually written as an integral and since we are looking at discrete valued even sense, everything has only 2 possible values x has 2 values, z has 2 values, u has 2 values. So everything becomes summation instead of integral here.

And therefore, we are now summing over all possible values of x naught instead of integrating over x naught. So let us look at this a bit by bit. So I am going to look at what is bel bar of x1 equal to open. So, I had to look at all values that x1 can take. So, this was some particular value of x1, here I am actually putting a, filling in a value here, the bel bar of capital x1 equal to open is given by, so the first value that I could should consider for x naught which is x naught equal to open is what we are considering here.

So belief of x naught equal to open, since what did I believe was the probability that x naught will be open times the probability that x1 is open, given x naught was open and I did nothing, action was do nothing. So I am looking at bel bar of x1 equal to open. So the first value we will consider is started off with x naught equal to open, that is x naught equal to open action was do nothing and then ended up with x1 equal to open. So what is the chance of that happening.

And then the second thing I have to look at is x naught equal to closed, I have to sum over all possible values of x naught. So, I looked at x naught equal to open, now we have to look at x naught equal to close. And then again, I have to look at the probability of x1 equal to open given that u was do nothing, and the x was x naught was closed. So, I had to add up these 2 probabilities.

So, if you remember our values that we had earlier, so, the belief of x naught equal to open and x naught equal to close to both half, so that is a 0.5 here, and the probability that x1 equal to open given that I started off with open and I did nothing, is 1, we saw that if you do nothing, the state does not change, so the probability is 1. And given that I started with closed and I did nothing, what is the probability that x 1 will be open? 0. We said the state does not change.

So, basically the bel bar x1 equal to open is basically 0.5, has not changed. And so, likewise, we do the same for bel bar of x1 equal to closed. Of course, you could always take it as 1 minus bel bar of x1 equal to open, but we will just walk you through the computation again. So, likewise, you look at both values of x naught. So you consider x naught equal to open first. So belief of x naught equal to open. And so you start off with x naught equal to open, you do nothing, what is the probability that you will end up with closed, so, that is 0.

And belief of x naught equal to open was 0.5 and then x naught equal to close, you start off with x naught is closed, then you do nothing. And then you end up with the x1 equal to closed and what is the probability of that happening, we know it is 1, so, that is 1 and belief of x naught equal to close is 0.5. So, the whole expression evaluates to 0.5. So, you have bel bar of x1 equal to open is 0.5, bel bar of x1 equal to close is 0.5. And so it really does not change from whatever bel of x naught was.

So bel bar x1 has not changed. It is not surprising, because it is essentially this is our prediction update and our ut was basically do nothing, do nothing is supposed to not change anything in the world. And that for whatever beliefs that we had on x naught gets transferred to bel bar x1. So the interesting update now is going to be what happens when we do bel of x1 after we incorporate this sensing. So let us go on to the next slide.

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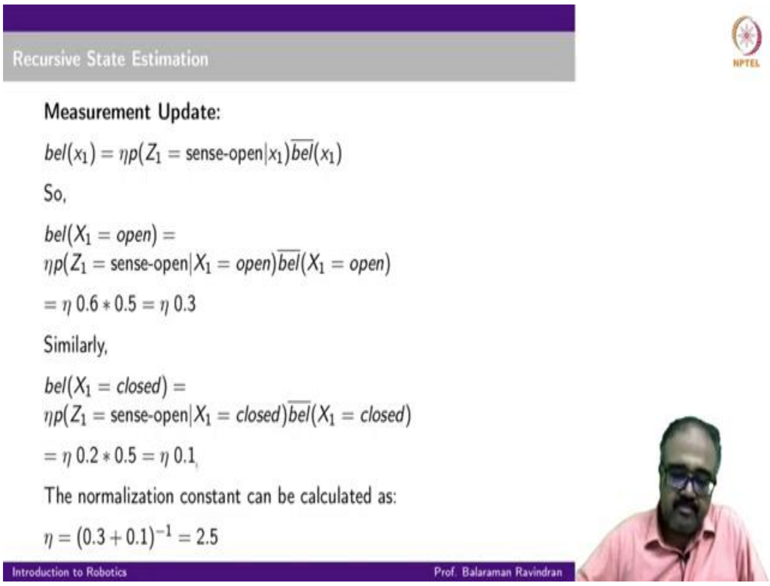

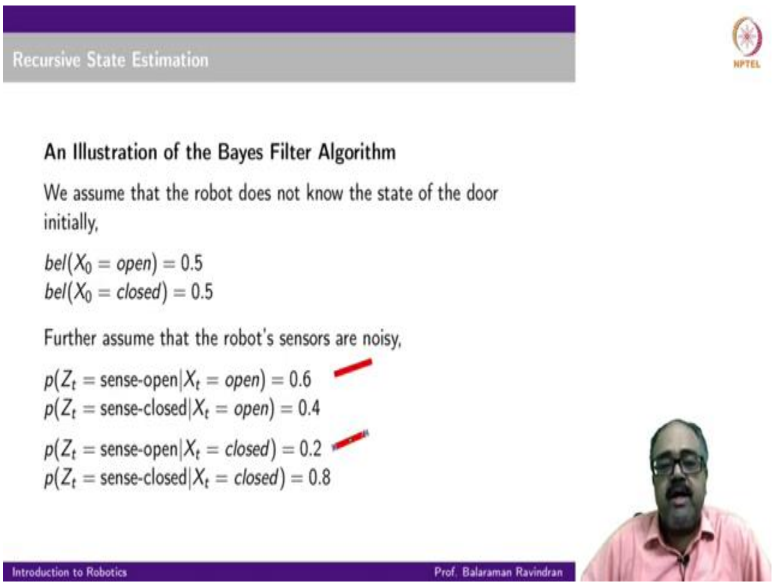

Alright, so now I had to look at the measurement update. So how does my measurement update work if you remember, so I had this normalization factor. And then there is my bel bar of x1, the bel of x1 is equal to well bar x1 times the probability that I will get my z1, whatever is the sensory action I had given x1. So that is essentially my measurement update. So let us start off with looking at, let us start off with looking at bel x1 equal to open. So remember, x1 can take 2 values, I have to look at x1 equal to open.

So that is equal to the probability of me sensing that it is open, given that x1 was open, times my bel bar, that x1 was open. So what does it really mean? Let us just wait for a second and think about this. So I want to know what my new belief is in the fact that my door is open. I want to know, what do I believe now about the door. So I take whatever was the bel bar, after whatever action I took. Yeah, so after whatever action I took the last time.

So what is my, what is the probability that x1 equal to open? What do I believe is the probability that x 1 equal to open, times what is the probability that I will actually get this sensory information or I will actually make this measurement given x1 equal to open. So notice that bel bar is still my estimate of what is the probability of x1 equal to open. It is not really the probability that x1 is open, it is only my estimate, remember that always. It is not a system parameter, it is your estimate.

And since we are updating in a very consistent fashion, very soon you will know what is the true state of the world assuming that you have enough interactions with the, with the world. So now, if you remember, our probabilities, like bel bar x1 equal to open was 0.5. And now if you remember, I said when x1 is open, we will sense it to be open with a probability of 0.6.

So if you remember the previous measurement updates, this is the measurement update that we had. So when the door is open, we will sense it, sorry when the door is open we will sense it to be open with probability 0.6. When the door is open, we will sense it to be closed with probability 0.4, we should remember these numbers. And when the door is actually closed, we will sense it to be open with probability 0.2. When the door is closed, we will sense it to be closed with probability 0.8.

Now notice that we have given that the robot has sensed the door to be open. So we know that the robot has sensed the door to be open, then the first instance, we have sensed the door to be open. Therefore, the 2 relevant probabilities for us are these 2. So door is open, what is the probability I will sense it to be open and if the door is closed, what is the probability I will sense it to be open, because that is basically what we have a sensed now. Remember, sense it to be open, so we already looked at the prediction step, now we are going to the measurement step.

So the probability that I am going to sense it to be open given that it is actually open is 0.6 times 0.5. So this quantity evaluates to 0.3, I still have my normalization factor. And we will come to that in a bit. This is the first update. Now what is the next thing I have to look at? Yes, I have to look at what is a belief that x1 is closed, I have to look at the belief that x1 is closed.

And again, I look at the belief bel bar that x1 is closed. And the probability that I am going to sense something as open with some, sense the door is open, given that x1 is closed. So what is the probability that x1 is close for me it is 0.5 so that we already know that. This is our bel bar, bel bar is 0.5, the probability I will sense something open given that x1 is close, given that X is close, I will sense it to be open is 0.2.

Remember that the 2 probabilities I pointed out to you are 0.6 and 0.2. So that is the 0.2 probability. So if I simplify this, I get 0.5 times 0.2 is 0.1 times eta. So the normalization constant, if you remember, I said we will add up all the numerators and divided by the sum of the numerators. So my eta is basically 0.3 plus 0.1 raise to the power of minus 1, that is basically 1 by 0.3 plus 0.1. So that is 2.5. So my final bel of x1 equal to open is 2.5 times 0.3. And the belief that x1 equal to closed is 2.5 times 0.1. So that is basically what we have here.

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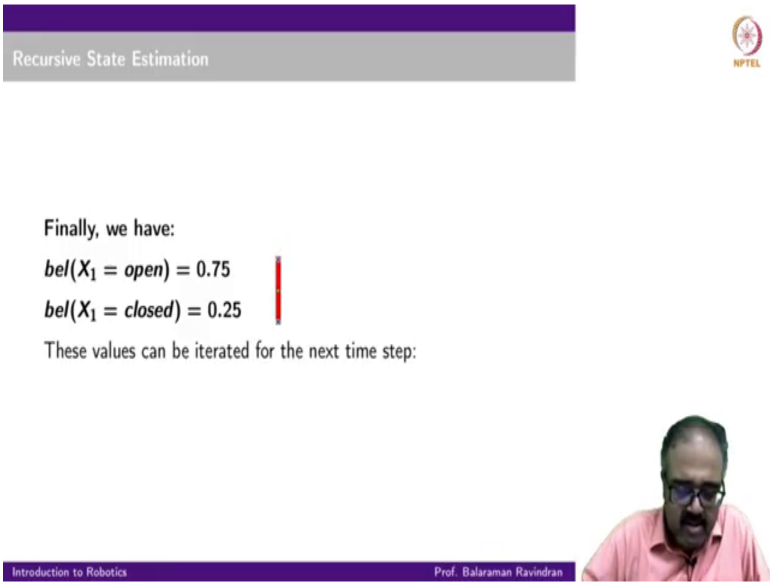

So finally, we have these 2 as our beliefs. So belief that x1 is open is 0.75 it belief that x 1 is closed is 0.25. So remember, what did we start off with, we started off with a belief state that said x naught is open with 0.5 or x naught is closed with probability 0.5, then what we did, we said do nothing, now make a sensory action, my u1 was do nothing and my z1 was sense the door to be open.

Now given that my sensor is reasonably reliable, so I update my beliefs now. So my new belief is belief x1 equal to open this 0.75, belief x1 equal to closed is 0.25, mainly because I have sensed z1 as door open. Now I can keep doing this. Now, this will be my new belief that I will start off with, I can iterate for the next time step. So let us go over one more time step of this update.

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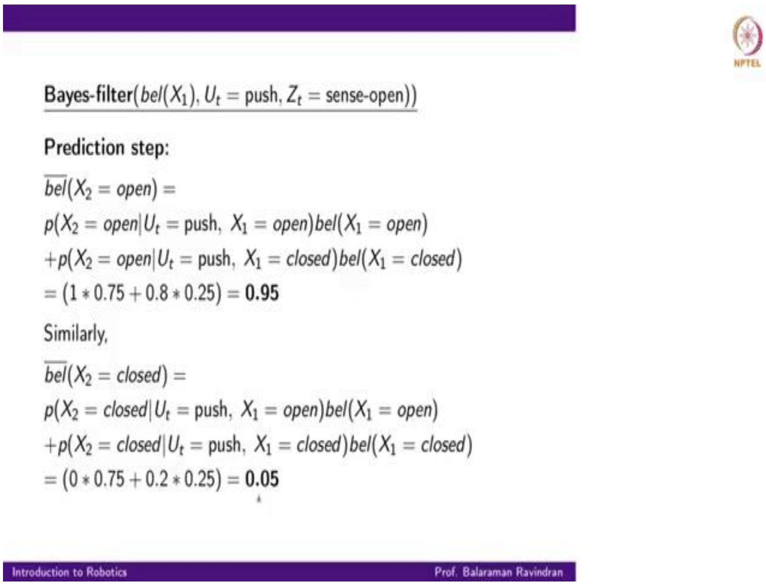

Now the new sensory information I am getting so I have starting off with my belief x1. And my next u, I am getting is push, I am going to push, just to make sure that everything is open. So I think the door is open with probability 0.75, but I still choose to push the door. And after I push, I sense the door to be open. So my u2 is pushed and my z2 is sense the door to be open. u2 is pushed and z2 is sense the door to be open. And so let us look at what our prediction step should be, I have to first look at bel bar x2 is open. So I have to look at bel bar x2 is open.

That is given by first start off with bel x1. So I had to look at both x1 open and x1 close as my starting points. So I could, x2 could end up as open. Because I started off with x1 equal to open and did something that made it stay open or I started off with x1 equal to closed and did something that made it open. So I have to consider both outcomes. That I started off with open, and then I made it open, I started off with closed and then I made it open, I had to look at both outcomes.

So what do I do now? I start off with x1 equal to open and then my action was push. So I look what is the probability that x2 would be open given that I started off with open and I pushed. So we know that if you push and you start off with open, the probability 1 you will stay as open. So that is what our model was earlier. And next component here is I look at, I start off with close, now I push what is the probability that it will be open. So we know that the push action is fairly reliable.

So if I push even when the door is closed, there will be a 0.8 be probability that will end up being open. So I take the first one, which is probability 1 times belief my current belief that x 1 is open, which is 0.75, and this is 0.8, the second term is 0.8 times 0.25 so I evaluate this I get a value of 0.95. Do not rush, this is still my bel bar, it is not my belief. So my belief has not become 0.95, my intermediate, my prediction update tells me that bel bar is 0.95 for x2 equal to open.

Similarly, I have 2 do this for x2 equal to closed. And like I said, because there is only 2 values, you could have done 1 minus, but we will not do that, we will just go through the full computation. So I start off with again, x1 equal to open that and I pushed. So I am looking at whether x2 can be closed. I know that that never happens. So that probability is 0. Even though my belief that x1 is open is very high. So that component contributes 0. And likewise, now the door is closed, I push, the door stays closed. That can happen.

So we know that if the door is stuck, or if you are not putting enough pressure or something like that, there are many reasons it could not work and the probability is 0.2. And what is the probability that I actually start off with the door being closed? That is 0.25, that is what my belief was. So I basically have 0.2 into 2.25, which is 0.05. This is my bel bar. So my bel bar that x2 is open is 0.95, and my bel bar that x2 is closed is 0.05.

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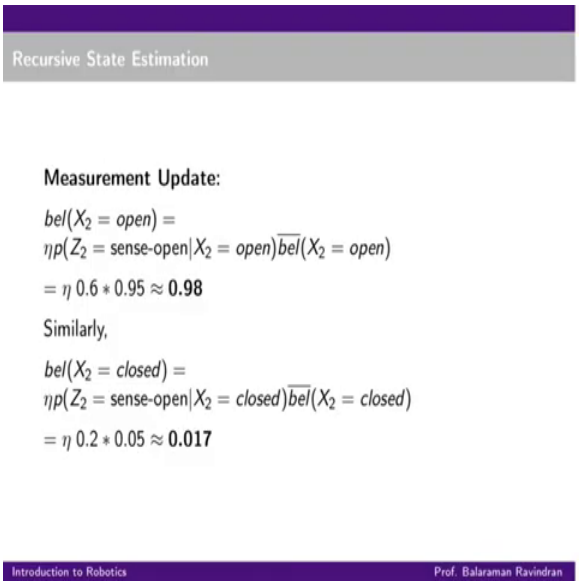

Likewise, let us do the measurement update. So I want to look at beliefs that x 2 is open. So that basically starts off with the bel bar that x2 is open times the probability that I will get sense open, given that x2 is open. So when x2 is open, I will sense that it is open with probability 0.6, we saw that already, where x2 is open, I will sense it is open with probability 0.6. Notice that both my first step and my second step I have sensed the door to be open, it could very well be that my sense, I could have started off with sensing it the door to be closed and now I could sense it to be open.

But in this particular example, we have chosen, the sensory input does not change. So it sense open. So these values that we are using are also the same values we used in the first step. So I sense it is open given that it is actually open, probability is 0.6. And the belief bel bar that it is open is 0.95 and then I take the product, in fact, here I have not done the normalization computation separately. So this whole thing evaluates to something like 0.98.

And similarly, I look at the bel of x2 equal to close. so now I take my prediction update, the prediction value, the bel bar x2 is closed. And then you look at what is the probability that I will sense something as open if it is closed, I know that probability is 0.2. And I know my bel bar value is 0.05. And I take this product, and then I adjust by the normalization constant, I basically get 0.017 as my, so you can see that these are all approximations.

So I get 0.98 as my belief that x2 open and it should be 0.02, if you round it up, now belief that x2 is closed, is it clear. So we will give you some practice exercise where you can try out different kind of update sequence for similar kind of problem. And so the thing to remember is, in fact, the door, even now, I do not know if the door is open or closed. So the robot does not know whether the door is open or closed. In fact, all of these computations, there is still a 0.02 chance, according to the robot's belief that the door is closed.

And in reality in the world, the door could still be close. There is nothing like saying that the door is open. So remember that, so when I am doing these belief updates, I could start off with really bad beliefs about the world and I could think that some, the actual outcome has very little likelihood of happening. I could start off by believing that the door is closed with probability 0.99 and door is open with probability 0.01, we started off with point 0.5, 0.5. Then it may take one more step of iterating through this before I am sure of what the new probability that time the door is open.

So right now, I think the door is open. But that might not be the case in the real world, so this is something we keep in mind and this is just an estimate of where I am. So, from the next set of lectures, we will start looking at making specific assumptions about the form of the transition function, form of the measurement model. And also the form of the belief distribution itself.

Here, I just looked at these as 2 numbers. Just like a set of numbers, the belief distribution was just two numbers, what is the probability it is closed, what is the probability it is open, the sensory distribution, again was four numbers, the transition was like, again eight numbers. But all of these were just numbers, we did not think of any functional forms. So next, we will look at a function that can describe what the motion model is, what the measurement model is, as well as what the belief function is, belief distribution is and then we will look at how to operate with those and that allows us to do more non trivial inferencing.