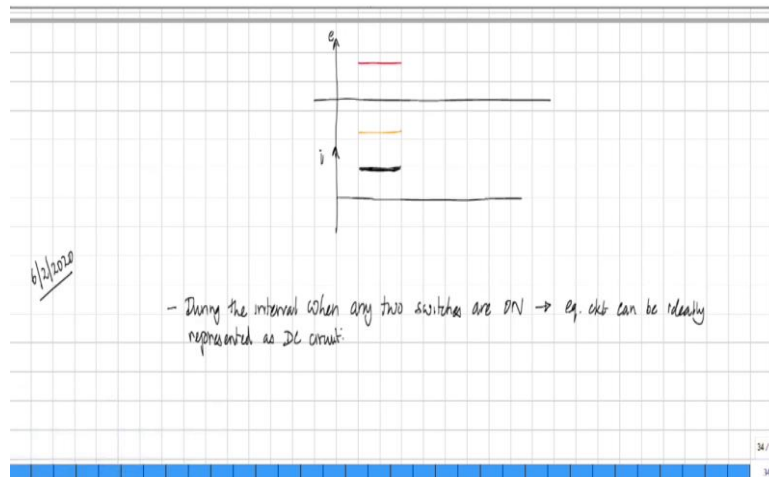


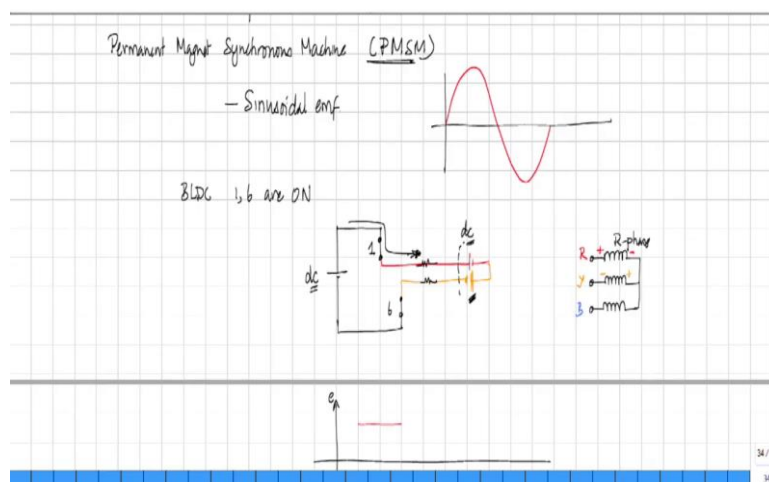
Introduction to Robotics
Professor Krishna Vasudevan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Lecture-24
PM Synchronous Motor (PMSM) and SPWN

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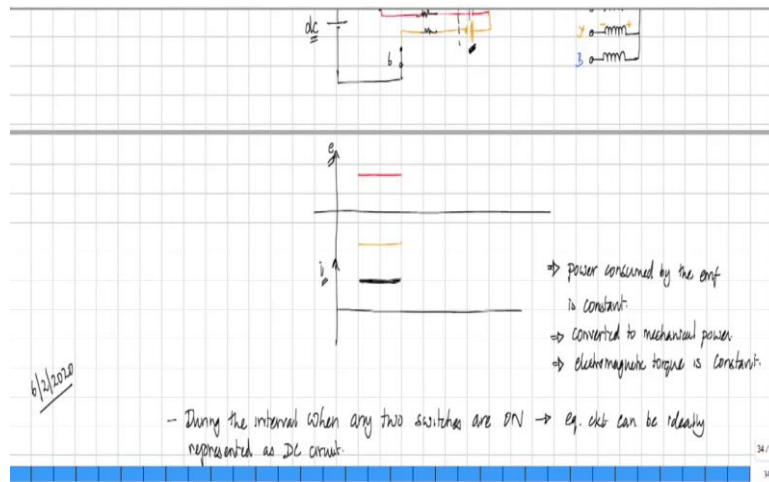
So, what we have seen is that during the interval when any two switches are on in that interval the equivalent circuit can be ideally represented as a DC circuit.

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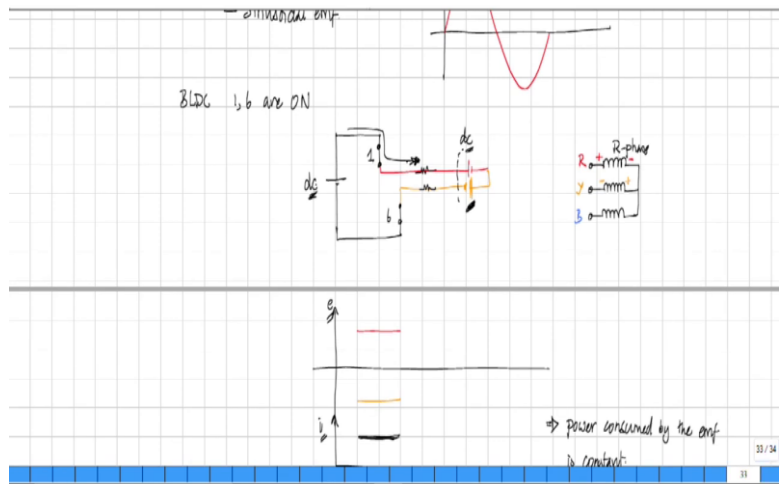
This is what we had seen here for the case of switches 1 and 6 being on what you have is the input DC source is now connected to the motor to those two phases of the motor, where the voltage looks induced EMF looks like DC and therefore, the flow of current is DC.

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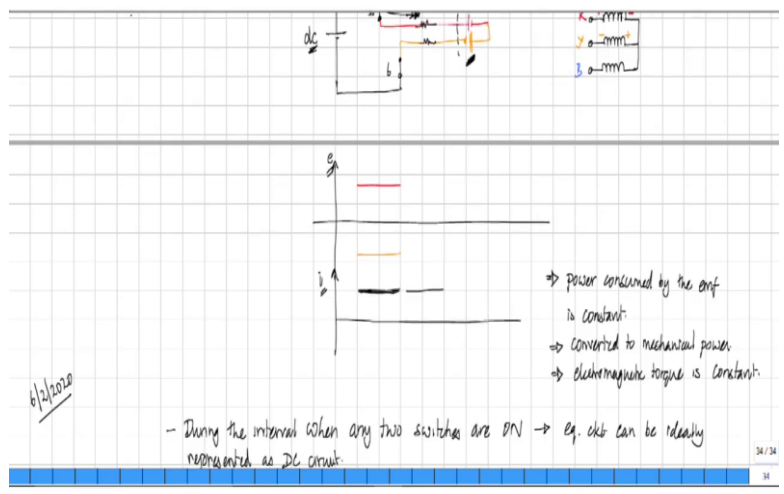
In that interval and that is what we have drawn here, the induced EMF is flat, flow of current is flat that means this, then means that power consumed by the EMF is constant E multiplied by I this is the induced EMF E and you multiply this by I E is constant I is constant and therefore, E into I is also a fix number and this is the one that is converted to mechanical power and therefore, mechanical power developed in the machine is also not going to vary within this interval and if therefore, the speed is not going to change this, then implies that electromagnetic torque is not going to change this is valid in 160 degree interval where two devices are going to conduct.

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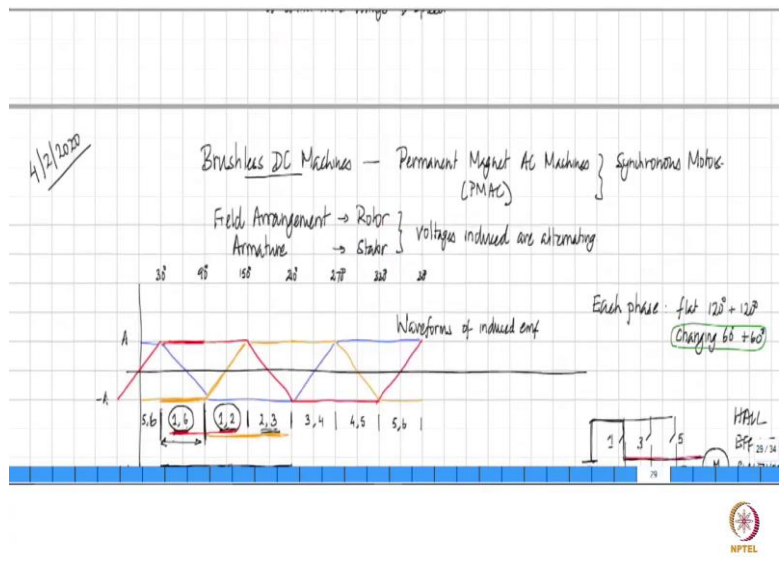
In the next 60 degrees another two devices will conduct again with the same value of induced EMF and you have the same applied voltage.

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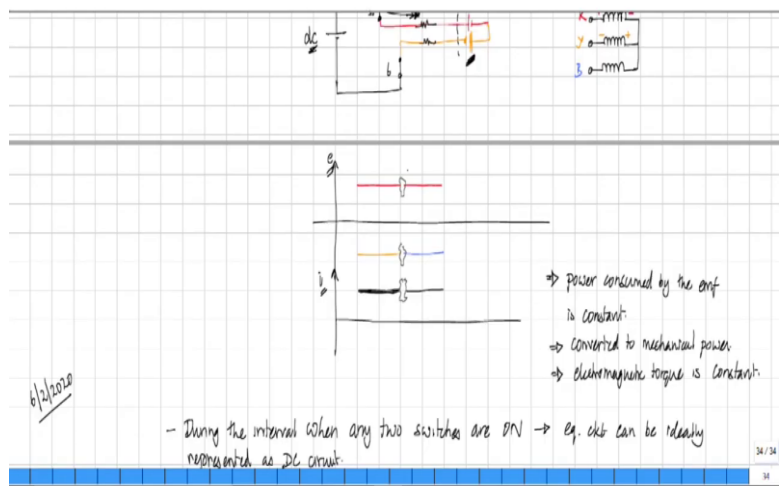
Therefore, in the next 60 degrees the same current will flow and you have some other.

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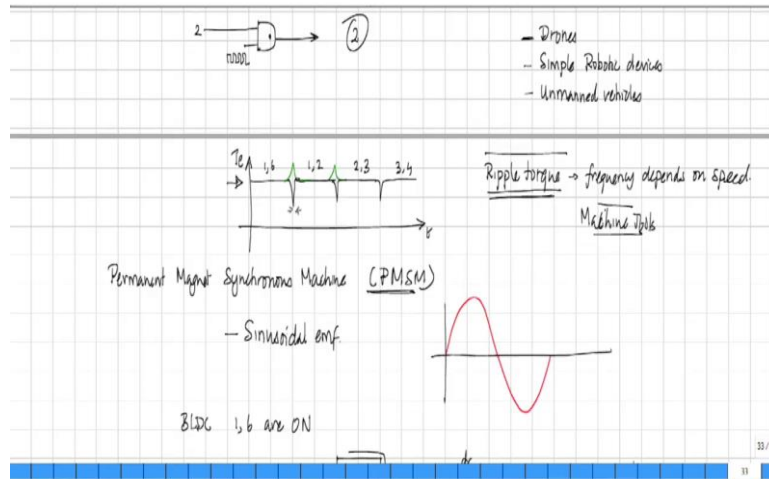
So, after red and yellow what do we have we have red and blue.

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So, in the next 60 degrees you have the same red that is going to continue and then here you have blue and through that the same DC current is going to flow and therefore, again electromagnetic torque is not doing very what we only said was that, when you go for 160 degrees to another 60 degrees then, there is an interruption something happens in this region where the EMF are not going to be the same because, one is falling another is going to increase and reach that level so, in that small interval there is a non ideal behaviour which leads to a ripple torque.

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As we had drawn in the graph that is shown there so, this happens in the intervening small interval when one set of devices finished conducting the next set is going to take over for the next 60 degrees.

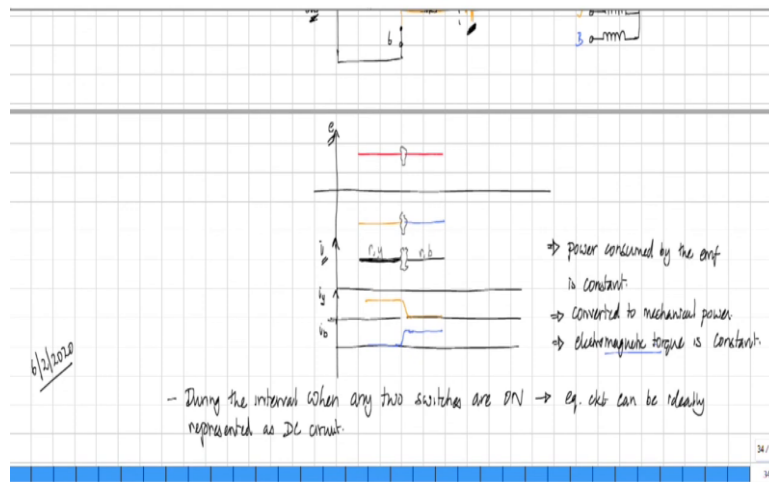
Student: will current have a ripple

Professor: Current also will have a ripple yes, during that small interval.

Student: does this arise because of the interval

Professor: Yes this arises because of the interval so, during that small interval you notice, that one phase which was off now has to start conducting so, ideally we assume that if.

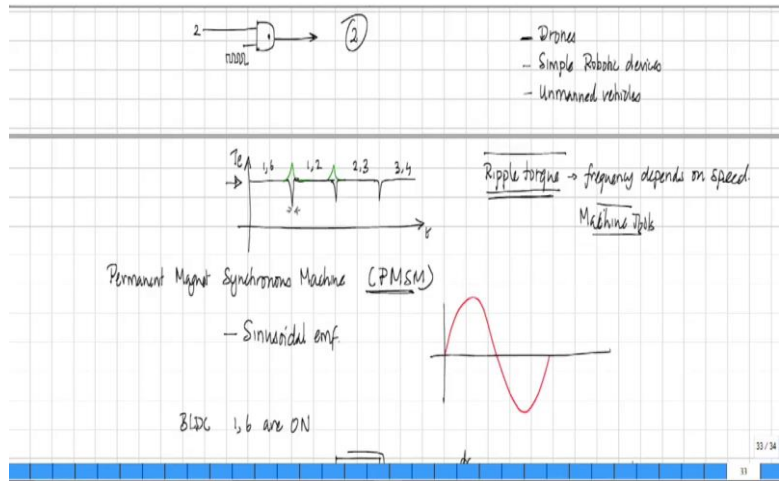
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So, here you have the red and the orange, red and yellow conducting here it is red and blue that is conduct that means, if you plot I_y and I_b I_y should have been DC until this time and it should go to 0 during this time because, y is no longer conducting whereas, b should have been 0 here and b will begin to conduct here but, the fact that there is an armature inductance it will not allow this current waveform to be like this if it were like this, then there is no ripple that is going to be produced electromagnetic torque whatever, was happening as one as it is simply being you know, implemented by another phase.

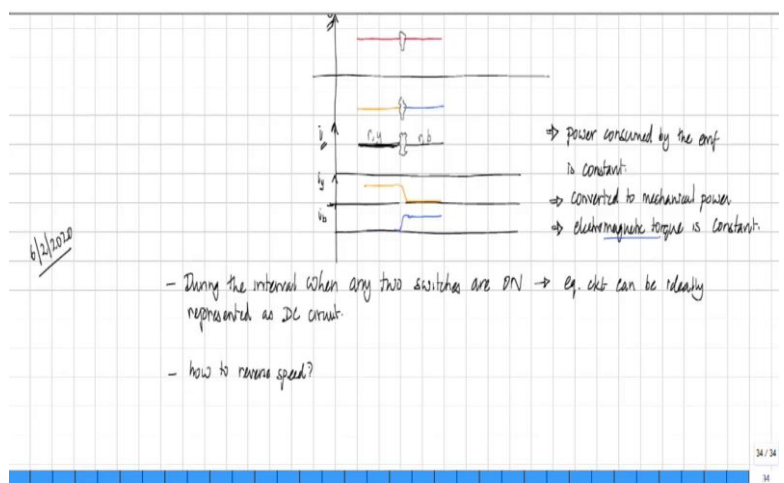
But, this is not going to happen because, of the inductance and therefore, the situation looks different what happens is, this current goes to 0 over some duration this current rises to this the desired level in some other duration the durations may not be the same but, that is governed by what is the speed at which the motor is rotating and because of this you have the electromagnetic torque not being flat some variation will be there depending on what exactly is happen.

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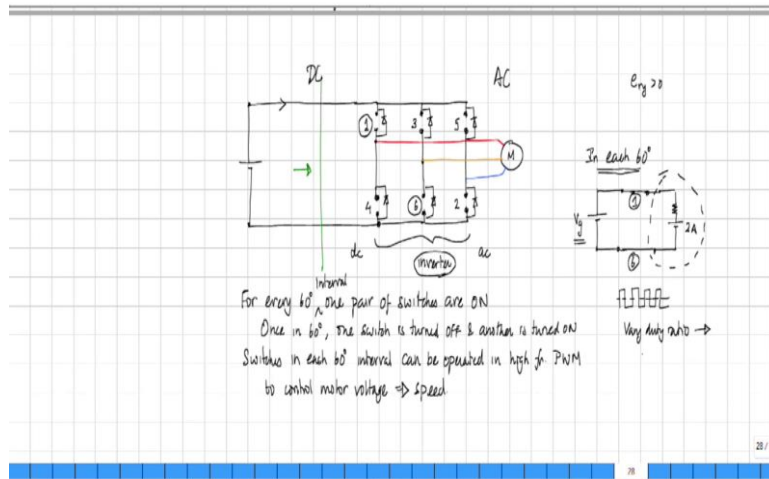
That is the origin of this ripple that we have drawn and that one has to contend.

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So, we need to understand one more thing in the operation that is how to reverse speed or reverse the direction of rotation in the case of a DC machine.

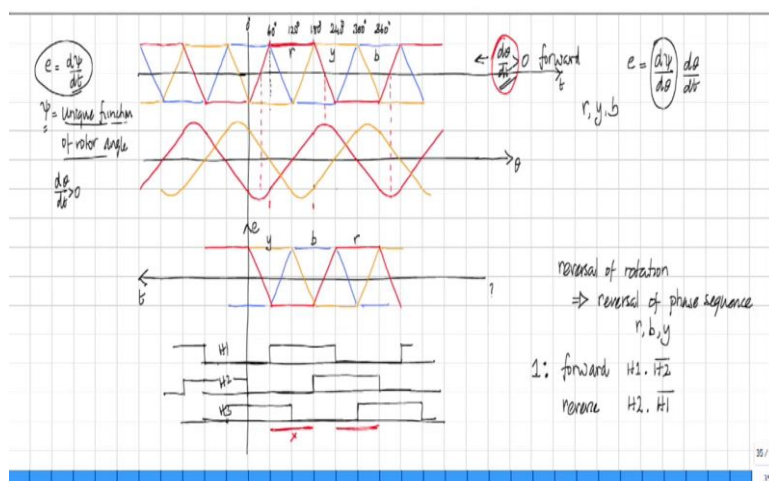
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In the case of a DC machine, if whatever was on the side after this line had you connected a DC machine here then, how would you reverse the direction of rotation you simply have the reverse the supply that is all but, unfortunately you cannot do that here, why? If you reverse the supply you have diodes that are connected here and the reverse supply will simply short circuit to the diodes and your whole circuit will simply blow so, it is not an option now, to reverse any sign of input voltage one has to figure out other ways of handling.

So, this is an AC machine and it is not a DC machine it is a brushless DC machine because, with the inverter from the DC source it looks like a DC machine that is all but, the motor itself is AC.

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So, to understand that, we need to remember one equation based on which we will develop, what is going to happen we should note that, the induced EMF is given by the rate of change of flux linkage there is a fundamental equation in physics so that is not too difficult and flux linkage is the unique function of rotor angle if the rotor is the one where you have the field that is being made, you have the field generating system located on the rotor and therefore, depending on how the rotor is rotor angle is then, that flux that is generated is going to depend on that and therefore, flux linkage is a function of rotor angle.

So, what we will do is first draw the induced EMF waveform that may be familiar with so, that waveform is going to be looking like this now, what I will do is change this so, I will make this as 60 degrees 0 120 180 240 300 and 360 I am drawing 60 degree intervals so, we are going to have waveform looking like this then, you have y then, we have b .

Now, if this is going to be the case given the fact that, the induced EMF is the $\frac{d\psi}{dt}$ what will be the waveform of flux linkage so, if you draw the flux linkage for the R phase red one in this interval the induced from 60 to 180 degrees the induced EMF is flat that means, that induced EMF is given by the derivative of the flux linkage function which means that, the flux linkage function should be of nature that is increasing in a linear manner.

Now, note that flux linkage that is induced EMF is given by rate of change of flux linkage with respect to time so, what we can write is e equals b side by $\frac{d\theta}{dt}$ multiplied by $\frac{d\theta}{dt}$ and we said that flux linkage is a unique function of the rotor angles which means that $\frac{d\psi}{d\theta}$ must be a unique function and that gets multiplied by $\frac{d\theta}{dt}$ which is then dependent on the direction of rotation if you are going to define that for anticlockwise rotation $\frac{d\theta}{dt}$ is defined to be more than 0 then for the other direction of rotation $\frac{d\theta}{dt}$ will be less than 0.

So, let us assume that we are going to be looking at the $\frac{d\theta}{dt}$ greater than 0 and that is this induced EMF is drawn for $\frac{d\theta}{dt}$ greater than 0 that is assumption that have made then, how will we draw the flux linkage waveforms, how will you draw the flux linkage waveform during this interval flux linkage should be linearly increasing with respect to rotor angle so, now what I am going to do is plot this with respect to rotor angle because, we are just going to multiply that by $\frac{d\theta}{dt}$.

So, during this interval then, you are going to have flux linkage linearly increasing and during this interval we are going to have flux linkage linearly decreasing obviously, because $\frac{d\theta}{dt}$ is negative similarly, here it is decreasing and here it is increasing what happens in this 60 degree interval there is an instant way $\frac{d\theta}{dt}$ is equal to 0 which means that the flux linkage sorry $\frac{d\psi}{dt}$ is equal to 0 which means that at this instance the flux linkage should have 0 slope and during this interval $\frac{d\psi}{dt}$ is still negative $\frac{d\psi}{dt}$ becomes greater than 0 only in this part and therefore, you will get a plot that is still negative here becomes 0 and then, starts becoming greater than 0 similarly, this plot will go like that and come over here where, this is the 0 instance this will then come like this and go where this is 0 instance and so on.

So, this will go like that, like this what will happen to the waveform of white that will also be very similar to this except that it is phase shift so, why will then be so, this is the region where it is negative and then, it is positive here, negative here so, let us not throughout the waveforms too much so, I am going to stop at this point.

So, now the question is that, we have drawn this induced emf for $\frac{d\theta}{dt}$ greater than 0 what will happen if $\frac{d\theta}{dt}$ is less than 0 now, $\frac{d\psi}{d\theta}$ is still going to be defined by this slope and they will now have $\frac{d\theta}{dt}$ less than 0 let us draw another plot so, the side by $\frac{d\theta}{dt}$ for the red one in the interval 0 in 60 to 180 degrees $\frac{d\psi}{d\theta}$ as greater than 0 but, $\frac{d\theta}{dt}$ is less than 0 that means, this induced emf will now be negative and here, it would be positive this will be positive so, you are going to have a waveform that is looking like this, what happens to y?

y will now be negative here y will be positive here and then, you have b that is going to come how will you mark the x-axis now this is still induced emf, what will be x-axis $\frac{d\theta}{dt}$ is negative that means, you are going to be moving along this direction see if this is the direction of increasing rotor angle but, if you are going to be rotating in the opposite direction you will have to be moving along this direction.

So, the induced EMF will have to be looked at as moving along this direction and there is now therefore, a difference in the earlier case where we plotted this with respect to time, this is now with respect to time because, the rotor is rotating in the opposite direction and what we notice is that, if you are going to have the induced emf greater than 0 for r phase that is

followed by the induced emf becoming greater than 0 for y and then, it is b whereas, now what you have is r phase is 5 followed by b followed by y.

So, the sequence in which you encounter greater than 0 induced emf now has reversed because, you are rotating in the opposite direction so, what we now need to do so, this means that opposite direction or reversal of rotation means reversal of phase sequence here, it was r y b sequence whereas, here it is r b y sequence.

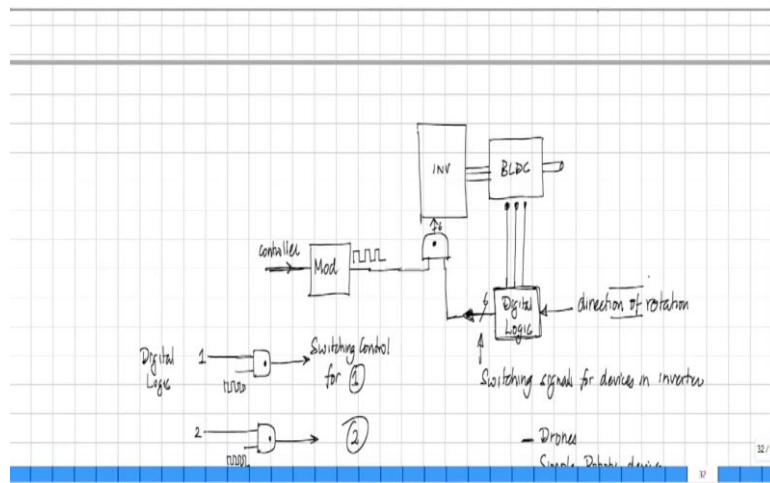
Now, we had hall sensors, hall switches that were put in order to detect the magnetic field and thereby give an indication to the circuit as to where the rotor is and we had put hall sensors and hall switches such that we had H1, H2 and H3 and we had put the position of H1 in the stator remember, the hall switches are positioned on the stator the rotor is rotating therefore, the hall switches would encounter varying magnetic fields as the rotor rotates and that is why it is able to indicate when the magnetic field under its influence becomes north or becomes equal south.

So, we had put the hall sensors in such a way that H1 was going high at the time when the induced emf or r phase was becoming flat so, this was the position of H1 and it remains high for 180 degrees and then becomes low this is how H1 would behave and H2 was at this position and H3 was further shifted by 120 degrees this was how it was and we said that, if you want to look at the region where device number 1 for example is on 1 would be on whenever r phase was high and so, this was the region when 1 is on as per the direction of rotation greater than 0.

But now, if you reverse the direction of rotation, where do you want 1 to be on 1 should be on whenever the induced emf becomes greater than 0 and therefore, the reason when 1 should be on now shifts it should now be on during this time and not here for reverse direction of rotation.

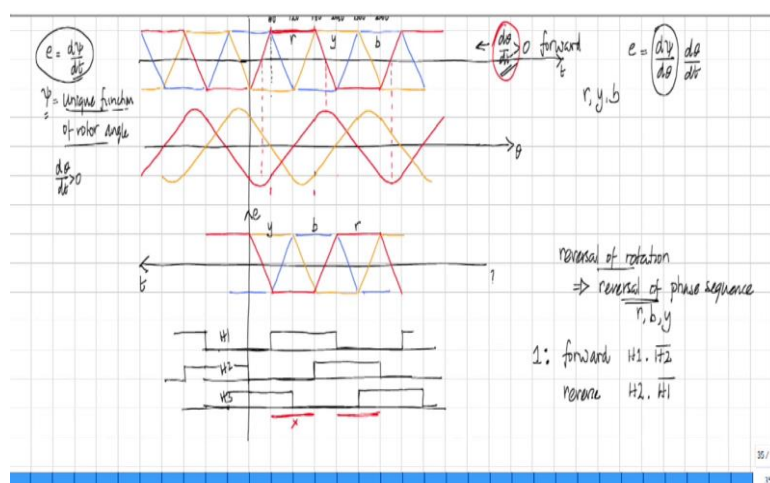
So, if you want to give the signals for switch number 1 for forward rotation, so, let us assume that, this is forward if it is forward then, we had written the function as H1 and H2 bar whereas, if you now want to reverse it what we need to do is H2 and H1 bar so, for reversing direction of rotation the signal to be generated using the hall switch outputs has to be redefined so, when we do the overall control structure, we said that there is going to be.

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We had a control structure that looks like this you had a digital logic and the digital logic accepts the hall switch inputs and then delivers the signals to be given to the devices. Now, this block requires one more input which is pertaining to the direction of rotation so, depending on what the direction of rotation is, you have to select an appropriate function which will determine when a particular switch has to be on so, for example, for switch number 1 you have defining functions which are either H1 and H2 bar or H1 bar and H2 which one will you select that depends on what is the direction of rotation you are going to have.

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So, based on that, then you have to select the switching instance so, reversal of rotation means reversal of phase sequence so, if you now write down the sequence in which the switches are going to operate, you will find that, the sequence is altered so, that is what will happen in this case so, please go through define that is formulate definitions for all the six devices formulate these explorations for both directions of rotation and then see what has happened. So, I will leave that, as an exercise to you do it.

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- Ripple Torque
- Need rotor angle information since in 60°

of rotor angle
 $\frac{d\theta}{dt} > 0$

reversal of rotation
 \Rightarrow reversal of phase sequence
P, b, y
1: forward H1, H2
reverse H2, H1

- Ripple Torque
- Need rotor angle information since in 60°

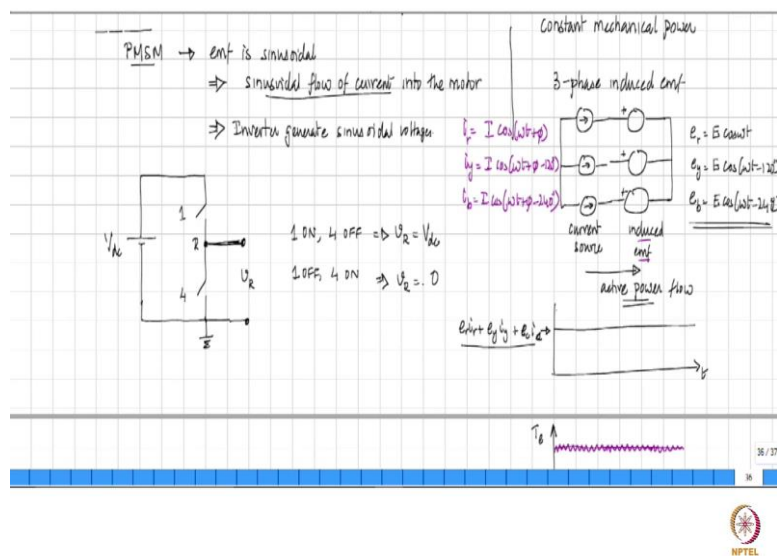
Now, as I mentioned in the last class, the brushless DC motor has a ripple torque however, what is interesting about the brushless DC motor is that in order that you do closed loop control, in order that you define what switches need to be turned on, you only need rotor

angle information once in 60 degrees that is you require the rotor angle information whenever, a particular EMF waveform becomes flat.

So, you need information regarding whether this specific rotor handle has been reached, if it has been reached than you need to know that the switch pertaining to this region, that is the r phase will need to be turned on. Similarly, after this you need information when this angle is reached, then you need information when this angle is reached. So, it is enough if you know that the rotor has reached a position where an emf is becoming flat.

So only once in 60 degrees it is sufficient if you know the angle. Because in the interval of 60 degrees the circuit simply looks like a DC cirpute, DC current is suppose to flow there is nothing you need to specifically control. Because the only thing that you need to control is the magnitude of DC voltage to be applied that you are already doing by having a high frequency switching wave form which you use to define the duty ratio during that time as we discussed in the last class.

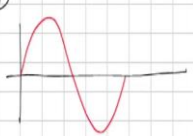
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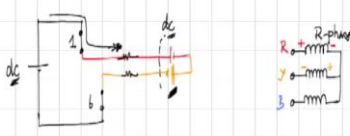
\rightarrow $\frac{1}{2}\pi$ $\frac{3}{2}\pi$ $\frac{5}{2}\pi$ $\frac{7}{2}\pi$ \rightarrow Kipples torque \rightarrow frequency depends on speed.
 Machines 1000

Permanent Magnet Synchronous Machine (PMSM)

- Sinusoidal emf




BLDC 1, 6 are ON



R-phase
 Y-phase
 B-phase

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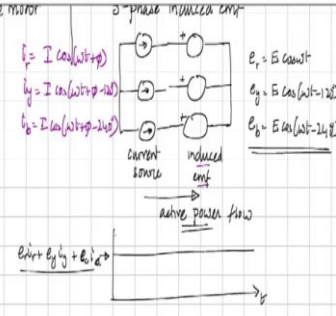
NPTEL

\Rightarrow sinusoidal flow of current into the motor

\Rightarrow Inverter generate sinusoidal voltages

$i_f = I \cos(\omega t + \phi)$
 $i_y = I \cos(\omega t + \phi - 120^\circ)$
 $i_b = I \cos(\omega t + \phi - 240^\circ)$

3-phase induced emf




$E_1 = E_m \cos \omega t$
 $E_2 = E_m \cos(\omega t - 120^\circ)$
 $E_3 = E_m \cos(\omega t - 240^\circ)$

$E_1 i_f + E_2 i_y + E_3 i_b$

T_g A

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NPTEL

But, when we go to a PMSM, that is a synchronous machine as it is called, we said that the emf is sinusoid. Now if it is going to be sinusoidal this poses some difficulty that first so the emf is sinusoidal which then means that you need to allow sinusoidal flow of current into the motor is necessary.

Why sinusoidal flow of current is necessary that is because as we have saying all along you want a constant mechanical power output form the machine and if you have a three-phase induced emf that means you have, let us say three sources which are looking like this. Let us say they are connected at one end and these are emf sources than if you send three flows of current, if you do this, this is a current source, this let us say induced emf.

So from the source to the induced emf is active power flow. How do you find out how much active power flows in this electrical circuit? So, RMS is one way but RMS means that you are looking at one cycle of the wave form. And then you take a RMS of that, RMS does not say how the wave form is going to be with respect to time. RMS is a sort of lumped array, lumped information about the entire thing leaving out the details of what is happening. But what we want is that the active power flow with respect to time should be constant; this is our desire.

And by circuit analysis what one can understand is that if they induced emfs are of the form e_r is equal to $\sum E \cos \omega t$, e_y is equal to $\sum E \cos(\omega t - 120^\circ)$ and e_b is $E \cos(\omega t - 240^\circ)$. If this the induced emf set, such induced emfs are called as balanced induced emf where the magnitudes are all the same, amplitudes are the same and they are all well defined sinusoids and they are phase shifted by 120 degrees.

So, if the induced emf form a balance set than if the flow of current is such that i_r is $I \cos(\omega t + \phi)$ and i_y is $I \cos(\omega t + \phi - 120^\circ)$, that is 120° . And i_b is $I \cos(\omega t + \phi - 240^\circ)$. So, if you have such a set than if you draw the wave form of e_r into i_r plus e_y into i_y plus e_b into i_b , note that I have used lower case in all these things that means we are taking instantaneous values instant to instant. Every instant you take the value of e multiply by the corresponding value of overall the phases add all of them up.

So, we are talking about instant to instant variation. If you plot this, this will be the nature of wave form. Therefore, if you are going to have a motor with a sinusoidal emf than it is necessary to send a sinusoidal flow of current into the motor so that the mechanical output power is flat with respect to time. If mechanical output power is flat with respect to time then it means also that if you draw, if you draw electromagnetic, electromagnetic torque that will also be constant with respect to time for constant speed operation.

Because of this you want the flow of current to be sinusoid and if you want sinusoidal flow of current into the source, into the induced emfs. What should be the nature of output voltage generated by the inverter, that should also be sinusoidal. This therefore demands that the inverter generate sinusoidal voltage.

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For every 60° , one pair of switches are on
 Once in 60° , one switch is turned off & another is turned on
 Switches in each 60° interval can be operated in high-f ω PWM
 to control motor voltage \Rightarrow speed

$E_g > 0$
 In each 60°
 V_{avg}
 $2A$
 Very duty ratio \rightarrow

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\Rightarrow Simultaneous flow of currents into the motor
 \Rightarrow Inverter generate sinusoidal voltage

3-phase induced emf
 $i_1 = I \cos(\omega t + \phi)$
 $i_2 = I \cos(\omega t + \phi - 120)$
 $i_3 = I \cos(\omega t + \phi - 240)$

$E_1 = E \sin \omega t$
 $E_2 = E \sin(\omega t - 120)$
 $E_3 = E \sin(\omega t - 240)$

current source induced emf
 active power flow

$E_{11} + E_{22} + E_{33}$

$T_e A$

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You know the inverter is, the inverter is this circuit where is that circuit, this circuit we have already said this is the inverter. Now the problem is that the inverter consist of switches and it cannot generate a sinusoidal voltage, it cannot generate a voltage wave form that vary smoothly over time, it is not possible.

So again, like what we said before when we say sinusoidal what we really mean is that if you take the switching wave form generated by the inverter on an average it looks like a sinusoid. That is what we really have therefore the inverter will generate a sinusoid plus lot of switching repel. And therefor you will not get a wave form that is smooth electromagnetic torque like this, what you will get is a wave form like that.

And we already concluded that this wave form is acceptable, provided you are able to keep the amplitude small and the frequency of the ripple high. So, the question then is how does the inverter generate this. So if you take, for example this particular leg. Let us take one leg of the inverter and you consider the voltage at this point with respect to this potential. So, let us call this node as R and let us designate this ground. What will be the voltage V_R at any instant of time, what will be the voltage V_R ?

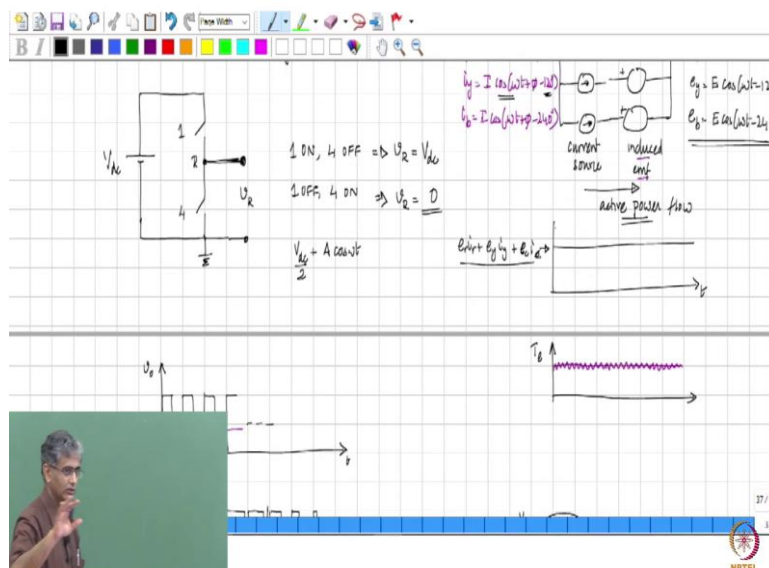
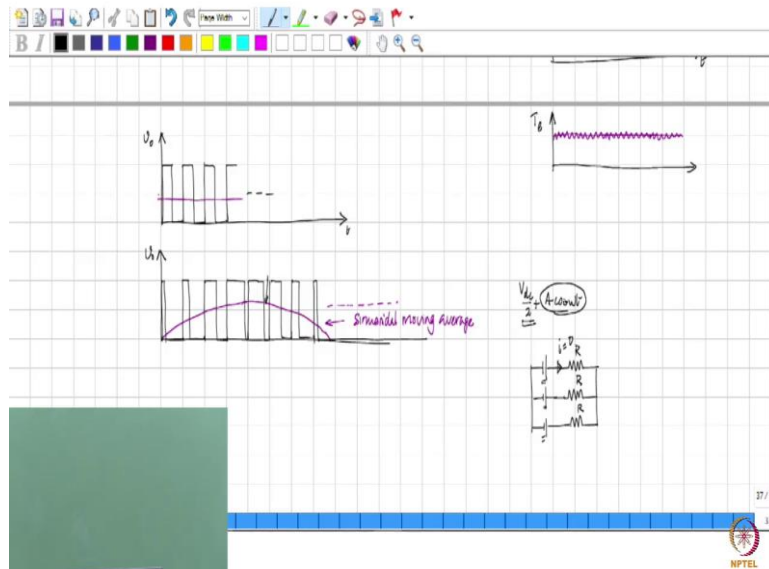
The switches have to be operated necessarily that either this switch is on or this which is on, you cannot have at any given instant of time both the switches being on because that will short circuit resource. You can of course have a situation where both switches are off, nothing will go wrong. But if you leave both switches off, then you do not know what this potential is, so nobody operates it like that.

So, the switches are operated such that this potential is always well defined, which means that at any given instant of time either the switch is on or this which is on and if one of the switches are on the other one is necessarily off. So, that is the manner in which the switches are operated, which means what will be the potential of this node R with respect to ground. What do you think it will be?

So, you have these two situations, call this as which one and let us say this is switch four, we have situation one is on four is off or one is off four is on. These are, these are the only states that are allowed. So, in this case, what is V_R , so let us say this is equal to VDC, V_R will be VDC. In this case what is V_R , 0. So, V_R will either be VDC or 0, that is all. So, you can never get a negative voltage of this, it is not feasible. You cannot go to a voltage lower than 0 because the structure does not allow it. But if you want to generate a sinusoidal voltage you have to have negative voltage, voltage has to be negative.

So, what you do is you do not generate a sinusoidal voltage the 0 mean, you generate a sinusoidal voltage such that it is VDC plus some $A \cos \omega t$, VDC by 2 plus $A \cos \omega t$. So, you are generating a level shifted sinusoid, that means the output at this point will generate a DC quantity plus a sinusoid of certain amplitude at the low frequency entity

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So that means, if you know, we have seen in the earlier lectures that if you are going to generate a wave form, some V naught with respect to time, which looks like this and so on. This contains an average value which can be realized as, you take the average of this and that is going to be the average.

But now instead of this, suppose you develop a waveform which is like this. What will happen to the average, the average will be a waveform that increases, goes to some maximum and then what I do is generate something like this. So now the average goes down and goes towards 0. So, now what we are doing is we are generating a sinusoidal moving average. As you move the window along, then you find that the average defined over one cycle of switching is varying along this is, along this x-axis and that variation can be made to define a sinusoid.

But nevertheless, you cannot get negative variation because the topology limits you from generating negative voltages. And therefore, if you generate not just the sinusoid but the sinusoid shifted by a DC offset and you define these variations in accordance do that shifted by a DC offset, then it is feasible to generate a sinusoid completely shifted by that offset. So, you will have the full cycle being defined like this. And therefore, what you get is V_{DC} by 2 plus some $A \cos \omega t$ generated at this node R.

How do you now generate another output voltage which is phase shifted by 120 degrees, all you need to do is phase shift this sinusoid by 120 degrees for the next phase and further by another 120 degrees for the next phase. Therefore, you are generating three sinusoidal voltages of same amplitude phase shifted by 120 degree each. And you are also adding a DC component to it how will that affect the rest of the circuit; will you see a DC current also flowing in addition to sinusoidal currents.

Let us say that you have a circuit and consider resistors here. All these DC voltages are equal. And let us say, all these resistances are also equal. How much will be the flow of current here, zero current will flow. And therefore, even though we are generating DC voltages here since the DC is same on all three phases it will not pass a DC current into the circuit, into the motor what will flow is only a flow of current due to this excitation. And therefore, we achieve our goal of creating constant active power with that switching is.