

**Introduction to Robotics**  
**Professor. Krishna Vasudevan**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Madras**  
**Lecture No. 21**  
**The H-Bridge and DC Motor Control Structure**

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$V_g$   
 $L_m$  Armature Ind.  
 $E_b$  Induced EMF  
 $L_m \frac{di}{dt} = V_g - E_b \Rightarrow \frac{di}{dt} = \frac{V_g - E_b}{L_m}$   
 $L_m \frac{di}{dt} = 0 - E_b$   
 $= -E_b \quad \frac{di}{dt} = -\frac{E_b}{L}$   
 Mag & freq of ripple is in your control  
 $\Delta i = \frac{V_g - E_b}{L} \cdot t_{on}$   
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So, in the last class we looked at the circuit as we have drawn and we found that one can determine what is the magnitude of this increase that is, if you call this as delta i, then delta i is the magnitude of the ripple.

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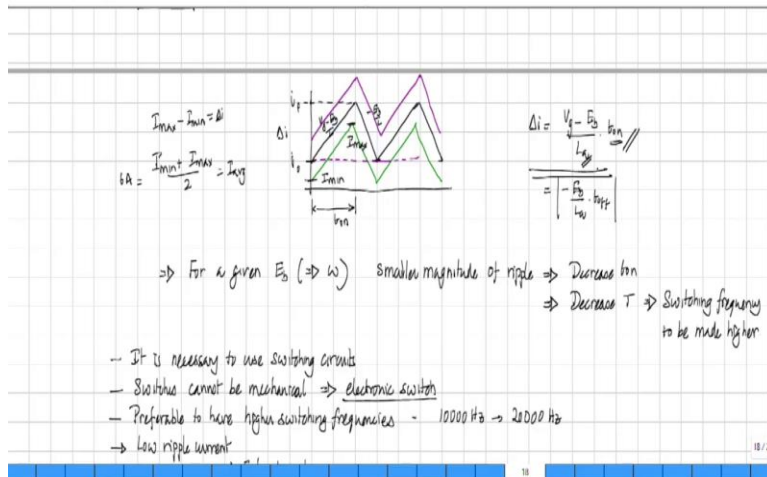
$L \frac{di}{dt} = V_g - E_b \Rightarrow \frac{di}{dt} = \frac{V_g - E_b}{L}$   
 $L \frac{di}{dt} = 0 - E_b = -E_b \Rightarrow \frac{di}{dt} = -\frac{E_b}{L}$   
 Mag & freq of ripple is in your control.

$\Delta i = \frac{V_g - E_b}{L} \cdot t_{on}$   
 $= \frac{E_b}{L} \cdot t_{off}$

And one can determine the magnitude of the ripple if you know what is the duration for which the switch is on and the value of the inductance and the difference between the supply voltage and the induced EMF. So, this only determines the magnitude of the ripple, but you have no idea whether it is going to be a waveform like this or is it going to be a waveform like this. In either case, the magnitude of the ripple in all these 3 cases is the same, the only difference is the average value.

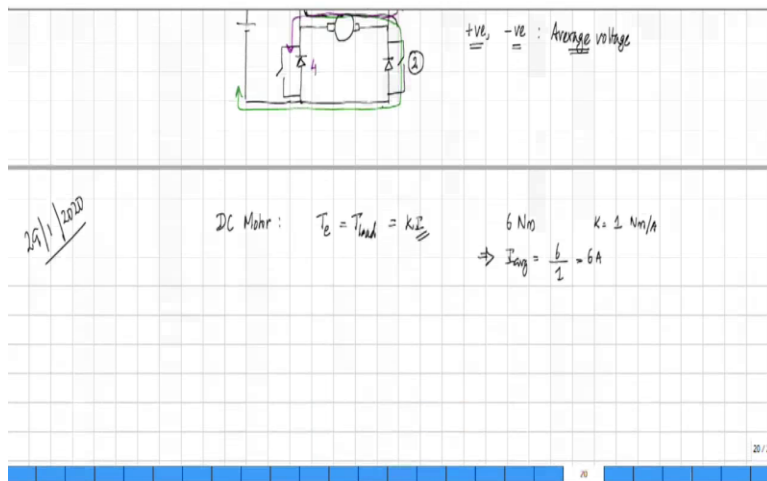
These equations that we wrote in the last class do not say about what average ripple value is going to be. How do you find out what will be then be the average value? No, because if you know the current waveform, then you can find out the average value, area of the triangle divided by the duration. But these equations do not tell you anything about what the area of the triangle is also going to be, this equation only says that the change between the minimum and maximum value of current  $\Delta i$  is determined in this fashion. So this is, this information is incomplete.

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So, in order to determine what the average value is going to be, you need to understand that this input voltage or applied voltage is being given to a DC motor.

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And we need to understand what the DC motor will do? The DC motor or any other motor will always attempt to meet its load requirement. So, if it so happens that the motor has to rotate a particular load, demanding a certain amount of mechanical torque to be developed at this particular RPM it will attempt to generate them and it is from that you can determine what the average value should be.

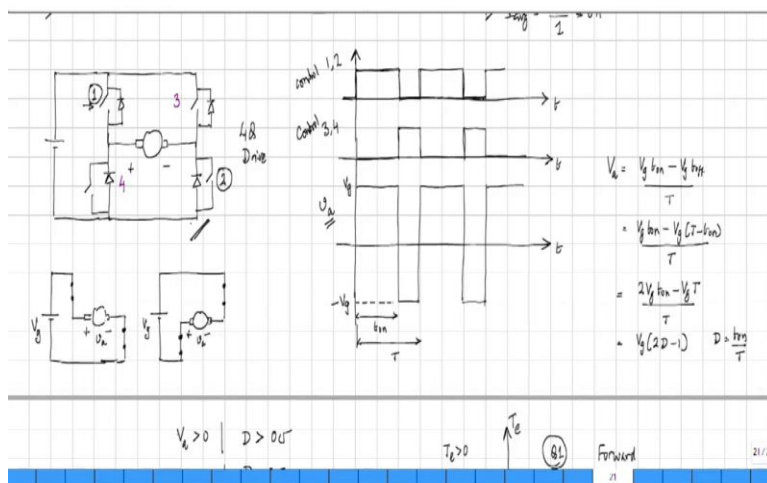
And we know of that, the electromagnetic torque which the motor generates at the specific speed of operation must be equal to the load torque, this we have already seen from the graphs that we drew and this is equal to K times I. This I is the average value of flow of current Armature current into the DC motor and therefore the average value of flow of current is determined by what is the load torque required.

So, if the load torque is, that the machine needs to supply to the load for that particular speed of operation is equal to 6 Newton meter and the value of K is equal to 1 Newton meter per ampere. Then, average current will then be equal to 6 divided by 1 which is 6 ampere. Now on top of the 6 ampere you are now going to have this ripple.

So, obviously as you can see from this shape if this value is going to be I min and this value is I max then the average value is simply I min plus I max divided by 2, that is the average value because that is seen from the wave shape of this graph. So, we are now saying that this average value is equal to 6 ampere and then we also know what is I max minus I min which is delta I. That we know from this equation, that we have written.

Therefore making use of these two equations one can determine this waveform fully. That means you can determine what is I min? What is I max? What is average? All that can be determined. So, the average value needs to be found out from what is the requirement from the motor.

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Now, one must then understand if we are going to have a circuit like this that we have drawn yesterday. What is the average value of the applied voltage that is going to come across the motor? In the earlier case, we saw that the average value is simply equal to  $V_g$  multiplied by the duty ratio, what happens in this situation? So, let me copy this. So, this is the circuit that we have that means we have, that is the circuit, we have switches 1 2 3 and 4. Now let us say that I am going to operate the switches in this manner.

Switches 1 and 2 and then switches 3 and 4 that is control signals for 1 and 2, control signals for which is 3 and 4. This is drawn with respect to time and what I want to draw is the waveform of the voltage across the motor.

So, we will figure switches 1 and 2 in this manner, this is the waveform and the control for 3 and 4 goes like this. That means what I am doing is, switching on 1 and 2 at the same time and when 1 and 2 are on, switches the other two switches are off and when 1 and 2 are off the other two switches are on, that is the manner in which I am going to operate. One may operate it in many different ways. But this is what we take, let us take this as an example.

So, if this is the case, what will be the wave shape of  $V_a$ . So, if 1 and 2 are on, then it means that the circuit reduces to one that looks like this, you have 1 is on and then you have the motor and then 2 is on and the voltage across the motor that we are looking at is this  $V_a$ , so how much will be equal to  $V_a$  if this value is  $V_g$ ?  $V_a$  is equal to  $V_g$ , it is simply connected to across  $V_g$ , so that means the voltage here is  $V_g$  during this time and in the next interval, what we are doing is switching off 1 and 2 and the other two switches are coming on which means that the circuit now looks like this you have  $V_g$  and then you have 3, you have the motor and then 4 is on.

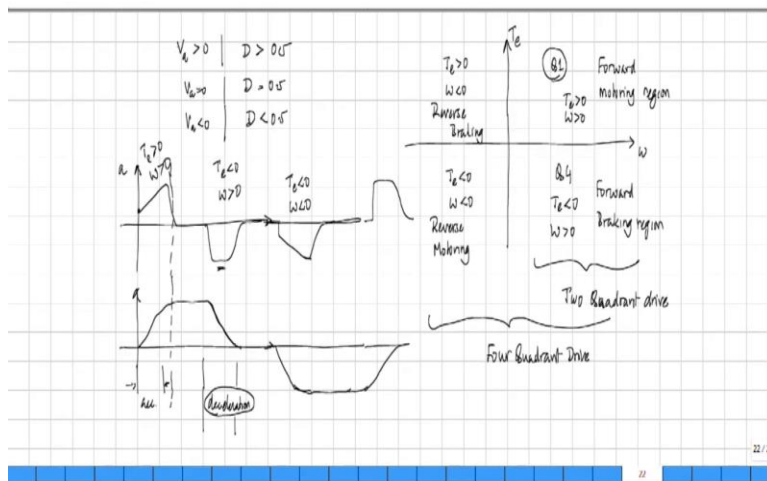
This is  $V_a$ , here  $V_g$ . Now how much is  $V_a$ ? Minus  $V_g$ . So, you now draw minus  $V_g$  and after this the switching pattern repeats. So, you keep going like. So, let us talk this duration as  $t_{on}$  and this duration as capital  $T$ . So, the level here is  $V_g$  and the level here is minus  $V_g$ . What is now the average voltage that is coming across the motor?  $V_a$  which is the average value of this lower case  $V_a$ , this may form as given by  $V_g$  into  $t_{on}$  minus  $V_g$  into  $t_{off}$  divided by capital  $T$ .

We are just taking the area of the graph, which is  $V_g$  into  $t_{on}$  minus  $V_g$  into  $T$  minus  $t_{off}$  divided by capital  $T$  which is two times  $V_g$  into  $t_{on}$  minus  $V_g$  into  $T$  divided by capital  $T$  which is equal to  $V_g$  into 2 times duty ratio minus 1. Where duty ratio is  $t_{on}$  by  $T$ . Now you have a

different expression for the average voltage that is applied, so the value or the expression for average voltage is not something that is fix, it depends on what sort of converter is being used and how it is going to be operated. In this case, there is the expression that you get. Why this converter is interesting?

Is that with this expression if you want the average voltage to be greater than 0, how can you make a greater than 0?

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Duty ratio must be greater than 0.5. What happens for duty ratio equal to 0.5, you get average value equal to 0, what happens for duty ratio less than 0.5.  $V_a$  is negative, therefore you get a variation in the average value, all the way from negative to greater than 0 by simply varying the duty ratio without any discontinuity have happened, smoothly it can be very and therefore you can now get this motor to rotate in either direction or you can bring it to 0 speed as well. So, that is why this topology is very advantageous.

Usually, whenever you talk of electrical motor drives one refers to whether if you draw a graph like this, where the X axis is P, Y axis is the generated electromagnetic torque. Then if you denote this operation here. Now let us say you are going to have a robotic arm. That is a nice example. So, you have a robotic arm that is to pick up an object from here and go and put it there that means the motor has to rotate in a certain direction it will have to go like this and then come down.

Now obviously the motor you may want it to be increasing speed all along and as you reach the place where it is going to be put down, the speed has to be decreased and it has to come to 0 right at the point where the object has to be released. You cannot have a non-zero value of speed when the object is going to be located there, because if you are going to go with the non-zero value of speed and keep it there you are creating a mechanical impact and it is likely that the object will break or if it is going to be strong enough, it may not break, it may withstand.

But in general, we would like the speed to go to 0 just at the point where you are going to put it down. That means you are going to have during the, during the operation if you are going to plot the acceleration, that is there in the electric motor the acceleration would perhaps start at a non-zero value, at the beginning and then this acceleration may increase.

After sometime maybe if you went to pick up the object from here, you have accelerator then maybe you move with a fixed velocity. So, then acceleration could perhaps become 0 and then you go with 0 acceleration and when you are going to put the object down you have to decelerate and then go to 0 velocity. So, after some time you may have to have a negative acceleration and maybe maintain the negative acceleration and then go to 0 acceleration, when you place the object at the desired location. So, this is acceleration.

As far as looking at speed is concerned, you know how to derive speed from acceleration. You can do some integration and get it. But the speed will then have to start from 0 in some manner and then the speed remains fixed and then the speed goes down and bring with 0. So, in this duration you are having a deceleration. In this region, you are having acceleration. So, you need to have a circuit here. This for example this circuit. Maybe it could be as an alternative it could be a circuit like this. You must see whether the circuit allow these kind of modes of operation. For example, this circuit will not allow you and ability to decelerate in a well determined manner.

So, what you want is in the first case, that is the in this region, you want to operate such that electromagnetic torque is greater than 0,  $\omega$  is also greater than 0. So, you are really operating here Q1 both  $\omega$  greater than 0 and electromagnetic torque greater than 0, you can call this as forward motoring region. Now in this zone, speed is still a value greater than 0. But your acceleration is negative. How does the acceleration become negative? Acceleration is negative only if the generated electromagnetic torque becomes negative.

And therefore, in this region of operation, you have a situation where  $T_e$  is less than 0 and  $\omega$  is still greater than 0, therefore you are operating in Q4  $T_e$  less than 0 and  $\omega$  greater than 0, this is then called as forward braking region. An electric circuit that is able to achieve these two regions of operation is then called as a two-quadrant drive.

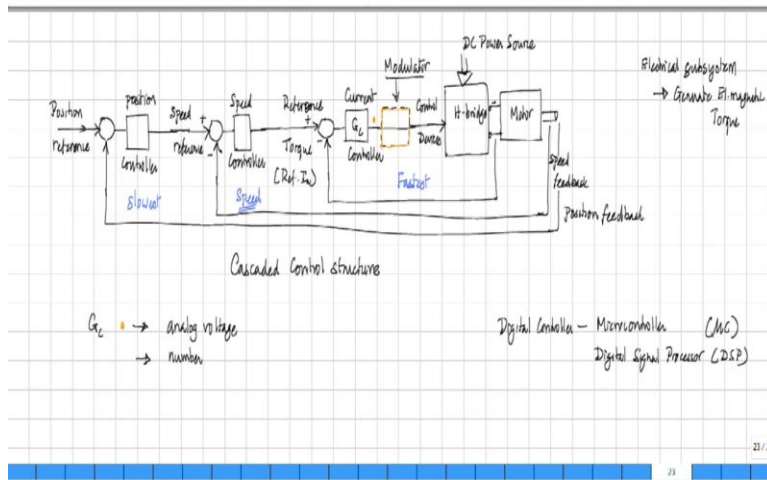
Now after you do this, this arm has to, having left the object there it now has to go back to the earlier place to pick up the next object and how does it go back to the next place, it now has to accelerate in the reverse direction speed increases in the negative direction. So, you still have now an acceleration in the reverse direction and then maybe go with the fixed velocity decrease acceleration and then after some time you may want to break that and then go something like that speed will then increase in the negative direction.

So, go to 0 and therefore after coming here the speed remains fixed and then at this point the speed now starts to decrease and then go to 0. So in this region, then  $T_e$  is less than 0 speed is also less than 0. This means you are operating here  $T_e$  less than 0 and speed less than 0. But this is really making the motor rotate in the other direction. So, this is reverse, it is reverse but acting as a motor and then here is where you have the  $T_e$  greater than 0, but  $\omega$  less than 0 this is reverse braking.

So, a circuit that allows you all these 4 reasons of operation, is then if you have everything available that is called as a four-quadrant. So, this circuit which we discussed last this circuit is really a four-quadrant drive capable of rotating the motor and braking the motor in either direction. So, having seen that, what we now need to understand is what is the overall motor drive closed loop system going to look like.

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So you have the motor, this is motor you have to input, that is one input port that is electrical input port and you have the shaft which is the mechanical output port and to this electrical input port you connect a drive circuit. This is your H-Bridge and this is H-bridge gives gets a DC Supply, DC input power source and this H-bridge requires, in order for the switch to operate you require signals to control devices.

That means it needs to be given explicit signal stating when the switches should turn on, when it should turn off, that is what you required. How do you get that? Now, this break this all is going to apply a DC voltage to the motor and when you apply a DC voltage you note that you are applying an electrical input.

The DC machine has an electrical subsystem and then a mechanical subsystem systems, the electrical subsystem is the part that contains all the armature windings and all that and as a result of flow of current intake, there is an electromagnetic torque which acts on the mechanical subsystem which contains the rotor inertia and whatever else is connected to. Therefore when you apply an electrical signal that is an applied DC voltage, what responds first, the first respond will be there from the electrical circuit.

So, the fastest responding element in the DC motor is the electrical circuit, the electrical subsystem let us say, that is the one that responds first and therefore unless you have a good hold on the behaviour of electrical subsystem, your overall response will not be very good and therefore and this electrical subsystem is the one that is responsible to generate electromagnetic,

electromagnetic torque. If you look at the behaviour of the motor itself in order to make it move, the first thing that it has to generate is an acceleration.

If it cannot generate an acceleration, there is no way the rotor is going to rotate and therefore if you want to control the movement of the motor the first variable that you need to control is the acceleration or the electromagnetic forces that are happening in the machine and therefore the first thing that one does in this motor control system is that you require in some sense a reference, reference value for torque and in the case of DC machine since you know that  $T_e$  is equal to simply  $K$  multiplied by  $I$  this is also reference Armature flow of current  $I_A$ , same as that.

And this reference torque or reference  $I_A$  is then compared with the actual Armature current that is flowing from this and this determines at any point of time, what is the acceleration required in order that you go where you want probably at the fastest time or whatever and this difference therefore determines, this difference determine, how much Armature voltage needs to be applied.

Because if you want more torque, it means that you need more Armature current that your somehow the reference is more than what is actually flowing that means you require more torque and therefore, it means that you need to apply more voltage to the motor and this is therefore, this input needs to be given to the bridge. This is then a current controller.

We need to put something else here. We will come to that a little late. But now the question is, how do you know, how much reference torque has to be generated? That will usually come from something else. In the operation, you will not be determining the acceleration of the motor as it is, you may be determining the speed. For example, if you are going to move something from, if you need you have a mobile application, let us say and you want this to move from there to there. You will say move with this much speed.

So, speed is most probably the variable that as a user you will be likely to give and acceleration is something internal to the system. So, how does you how do you know then, what acceleration must be given or what reference for must be given, that comes from the difference between the speed reference and then you have a feedback of the actual speed at which the motor is running.

This is speed feedback and once you know, what is the difference between what you want as speed and what is the speed now. Obviously if the speed difference is large, what you want and

what the motor is now rotating you would like to accelerate more and reach that value of speed. So, depending on this difference you then have a controller that determines how much should the acceleration be. So, this is then a speed controller.

There are many applications, where speed is the reference value that is given by you directly, especially in application where you want something to move along a horizontal axis or so you will say move with this speed, but there are also many applications, where you cannot determine the speed. For example, if are going to have a robotic arm, that is a very nice example to say that, lift this object from here and place it there you will not be giving the reference in terms of speed.

Because you would like this object to move in a particular manner specified locations with respect to time and then place it exactly there, that means the reference that you give will not be in terms of speed but actually in terms of the actual location of the object. How that is varying with respect to time that means with respect to time, you will see at this instant I want the object to be here, at the next instant I want the object to be there that means what you will give is actually a position reference.

When I say reference, it does not mean it needs to be a fixed value, position reference may vary depending on where you want it to be and therefore what you do is, you take the position reference and then you get a feedback of the rotor position, position feedback. Note that even if you are looking at a robotic arm, which is going to take an object move it in space and then put it down somewhere, the entire motion of determining where the object is at a particular location in space at a particular instant can be equivalent map back to the position of the motor shaft.

Because the motor shaft is what is going to cause rotation the motor shaft link with certain mechanical linkages is going to result in the motion of this object and therefore you can map that back to the shaft angle and that is what this motor control system will see. And depending on this position error, then a controller determines this is then a position controller, this controller then determine with what speed it has to operate.

If the error is large then to go to that position you need to go the greater speed, reach that position which in turn means a certain acceleration and a certain acceleration means a certain flow of current which then means a certain voltage. So, all that is then subsequently addressed.

So, this is the sort of control structure that you would need for a motor drive for these kind of actuation application. So, this is called as a cascaded control structure.

In this fastest way, fastest system variable is attempted to be controlled first and the fastest system variable is the flow of, flow of input current into the armature, that is an electrical system and electrical systems are usually much faster than mechanical systems, mechanical system have a moment of inertia, which is a big mass and it will take time for it to start rotating and move. So, fastest variable is controlled first, then you control the next faster variable, which is speed, which is an intermediate is neither too fast not too slow.

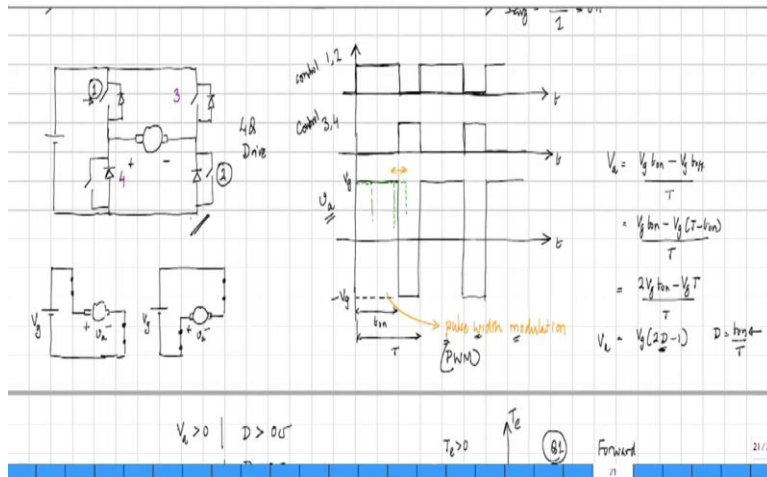
And then finally you have the outer most loop which is the slowest variable, this is speed is something that varies much slower than armature current. Now comes this system which is there in between here that is this one, what does this block do? Now the output variable that is given at this position. This is, this is what this block let us call it GC the output of GC is going to determine, how much voltage needs to be applied to the armature in order to get so much current flowing.

So, this variable at the output, if it is a analogue circuit implementation it is a simple analogue voltage, if you are going to implement this whole thing inside a digital system, a digital system, a digital controller, like a micro controller or a digital signal processors, so this is usually represented as new mu C and this is usually called at DSP. Now in this case this output variable will simply be a number.

You are going to do some arithmetic inside, implement some equations etc finally you get a number. That number represents how much voltage should be applied to the machine. Now, there is obviously a disconnect between, what the output given by GC is and what needs to be given to the bridge, what needs to be given to the bridge, is a signal saying switch on, remain on for sometimes switch off. It means the switching control signal with whereas, what GC gives you is a number or an analogue output voltage.

Therefore, you need something to convert this into the signals that the bridge really require and that job is done by a block known as modulator.

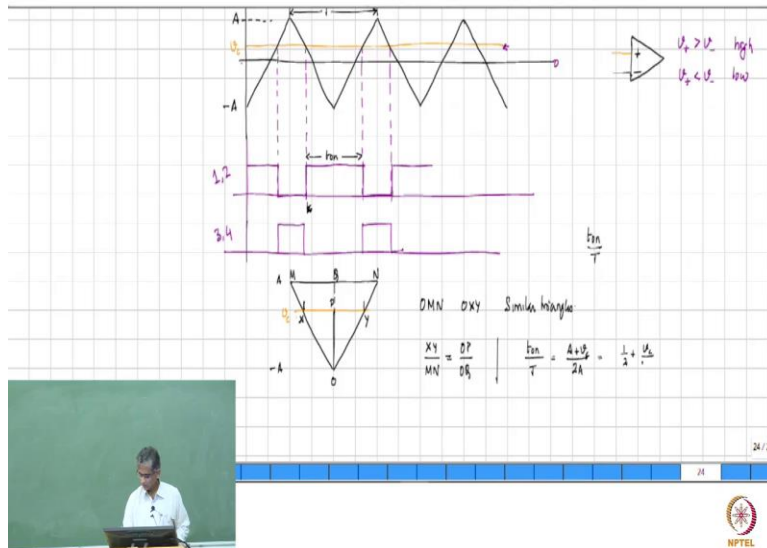
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So, if you look at this and let us say you want to vary the output voltage given to the armature, what you will vary? This is the average voltage, you are going to vary therefore duty ratio and duty ratio variation means you are going to vary  $t_{on}$  you are not going to vary  $T$ , though that can also be done if  $t_{on}$  fixed and varied  $T$  that will also result in duty ratio creation. But that is a nonlinear variation, because we have one over  $T$  sitting there. Whereas, if you keep  $t_{on}$  fixed and vary  $T$ , then it is a simple scalar relationship.

Therefore, what you would like to do is that you would like to vary the occurrence of this edge, such that maybe you would want to at some point control it to switch off here or at some point control it switch off here or you may want to switch off here, depending on that. Which essentially means that you are varying pulse width is being varied or in other words this width is now being modulated and therefore, broadly this scheme is known as PWM, Pulse Width and then Modulation. So, that is why this block is given this way. How do you do that? That is again very simple.

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What one does is let us say that you have a triangular voltage waveform, this is triangle and I got a Y axis here and this let us say if 0 of the triangle and this goes up to an amplitude of A and here it is minus A and then what I do is, we said that the output from GC, let us say it is an analogue output voltage, then let the voltage be somewhere here. Then what we do is you take this to a simple comparator. The comparator is a way of comparing these two signal.

So, here you give this value and then you give the other one here. The job of the comparator is to implement a very simple function if V plus is greater than V minus, output is high, if V plus is less than V minus, the output is low. So, if this is implemented what will be the output waveform from this comparator, this comparator will look at these points, places of intersection and then in these region, it will give a high signal and then it gives a low signal, then it gives a high signal, low here, high and low.

And then what we can do is, feed this variation as inputs to switches 1 and 2 and then give the inverse of this as inputs to switches 3 and 4. This is to switch 3 and 4 and therefore we have the switch pattern that we had drawn here, something very similar to this and the system will then work well. Suppose the value of this analogue voltage is equal to 0. How much will the duty ratio of these signals be? Half, which means the armature voltage applied is 0.

So, the reference that is the orange line, the level of that orange line is directly indicative of what is the magnitude and the sign of the Armature voltage that is then applied. So, this is a mechanism that allows you to convert the analogue output voltage is given by DC into signals,

that are given to the bridge for it to operate. So, if this value is  $V_c$ , then  $t_{on}$  is given by this interval and  $p$  is given by, this duration is capital  $T$ . How do we now find out what is the duty ratio  $t_{on}$  by  $T$ ? So, that can be found out by looking, we take this waveform of the triangle. So, this varies from  $A$  to minus  $A$  and then we have the DC level somewhere.

This is the DC and therefore what we have is the triangle here. So, let us call this as  $O M N$  and  $XY$ ,  $O M N$  and  $O X Y$  are? What are they? Similar triangles and we need to find out what is this duration? So, it is evident that  $XY$  divided by  $MN$  is equal to this high, whatever that is call it a  $P$  and let us extend this and this is  $Q$ . That is then  $OP$  by  $OQ$ . Which then means  $XY$  is nothing but  $t_{on}$ ,  $MN$  is nothing but  $T$  and  $OP$  is  $A$  plus  $V_c$  and  $OQ$  is 2 times  $A$ . So, this is nothing but half plus  $V_c$  by 2  $A$ . Now this term  $V_c$  by  $A$  is then given a called modulation index.

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$\frac{V_c}{A} \rightarrow$  modulation index  $m_a$

$$D = \frac{1}{2} + \frac{V_c}{A} = \frac{m_a + 1}{2}$$

$$V_g = V_g \left[ 2 \left( \frac{m_a + 1}{2} \right) - 1 \right] = V_g [m_a + 1] = m_a V_g$$

DC Motor Control Structure: Brush - Graphite

Brushless DC Machine BLDC

So, if you call this as  $m_a$  then duty ratio is half of half plus  $m_a$  by 2 or  $m_a$  plus 1 by 2. So, what can be done is this expression, what we wrote as  $V_a$  is  $V_g$  into 2 times  $D$  minus 1 instead you can cast it in terms of modulation index.  $V_a$  is therefore  $V_g$  into 2 times  $m_a$  plus 1 by 2. Minus 1, which is  $V_g$  into  $m_a$  plus 1 minus 1. Which is modulation index into  $V_g$ . So, one can therefore see that the output voltage is a linear variation of this.

So, in this manner one can then attempt to control the entire system. This is how the closed loop control system of the drive looks like. One then has to look at how to design these controllers, which is a subject matter, we will not discuss in this course, but that is what it is. Now in this

whole thing, there are certain important aspects that you need a mechanism to measure the speed. You need a mechanism to measure the actual position of the shaft and you need a mechanism to measure the flow of armature input current, those are additional stuff that one might need. So, this is the way a DC motor control structure looks like.

Now the DC motor, as we have seen consists of a stator and then a rotor which is sitting inside and drawing the cylinder as seen from the side and then you have here another cylinder with brushes, that are going to be sitting there and then you take these two wires out and this whole thing is enclosed in an outer shell, then you have the shaft that is sitting off. The field is generated in the stator. So, stator generates the field and field lines go through the rotor and come back to the state, that is the system.

Now as I mentioned right at the beginning DC motor is a very good motor for the purpose of implementing this kind of control structure, because whatever Armature current flows into the system flows into the machine does not affect the field. You do not change the field by sending an Armature current this happens because of the geometry of the system inside. That way it is very good.

But the difficulty with the DC machine is that it has a system here. Which is the brush and commutator. Now this brush is something that has to slide on top of the cylinder which is going to be rotating always and you very well know that if you are going to have a sliding arrangement there is bound to be friction and there is not only friction, you need to have the friction very small, which therefore means that there needs to be good amount of lubrication and if there needs to be good lubrication, you cannot put oil or grease there, why?

It would not conduct, you need to have good electrical conduction there, because you are looking at flow of current into the armature and coming out. So, you cannot put, afford to put oil there and say I will get good lubrication. So, that is impossible. So, you need to have a material where that will allow conduction and be a good lubricate as well, do you know such a material? Carbon is a very good material and therefore this brush is made of graphite.

But what is the difficulty with that graphite is a very soft material and as you go on rotating the shaft it will produce less friction never the less, but it will also erode the material away. So, if you install the DC motor in the system, allow it to operate and you had an initial length of this



brush as this much, after about a month you will see that the brush is only that much. That is not good system, because we will have to keep on opening the machine replace the brush. So, it is a nuisance.

It is not only that, the eroded material which is removed from this brush. How will it, I mean it would not just go away into the air. It has to deposit somewhere and in which place will it deposit? It will deposit on that cylinder and that is a very dangerous thing, if you allow it to deposit on the cylinder and do not remove it, then it will, it will simply make it a, it will make a dead shot between the two brushes.

You want armature current to enter through the brush, through one of the brushes go into the armature conductors and come out through the other brush. But instead what will happen if it will enter through the first brush short-circuit the whole thing and go out through the second brush it means your motor will stop working after sometime. It will cause an explosion inside the machine, therefore DC machines though they are very good for the purpose of understanding control systems and so on they are not used in robotics application especially.

Not to say that these machines are never used, they are used in industry for a wide variety of applications. But more because, until the last few decades the way until about 1990s to 2000s, there was no other very good alternative available. So, DC machines were the machines that are where used for all high precision, high performance control applications. But today if you look at it, nobody will use DC machines for this kind of control. Why did we then discuss DC machines at length? It is because they still provide the best way of understanding, what is required out of a motor control system and what are the basics of motor control operation.

There are other machines which now substitute DC machines, which we will see in a little bit of detail as we go along. So, the next best machine is called as a brushless DC machine or a BLDC machine, which obviously means the brushes are no longer there. Which is the main drawback with decades of the DC machines. So, what happens if you have a brushless DC machine and how does one operate that? We will see in the next class.