Introduction to Robotics
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Lecture 18
DC Motor Equations and Principles of Control

In the discussion yesterday, we were looking at the DC motor and attempting to understand what are the basic physics behind the operation of the motor.

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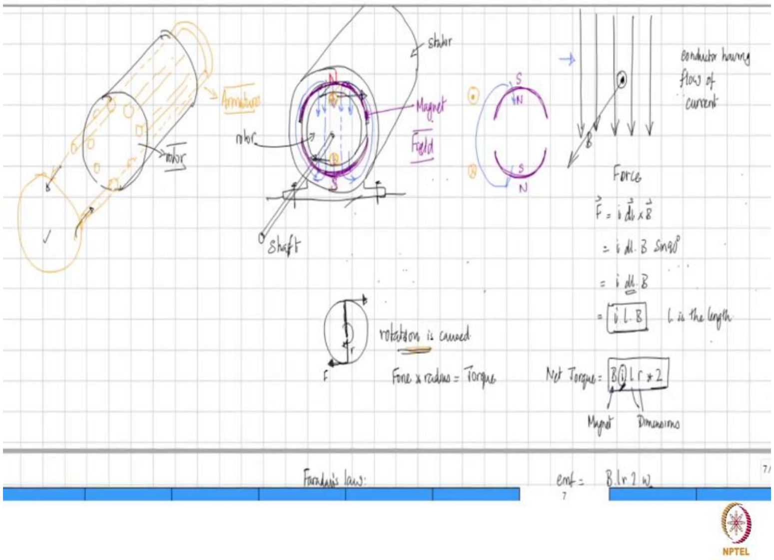
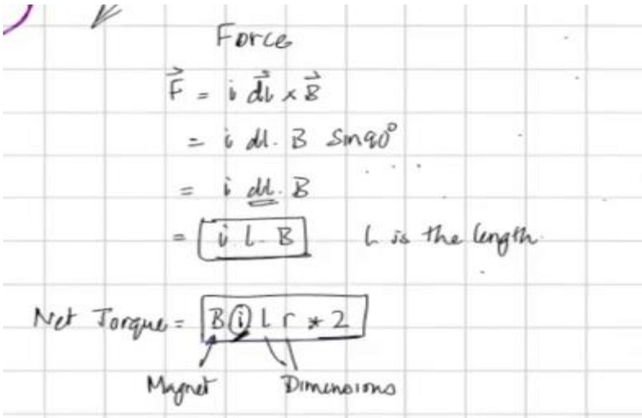


So, this gives an elementary view of how the motor is made and the physics behind the operation is given. Now, if you see here, we had put the rotor this is a drawing, rough drawing of the rotor and we had said that you will have an electrical conductor here and a conductor here, which is going to produce a force when you have an electric flow through it due to the presence of a magnetic field.

But, you have so much more surface available, where you can put more conductors and therefore, the ability of the field to allow the rotation increases much more, if you can therefore put more conductors all around. So, that is indeed what is done, so you have conductors everywhere and all of them are going to run alongside, along the axis and they all have to be interconnected in some appropriate way, so that the whole thing works in the way you want.



So, important that the interconnection be done appropriately otherwise, you would not get anything of use from the machine, so this whole arrangement of laying out the various conductors and interconnecting them in an appropriate manner that entire arrangement is known as the armature and the arrangement on the other member which is not going to rotate the main role of the other member which is not going to rotate is of course to house the entire thing and it is also the member that generates the magnetic field.

So, the assembly which is going to generate the field inside the machine is then actually called as the field itself. So, apart from the magnetic field, the entire assembly is also called as a field arrangement.

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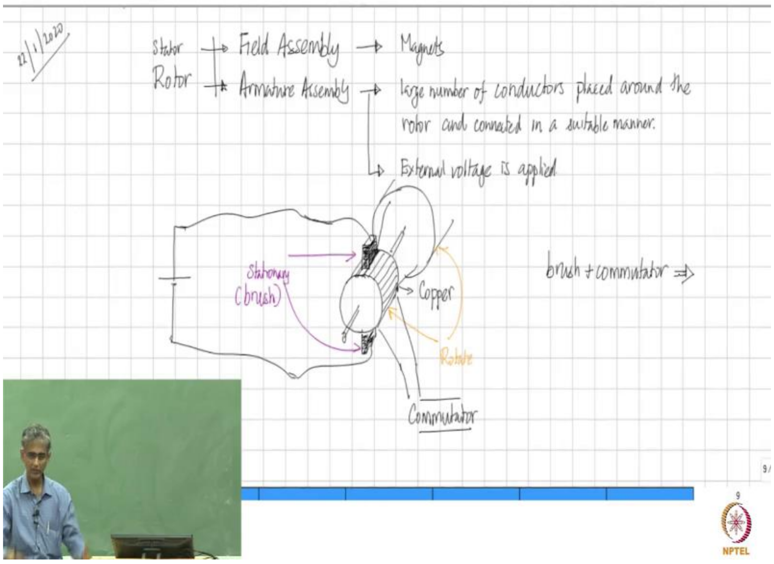

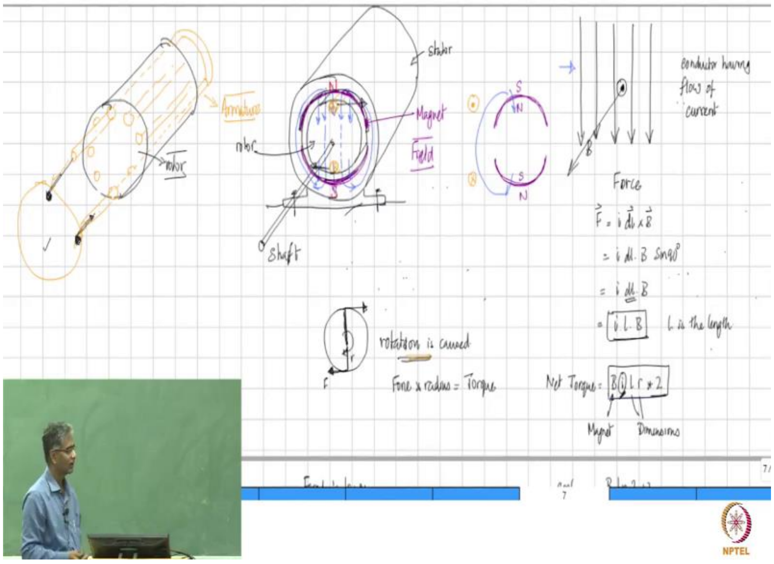

So you then have, in a sense you have the machine comprising of the stator and the rotor, which houses the field, the assembly and then, the armature assembly, so the stator has the field, the rotor has the armature.

Now, the field in the DC machine which is used for these kind of applications if at all the DC machines is used is obtained by having magnets and this is basically large number of conductors placed around the rotor and connected in a suitable manner, which is there and it is to this armature assembly that external voltage is applied, the external voltage that is applied to make the motor rotate or to make the rotor in the motor rotate is applied to the armature and that supply is a DC supply that you are going to apply, that is why it is called as a DC motor.

But, one would then see that, this is really a difficult job, because now let us say that, these are the two terminals, electrical supply terminals to which you have to supply a DC voltage and it is the rotor that is going to rotate, so how can you supply a DC voltage to an entity that is all the time rotating it will not feasible as it is, if you supply two leads to these two terminals and it is going to rotate, you will soon have a situation, where everything becomes intertwined and it will simply stop operate or something they simply snap off. So, in order to avoid this situation, there is a fairly ingenious arrangement that is done.

In the system that, you have another cylinder, which may be very short in length and it contains many segments like that, all these are insulated segments from each other, like these are all made of, these are made of copper and all these are insulated from each other, therefore each of these segments is an independent segment and then what you do is, this is linked to the shaft of the machine.

So, you have the shaft here it goes to the machine and the entire armature assembly fits here. So, you have the entire armature assembly fitting there and at the, at one end of the shaft near the armature assembly, you have this other cylinder as well on top of which there are many segments that are put.

So, it means that if the rotor is going to rotate, then this cylinder along with the many segments will also rotate, because they are all fixed on the same shaft and then you have these two outputs from the armature you have two terminals which are going to come out, you make a connection of that to these segments itself from where else it do not.

Now it mean that, as this is going to rotate this output from the armature lead that is going to come is actually linked to this cylinder and this is connected to another segment and therefore, the entire thing will rotate without any difficulty nothing will get you know intertwined or anything like that, this is one fit mechanical assembly and then what you do is, you place something else with slides on top of this, where you have another arrangement here which is a fixed block, rectangular or a block like that and you take out the lead from this.

Similarly, what you do is, you place another block here and you take out another lead from there. So, this block that you have put slides on top of the rotating surface and therefore, both this cylinder is going to rotate this block does not rotate, it is the sliding surface here. Similarly, this also slide therefore on the one hand this member and that member they rotate whereas, this member and that member are stationary.

Since, they are not going to be rotating you can take this wire lead and make a connection to a DC supply these two will not get intertwined because, the two blocks that we have put are stationary. So, now this arrangement the cylinder here, the cylinder here this is known as the, this is this one and these two blocks are called as they are known as brush.

So, the brush and the arrangement of commutator as it is called is the one that enables you to make a electrical connection of a DC source to an armature that rotates. So, with this arrangement then, you can have a DC machine that has a rotating armature to which you can connect a stationary DC supply to it through a wire and the whole system will then move.

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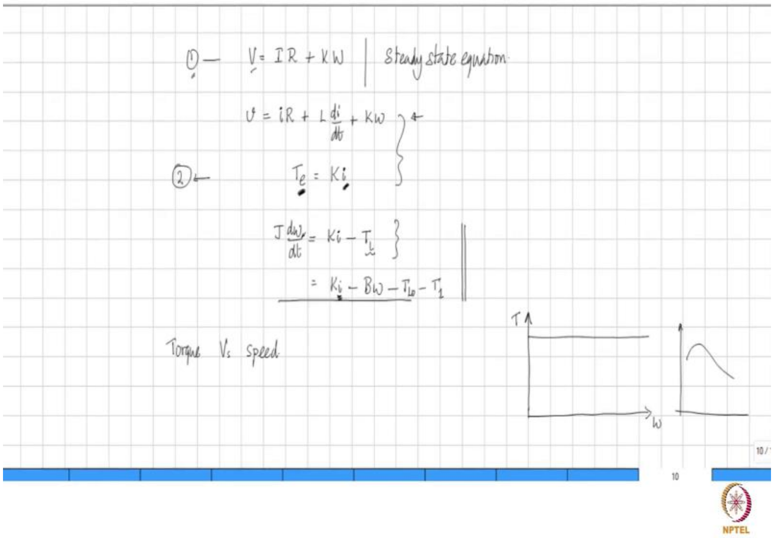
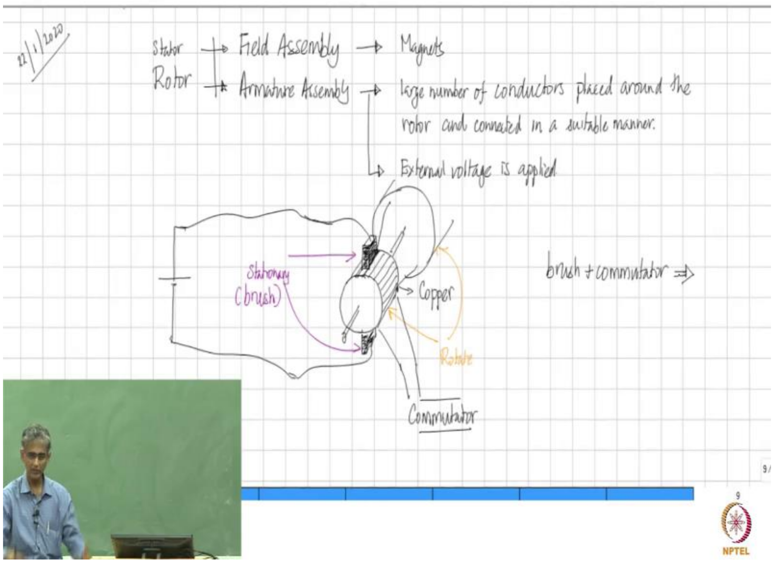
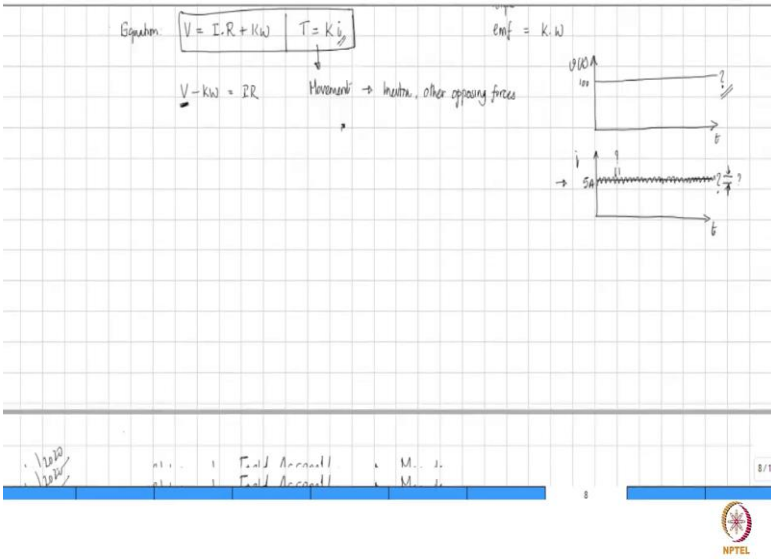


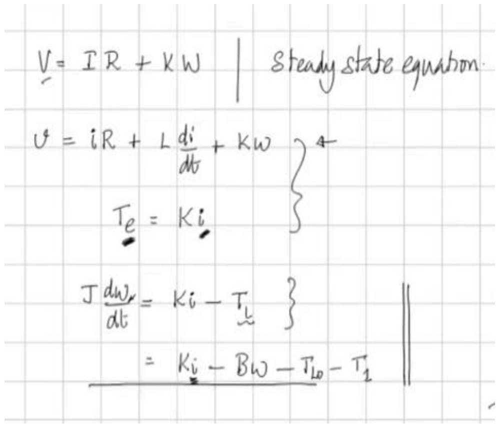




So, yesterday some of you are asking about the DC machine equation that we wrote, that says V equals I into R plus K times P. Now, this equation is a steady state equation which means that, if you give a supply to a DC machine, which is steady DC supply then, the motor will start rotating, initially if it was not rotating the motor will accelerate run up to some speed and then will continue running at some speed, at what speed it will run that is an issue that we have to address still.

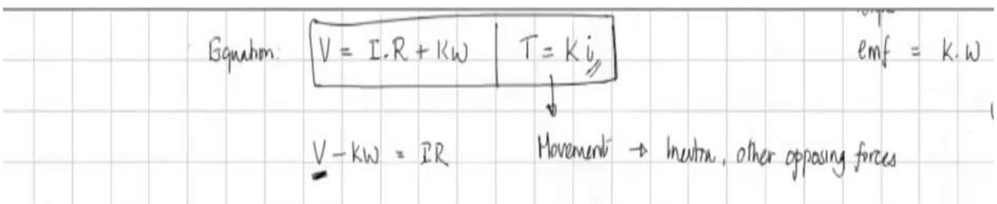
So, the motor will pick up accelerate and run at some speed. So, when it is running at that, some steady speed, which you call as omega that is given here then, it will draw a steady flow of amperes I. Now, the relationship between that steady speed, steady current that flows and the steady voltage that is applied is this equation. But, if you were looking at determining the behaviour of acceleration of the motor then, this is insufficient, this is not going to give you what will happen there.

Now, because you have an armature that is sitting there which contains, large number of conductors interconnected in some manner and so on. So, electrically the whole thing in addition to a resistance also looks like an inductor. Therefore, the inductance effect also has to be brought into the equation and then, the circuit equation becomes instantaneous voltage that you may apply is equal to a resistance current, resistance drop plus an inductance effect plus the induced EMF.



So, this is a equation describing the electrical circuit performance of this machine and then, you have the mechanical side which says the electromechanical forces or electromechanical torque is given by K time instantaneous current I. So, these two equations together describe, what is happening inside the machine and what is happening outside the machine is that, the total moment of inertia that is rotating into d omega by dt is the accelerating torque, this accelerating torque is equal to the generated torque Ki minus, whatever is the load torque TL.

The load torque is everything that opposes the motion, it may be due to friction, it may be due to damping, it may be due to a fixture that, you are applying outside if the robotic arm is going to move then by enlarge, it is an inertial torque that you are talking about because you have a mass that is accelerating in some manner.



So, that inertia will of course figure in the entity J. If for example, the robotic arm is going to lift the heavy object then that is a mass that exerts its own weight. Therefore, that weight part of it will then figure in the load torque that you are having, apart from that the mass may also affect the inertia. So, one has to look at what is the mechanical system to which it is disconnected and look at how that mechanical system is going to reflect in the movement of the motor as far as electric motor see this is all that is.

So, one has to determine that part of. So this equation, I mean for example you may have an expression like this K times i and it could be plus instead of the K times i minus some B times omega, if you want to have, if there is a viscous kind of drag effect, that is there and then, maybe some friction torque which is TL not, may not vary with respect to speed and then, maybe something else as well from T1.

It could be more involved also, what is this form of the equation as I said has to be determined based on what the application here. But, this mechanical equation is one of the reason why, we said that even though you may have an armature current that goes in the form of ripples.

This mechanical equation provides sufficient attenuation to the ripples that may be there in the flow of current i, so that the speed may not really see that kind of ripple. So, as I said in the first class itself one needs to then look at what is an acceptable ripple that is there in T and what is the amplitude of ripples that is allowable in i, so that one cannot say that I need always a pure DC to flow through the machine that is impossible.

So, based on this then, one can attempt to DC how the motor itself is going to behave with respect to the voltages that are applied under the flow of current across. So, these two equation, that is equation 1 and this equation 2 form the basis of an understanding of how one can go about attempting to control the motor operation.

So, the second equation says that the electromagnetic torque is decided by how much flow of current is there and the first equation then say how much voltage needs to be applied in order that a certain amount of current can be made to flow. So, it is important to understand or note here that, the electromagnetic torque is simply a scaled version of i, this may not be case in all motor.

In this particular motor, the design and the geometry of the machine is such that electromagnetic torque is simply proportional to the flow of current. Remember that, we said that, the motor operates only when there is a magnetic field and the fact that there is a magnetic field in the entire thing is embedded in this number K, because K was the strength of the magnetic fields multiplied by the geometrical parameters of the machine and therefore, you must understand that if there is no magnetic field, there is no torque.

Further, this expression two also says, that if you are going to send the flow of current i the level of magnetic field inside the machine is not affected by that we have lumped it together as a number of K which is not going to change which means, that we are saying that K is independent of i.

So, we are really saying in this expression that, if you send an armature current i it does not change the magnetic field inside the machine, it simply results in electromagnetic forces which cause the rotor to rotate, this is a very important aspect of the DC machine which lands the utility of the DC machine itself. In fact, there are other varieties of machines as we see as we go along and the manner in which one attempts to control those machine is to somehow get it to try and behave like a DC machine.

So, the DC machine is you know in a way that, this sort of sets the benchmark that this is how a motor should perform, why it is so is because of the form of expression to, which says that, if you send the current i it immediately reflects in an electromagnetic torque and the field does not change because of that.

Now, why I am emphasising field does not change is that you know that the flow of electricity, flow of certain amperes to a conductor also generates its own magnetic field. So, one may ask like there is a magnetic field sitting inside the machine already. Now, you are sending a flow of current which will also generate a magnetic field why will not this magnetic field alter the strength of the existing magnetic field inside.

Fortunately, what we are saying by this expression is that the design and the geometry of the machine is such that, that does not happen. The magnetic field that is generated, we are not saying that no magnetic field is generated by the flow of current i, physics says that if flow current there is a magnetic field you cannot change the laws of physics.

But, what is happening is that the direction in which that magnetic field is generated is not aligned in the direction of the main magnetic field that is at an angle 90 degrees, so no component of the generated field due to current i is affecting the main magnetic field, this is one important aspect of the DC machine arrangement which lengths its utility and advantage for the purpose of control.

So, having said this then, if you look at any actuator, what is of importance is that, how much of generated torque it will give you as a function of speed. Ideally, you would like to have a situation like this, this is my speed and this is toque, I would like to have maximum torque available at all speed from the actuator. Anybody is familiar with engines in automobiles.

So, this is a sort of engine torque that you would want that irrespective of what speed you are operating at, you would like the engine to give the same torque, so that you can accelerate irrespective of what speed you are in. But unfortunately, engines do not give this kind of ability. It depends on the sort of engine it may have something like that, depending on speed. So, you find that as speed increases the generated torque from the engine may go down.

Similarly, in the case of motors also you cannot expect that, the motor will generate the same torque will be capable of generating the same torque always by itself by itself. By itself meaning, how are you operating the motor you are giving a DC voltage to the motor and the motor starts rotating as the motor rotates at different speeds the question is, is it generating the same torque.

A look at this equation will say that, it is impossible because, the amount of electromagnetic torque which the motor will generate, depends on how much flow of i is going to be there and i is determined by the speed as well. So, if the motor is going to speed up, input voltage being fixed. Obviously, this i will have to reduce. So, you find that the generated torque in the motor will generally come down with T.

But however, this general and intuitive understanding does not suffice for really implementing something, you need to have some equation and therefore, what one can do is use these two equations and deduce, how the speed versus torque graph will look like

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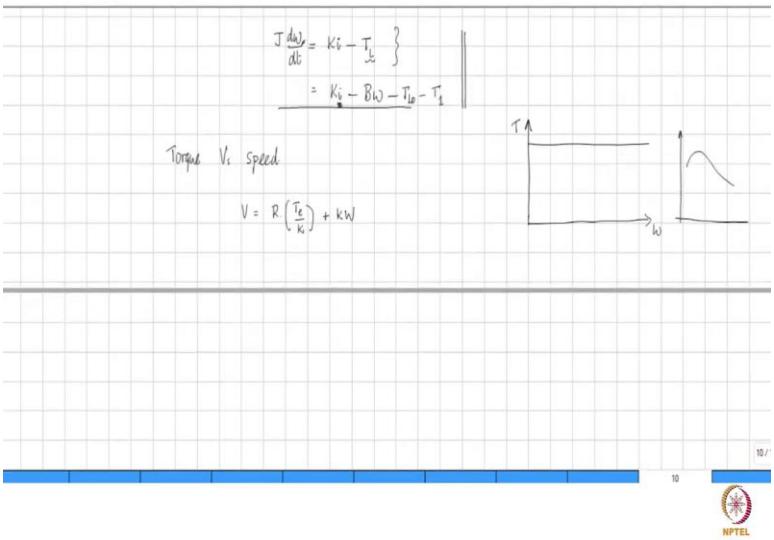

Professor: Sure. So, we will just come to that in a minute. So, if we want to get this expression, then what we need to do is to take this first equation V equals R into instead of i I will put it as Te by K and then, plus K times omega.

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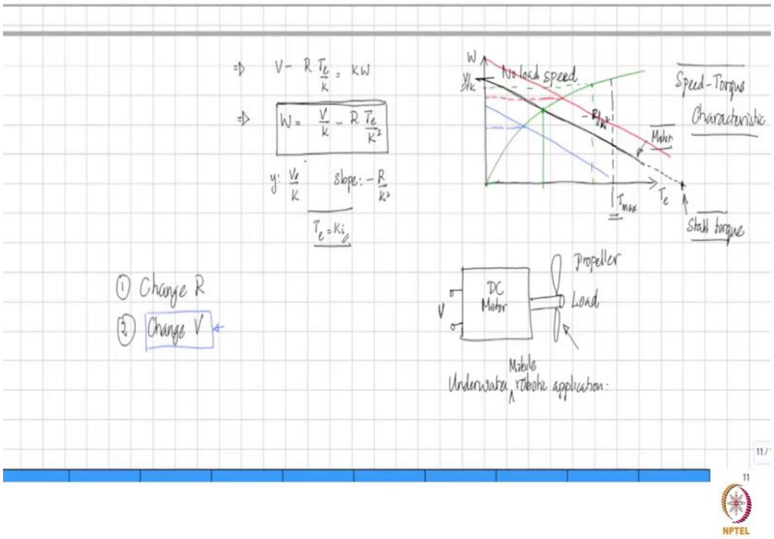
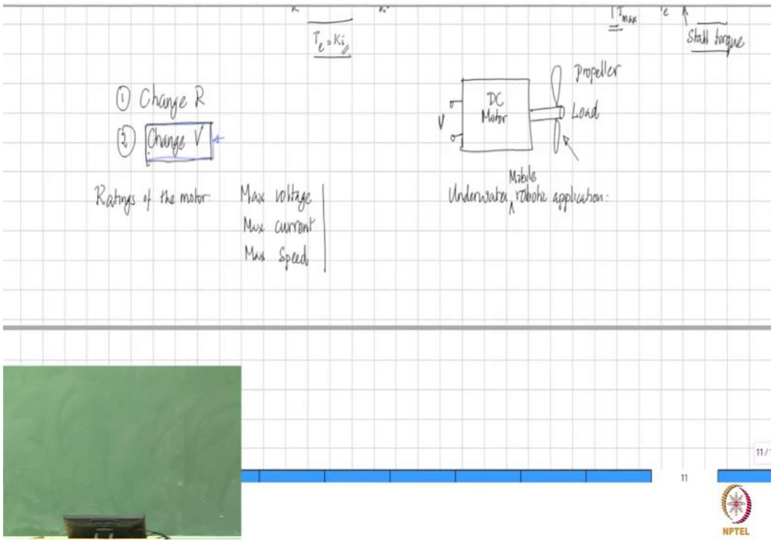
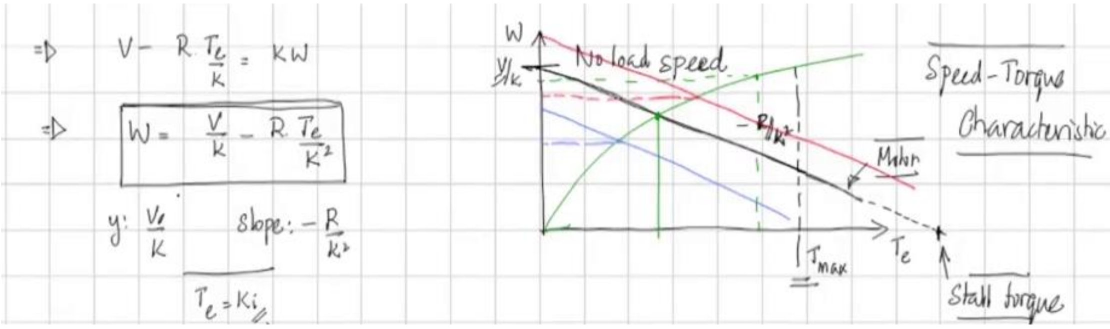




Which then means that V minus R into Te by K equals K times omega. Which then means that, omega equals V by K minus R into Te by K square. So, this then represents a relationship between speed and the generated electromagnetic torque inside the motor and one can draw this as a graph. So, if I put Te here and then P here, so this equation is a simple linear equation. You see that, this has a slope of minus R by K square and a y intercept of V by K. So, it is the V by K here and then, a line here which is minus R by K square will be flow.

So, one needs to understand certain things about machine. Any actuator but, in this particular case an electric motor. So, we have seen that, Te is equal to K into i. So, the issue is what is the maximum torque that this motor is capable of developing. Obviously, the maximum torque is dependent on how much flow of current you can give to the motor and how much current can they give to the motor it will depend on the size of these electrical conductors.

So, you have somebody has designed the motor with some conductor size or something and depending on that one will then, say that I can allow maybe 10 ampere to flow or maybe 100 amperes to flow, whatever that is. So, there is therefore, an absolute limit for this you cannot go beyond that, this is the maximum torque that can be generated.



But, never the less you can extend this graph and then go to the point, this point is then saying that speed will be 0 at that point and this is therefore, called a stall torque, stall means the motor stall means motor stop. So, if the motor were required to deliver that much torque then, it means that, the motor will simply not rotate.

Now, the question is whether, the maximum torque that the motor is capable of is less than the stall torque or more than the stall torque. For example, you may have designed the motor to take higher flow of current, the armature conductors may be higher, but due to some other reason, you are saying that no, I will not allow the motor to take more than certain value of current in which case the maximum current that if you take at 0 speed becomes a limit intake.

And then, this represents a certain value of speed and that is the speed at which the motor will run if there is no torque needed to be delivered by machine and therefore, this is called as no load speed. In an actual machine, it may not be feasible to get the machine to run such that, it does not generate any torque if it does not generate any torque that means, it does not require any flow of amperes into it i equal to 0 and a y equal to 0 and no torque if generated, the question is, how do you overcome friction and all those things?

So, in practise it may be necessary to allow some little bit flow of current in which case the speed, the no load speed is not really when T equals to 0, but then T equal to some small number, it is for response to friction equivalent.

So, now the issue is you have a motor and you are going to connect it to some load. Now, let us say that you are looking at a situation of underwater robot, underwater robotic application in an underwater robotic application, which is let us say you are looking at underwater mobile robotic application this fellow has to move first and how will you get it to move you will have, you will have to have certain, you will have to have a propeller.

Apart from that, the robot may be required to do something else, maybe take some layers of sand, maybe take some you know material that is there below etc, you may require other appurtenances to the robot like an arm and so on. But, if you look at this kind of situation where, first you look at the actuator which is going to make the robot move you need to have these blades that are connected to the shaft to enable it to move inside or move underwater and let us say this is a DC motor and let us say you are going to supply a certain voltage to it, this voltage V is what is given here V.

So, for this motor if there was no load it would have run at that speed, now the question is having put these blade onto the shaft at what speed will it run, reduced is a very good guess. But, the question is how do you determine how much speed

Student: intersection of speed v/s torque plot

Professor: Intersection of.

Student: speed v/s torque plot

Professor: Yes. So, first of all, you need to know what is the speed torque graph of this load. In order that, the blades rotate underwater at a certain speed the blades need to be supplied with a certain amount of mechanical torque. Just like the fan, if you want the fan to rotate at a certain speed it requires a certain torque, because the fan has to oppose the flow of air in this case, it has to oppose the flow of water.

So, you require for these blades, for these blades you require a certain speed versus torque behaviour. Only if you supply this much torque this blade, set of blades will run at that speed. So, now having connected both of them together on a single shaft how do you determine at what speed it will run, they both have to run at a particular speed such that, the motor will generate a suitable torque which is what is required for the blade to run at that speed.

If the motor is generating more torque it means the system will accelerate, if the motor generates less torque it means the system will decelerate. So, it must happen only at one point where the torque required for the blade is the torque generated by the motor, that obviously means, that the system will run at a point where the two curves intersect and therefore, this is the point at which the entire system together will work.

Remember that, the system together will work at a point where, the torque required by the load is the torque generated by the motor. It is not determined by the point where the two speeds are equal. Obviously, the two speeds are equal because you are connected them together that will always be the case.

Now, having seen this the question will then be if you want to make the robot move at a faster pace, what do you do? It means that the speed of operation has to be different. You want the speed to be higher that means, the intersection has to happen at a higher speed. So, then you start looking at what is it you need to do in order to get the intersection happen at a higher speed.

You cannot change the graph of the load, propeller is propeller you have somebody has designed it and fitted it you cannot go underwater and change the propeller when you want change the speed. So, you have to control the motor that means this graph which you have, this graph which you have, this is the graph of the motor and one has to change that graph in some manner, in order that the intersection occurs at a higher value of speed. If you want to run it at a lower speed, then you need to change it such that, the intersection occurs at a lower value of speed then the question is how do you do it.

So, we see that the slope is given by minus R by T square and the slope is one of the way of changing the points of intersection, if you make it slope more the speed obviously will reduce, if you make it slope less the speed obviously will increase. So, for adjusting the slope you can see if you can change the value of R. But, that is also very impractical because, you cannot go into the motor and say that I will now change the armature conductors in a motor that is rotating, that is not possible. But, you can add a resistance outside effectively increasing the value of R.

So, one of the ways therefore is to say that change R. But, what can you do to change that you can only increase the resistance you cannot decrease it, you do not have negative resistances that are available outside, so you cannot therefore decrease the value of resistance you can add resistors to increase. But if you increase it what will happen the slope will increase and the speed will only reduce.

The other thing that you can do is to change V, if you change V then looking at this equation of speed versus electromagnetic torque V is present only in the y intercept, slope is not affected by V and therefore, if you change V then, it means that, the y intercept alone will change. So, increasing V will result in a graph that looks like this with the same slope decreasing V will result in a graph that looks like this with the same law.

So, this means that either you get a speed of operation that is higher or you get a speed of operation that is lower this is certainly something that you can do. Because voltage is what you gave externally to the motor and you can give whatever voltage you want. So, the mechanism of speed control is the best method is to change the voltage to the motor.

If you change the voltage the motor and you want to increase the speed, obviously increasing speed means acceleration, that means you generate more torque. So, if you increase the voltage more current will flow into the machine motor will accelerate and the speed will settle down at a new point at which some required flow of current will happen. So, that is going to be there for the dynamic. But, as far as the variable to control, so that will be speed that will be applied DC volt.

So, this graph then provides you the basis of determining what is it you can control in the motor in order to change the speed of operation. So, this graph is called as the speed, torque, characteristic of the motor. So, just like we said that there is a maximum flow of armature current for which the machine has been designed for. Similarly, there is a maximum voltage that the machine has been designed for, you cannot say that I will increase the motor speed by increasing the voltage and go on increasing the voltage. Somewhere it will flash over and the motor is no gone after that.

So, there is an upper limit. So, one always has to contain with whenever, you select an actuator, electric motor or some other actuator, one has to contend with the ratings of the motor. The rating will say, what is the maximum voltage that can be applied, what is the maximum flow of input current that the motor can take, what is the maximum speed at which the motor can run. So, these three are going to be limiting condition. Whenever, you look at an application and you need to select an actuator, you need to keep these things in mind.

So, we have said that looking at this we have concluded that change in applied DC voltage is the most appropriate one for controlling the speed of the motor. There are other ways if you take any book, if somebody is going to take a book on DC motor control apart from changing the applied voltage to the armature people talk about field control, where you can change level magnetic field inside the machine that is also a feasible. So, there is a way to change in this geometry whatever, we have drawn how is the magnetic field established? By means of magnet and will you be able to change the field generated by a magnet?

Once you fix with that is it, it generate whatever field that generate. So, in this geometry this is not feasible to do any field control. But, if you want to do field control it is possible by having a magnetic field generated through electric current, then you can control that current and vary the magnetic field, the way you want. But, that is not normally done in these kind of applications. So, we are not going to discuss that aspect of it.

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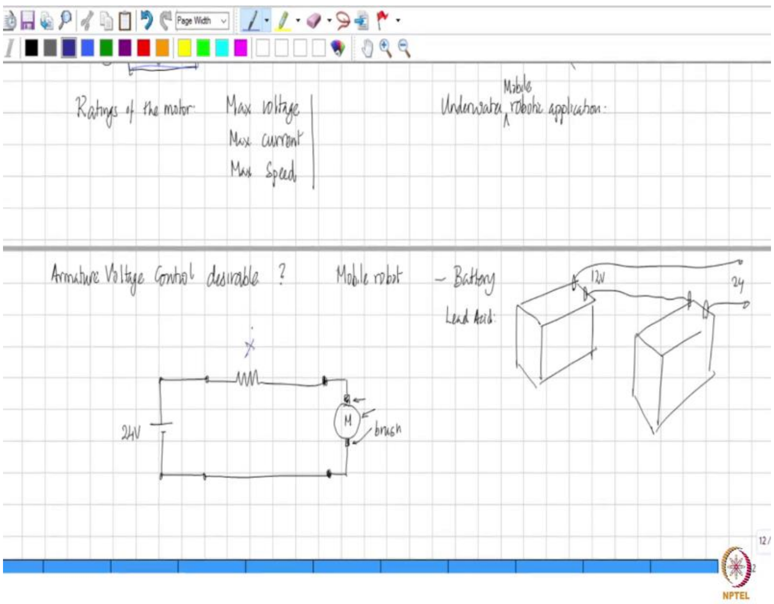

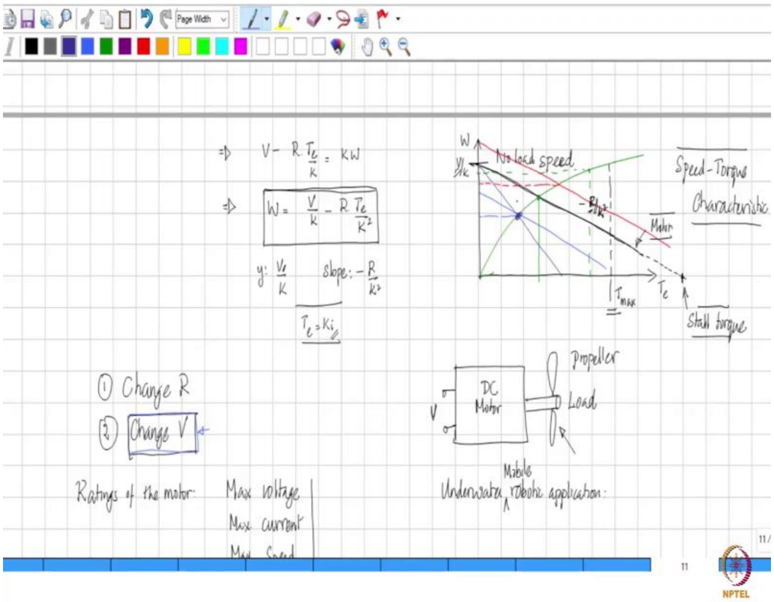

So, now the question is we have said that voltage control or more specifically armature voltage control is the desirable mode of operate. The question is, how? Now, why are we asking how? Why can we not say that yes, you have a motor you require 5 volt now, apply 5 volt, you want 15 volt now, apply 15 volt is there any difficulty with you see?

Student: it depends on the application

Professor: Yeah. So, you will have if you are going to look at a application that is let us say a robot that is mobile, I have mobile robot. How do you get the source of energy on this, you will have battery, battery is the source of electrical energy on both, battery will come if you are looking at let us say, lead acid battery. Lead acid battery is a box that you get with one terminal here, another terminal here, how much voltage will it give at this terminal?

Student: 12 volt

Professor: 12 volt it will give, if you want you can take one more box, there also you get the same output voltage, you can connect them in series and then how much voltage you get here? 24 volt you will get.

So, maybe what you can do is, you can say that in order to operate the motor I will connect either one battery or I will connect two batteries in series, you can have some switched arrangement by which you say that I will connect only one or I will connect both. But, you have a big problem there, that is you will get either 12 volt or you will get 24 volt, what if your motor for that particular point of operation require 18 volt, what will you do?

You cannot get it by this kind of switching the bank of supply arrangement. So, one of the ways we do is will be let us say for example, you have a source of 24 volts which maybe two batteries in series, do not alter that interconnection keep this way we have connected the batteries is fixed and you have a motor here by the way, the symbol of a DC motor is this. So, these two rectangle, they represent the brush and the circle of tools represents the rotor M stands for the fact that, it is a motor. So, that is a representation of DC motor that is how you draw it in a circuit diagram.

So, the simplest way is to simply connect them together, which means you are supplying 24 volt for the motor. But you suddenly want to decrease the speed of the motor and therefore, you cannot give 24 volt anymore. So, what you will do is maybe, what you can do is connect a resistance here, which is like adding an extra resistance to the circuit, so adding a resistance can be seen some two view point, one is you are changing the slope here or you are also reducing the voltage that is available here.

So, either way one can look at it is just a view point. But, affecting the slope is a more holistic way of looking at. But, if you do this, not only if you now want to reduce the speed of operation, this will work what it will do? It will give you a new graph that looks like this, it will give you a new graph that looks like this and you can get the point of intersection what you want. But, do you see any difficulty with this mode of operation, what do you think is an advantage, what do you think it is an disadvantage

Student: its a disadvantage

Professor: Yes. Now, that you are having a resistance and there is going to be current flowing through it, it will dissipate heat, it will generate heat and dissipate heat which means, that in order to run the motor at the particular speed that you want. Let us say you require that, the motor just needs to generate 50 watts.

Ideally, you would expect that, if the motor is going to give 50 watt of output the source needs to give 50 watts to the motor. Maybe the motor will have some loss to instead of a 50 watts you give 60 watts, I have some loss. But, if you put a resistor there instead of 50 watts, that you give to the motor you may have to give 100 watts. So, it was very lossy, very inefficient way of doing things. So, one does not normally go for this kind of an approach this is not desirable at all. So, the question is if that is not desirable what next? So, that we will see in the next class.