Introduction to Robotics.
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Lecture 2.8
Forward kinematics

So, in the last few classes we discussed about the DH parameters and then saw how do we actually assign coordinate frames and then using coordinate frame how do you, how do you find out the DH parameters basically the four parameters theta, a, d and alpha.

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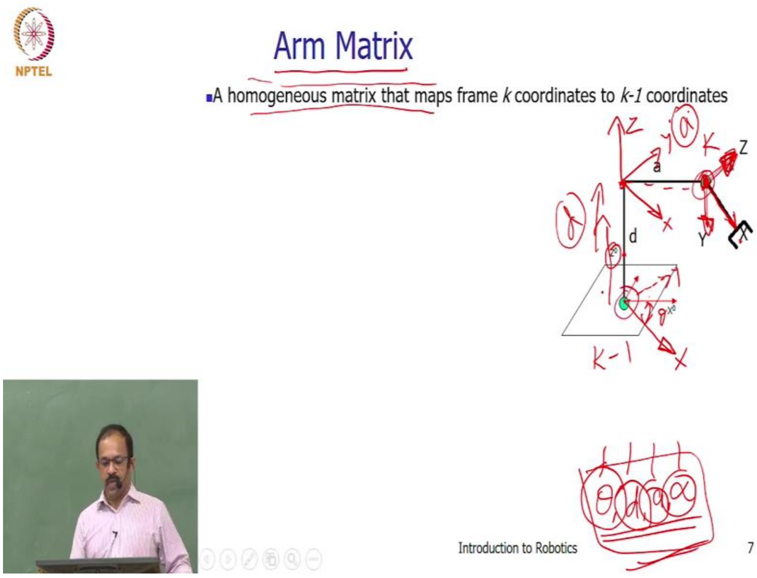

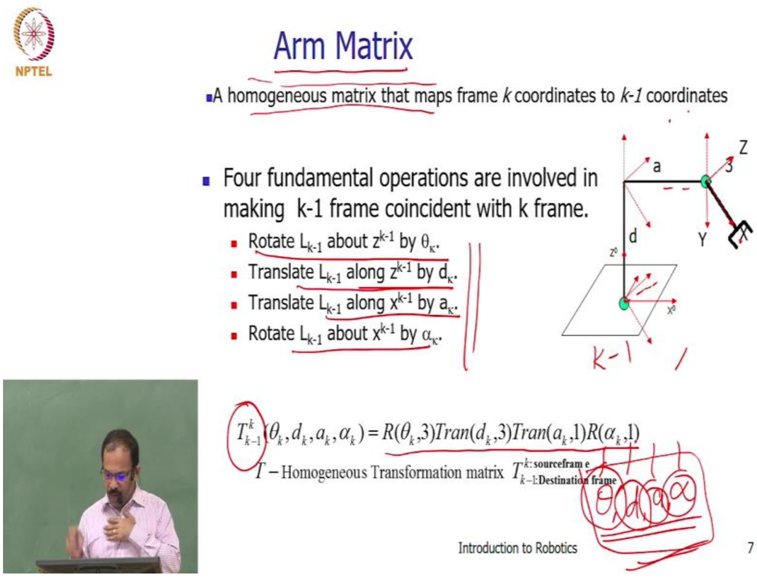

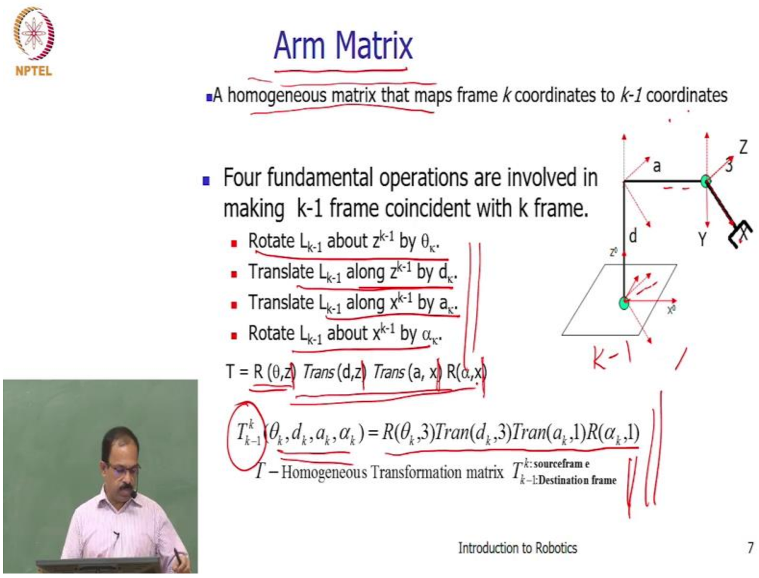

So, today we will see how do we actually use this one for developing the forward kinematics relationship for an industrial manipulator. So, the purpose of forward kinematics is basically to map the coordinate frame at the tooltip to the base frame here. So, if any point represented with respect to the tool frame, you should be able to represent that with respect to a base frame and there will be many joints in between the manipulator tooltip and the base frame.

So, how do we use the DH parameters and then using the DH parameters, how do we develop a matrix called Arm matrix to map the coordinate frames and then use these coordinate frame to get the forward relationship is the question that we are trying to address. So, the Arm matrix is defined as a homogeneous matrix that maps frame k to k minus 1 coordinates. So you have a kth frame, you have a kth frame and a k minus 1 frame. How do we actually map this two frames using a matrix is and that matrix is known as the Arm matrix.

So, we use a homogeneous transformation matrix, which is a 4 by 4 matrix which includes the all the joint parameters and the link parameters as well as the DH parameters, what will be using for getting the transformation and we saw that we can use the four parameters that is d, theta d, a and alpha. So, these are the four parameters, that we are, that is of interest between two coordinate frames. We will see how can we actually develop this arm matrix using this formula.

So, assume that this is the base coordinate frame, and then you have another coordinate frame at this point given by XYZ coordinate here. So, now here you can see this is the Z axis, this is the X axis and this is the Y axis. So this coordinate frame, base coordinate frame and this two coordinate frames we want to map or we have to find the transformation how this coordinate frame is transformed to this coordinate frame using this parameters. So, how do we actually represent this.

So, what we can do is we can first look at, assume that this two coordinate frames are initially aligned together, that is this X0, Y0, Z0 and XYZ are initially aligned, so initially they were actually the same XYZ here. Now, assume that this coordinate frame rotated by an angle theta, assume that this XYZ rotated by an angle theta with respect to the Z0 axis. So, if it is rotated, then you will see that this will become your X, the rotated X and it is rotated with respect to Z axis. So, Z remains same and then your Y will be something like this.

That is the first parameter theta. You can see that the transformation of this two coordinate frames, first it is rotated with respect to Z0 axis by an angle theta and then it has translated along the Z axis by a distance d. So, it reaches here. So, first it rotated by an angle theta. So by an angle theta it rotated X0 to X, with respect to Z0 it rotate by an angle theta and then it translated by an angle d to reach here and then it translated again along its X axis.

So, this frame I am just showing it like this because, it will be along the link, so it is translated along X by an angle by a distance a and then it rotated by an angle alpha with respect to X axis. So it rotated by an angle alpha, so Z axis has actualy came here and you got this frame here.

So, the four transformation takes place for this coordinate frame to reach here with this orientation. So, it rotates by an angle theta with respect to Z0 axis and then translates along d axis and the Z axis by d and again translates along X axis by a distance k and then rotates with respect to X by an angle alpha to get the Z axis.

- Rotate L_{k-1} about z^{k-1} by θ_κ.
- Translate L_{k-1} along z^{k-1} by d_κ.
- Translate L_{k-1} along x^{k-1} by a_κ.
- Rotate L_{k-1} about x^{k-1} by α_κ.

So, by this four transformations, you are actually getting this four transformations, individual transformation for this coordinate frame to get this coordinate frame. So the mapping between this two coordinate frames can be represented using these four transformation. So, that is why we need these DH parameters to understand how this particular coordinate transformation happens from the original coordinate frame.

So, this is what we are trying to understand here. So, we have this transformation. So, it moves and then it finally reaches here. So, rotate Lk minus 1 about Zk minus k angle Theta and translates along Zk minus 1 by dk and then translate along Xk minus 1 by ak and then Lk minus 1 about Xk minus 1 alpha k. So, all with respect to its own frame, so all the transformations are with respect to its own frame or with respect to the base frame.

So, all are with respect to its own frame and therefore, what we will do to get the full transformation it's a composite transformation. So, what we need to do, do a post multiplication. So, you will be getting this k to k minus 1 transformation that is the kth to k minus 1 transformation, how the k minus 1 and k are related or any point in k can be related to k minus 1 by using this transformation k to k minus 1, which is R, T, T, and R.

$$T = R(\theta, z)\ \mathit{Trans}(d, z)\ \mathit{Trans}(a, x)\ R(\alpha, x)$$

$$T_{k-1}^{k}(\theta_k, d_k, a_k, \alpha_k) = R(\theta_k, 3)Tran(d_k, 3)Tran(a_k, 1)R(\alpha_k, 1)$$

$$T - \text{Homogeneous Transformation matrix } T_{k-1:\textbf{Destination frame}}^{k:\textbf{source frame}}$$

So, these are the four individual homogeneous transformation we will be using in order to get the composite transformation of the coordinate frame from k to k minus 1. So, this is known as the Arm matrix or the homogeneous transformation matrix from k minus 1 frame to kth or the source frame to the destination frame.

So, this is what actually the coordinate transformation matrix. So, between any two coordinate frames of the manipulator, any two adjacent coordinate frames we will be able to find a transformation matrix with this four parameters included. Out of these four, only one will be a variable. So, finally, you will be getting a matrix with one variable and that relates the coordinates frame k minus k to k minus 1.

So, this is basically that Arm matrix or the homogeneous transformation matrix between two coordinate frames of the matrix. So, how do we get this, so first you get this matrices. So, you know how to get the fundamental homogeneous rotation matrix with respect to third axis, then translation matrix again with rotation matrix with respect to X axis rotation matrix, translation with the X axis and rotation with the X axis. So, all this four transformation matrices we know the fundamental rotation matrices and translation matrices. Multiply these matrices.

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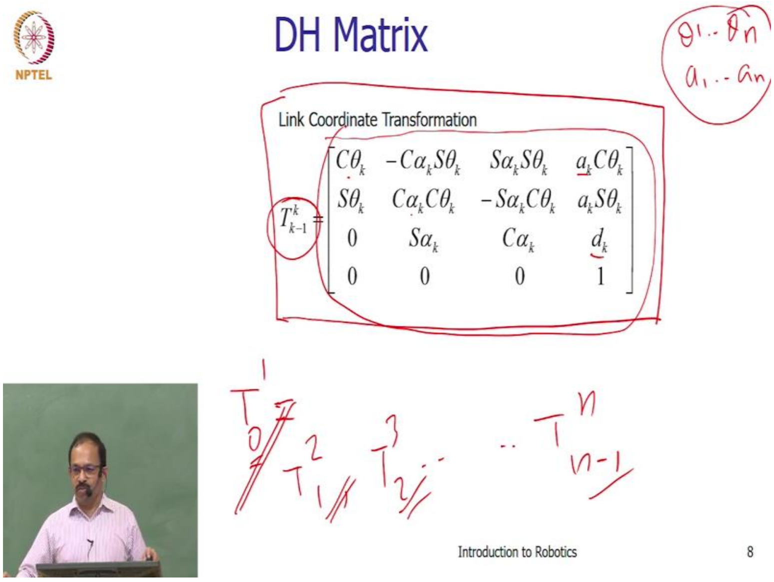

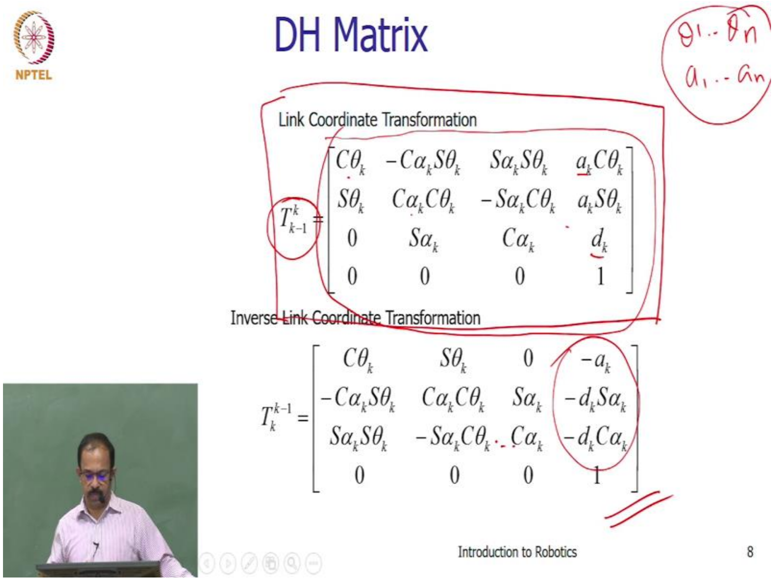

And then you get the final transformation matrix which is this one, k to k minus transformation. So, this is the final DH matrix which relates to adjacent coordinate frames with the parameters, all parameters theta, alpha a and d. So, you can see this is a function of all this parameters theta, a, Alpha and d and k to k minus 1 is obtained by this relationship and we saw that if we have multiple coordinate frames, you will be able to get the theta 1 to theta n.

Similarly, a1 to an, etc, for any and manipulated frame and therefore we will be knowing all this information and therefore, we can actually find out T1 to 0, that is if you have a manipulator and the base frame is 0, the next joint is 1 then we can find out the transformation from the first joint to the base frame by as a T1 0 and similarly, we can actually get T2 1, T3 2, etc., Tn to n minus 1 we will get because we know these parameters and therefore, we will be able to get these transformations, between adjacent frames you will be able to get as a transformation matrix n to n minus1.

So, the only thing what you need to remember is this matrix. If you cannot remember then you need to go to the fundamental matrices and then multiply it, otherwise if you can remember this then no need to worry. Just what we need in forward kinematics is only this particular matrix. Once we know this matrix everything is done, because it is only substitution of the DH parameters into this matrix then you get the transformation.

Any questions? So, now you know about coordinate assignment, you know how to get DH parameters and we know how to, if you know DH parameters, how do we get that transformation between two adjacent coordinate frames. So, that is the basic requirement for forward kinematics. Now the forward kinematics is basically.

So, if you want to do the inverse also you can get. So, if you want to k minus 1 to k if you are interested in getting k minus 1 to k, it is basically an inverse of this one and the inverse of homogeneous transformation matrix can be obtained by taking this R transpose and minus R transpose P that is what we saw in the previous class.

So, the same application can apply here and you will be getting it as inverse transformation also k minus 1 to k also we can get. That is known as the inverse transformation. Again, no need to, if you know this then you can actually get it directly by taking the transpose, R transpose n minus R transpose T, you will be getting this vector here.

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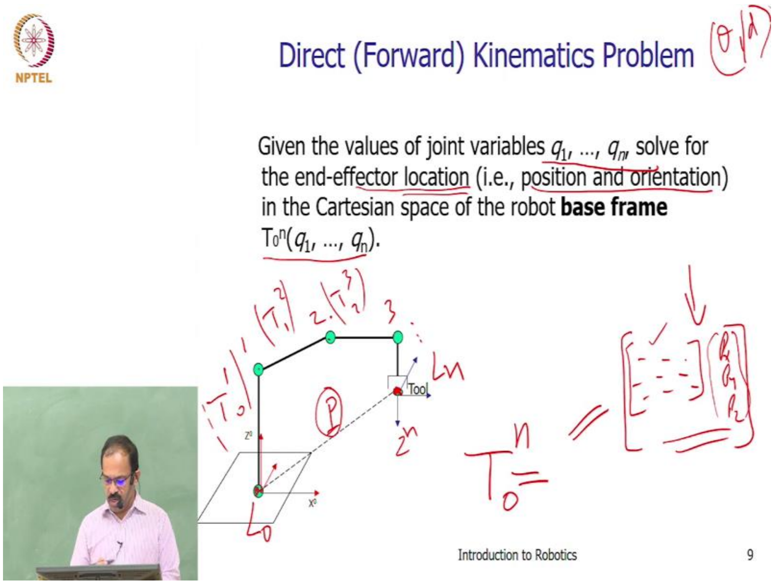

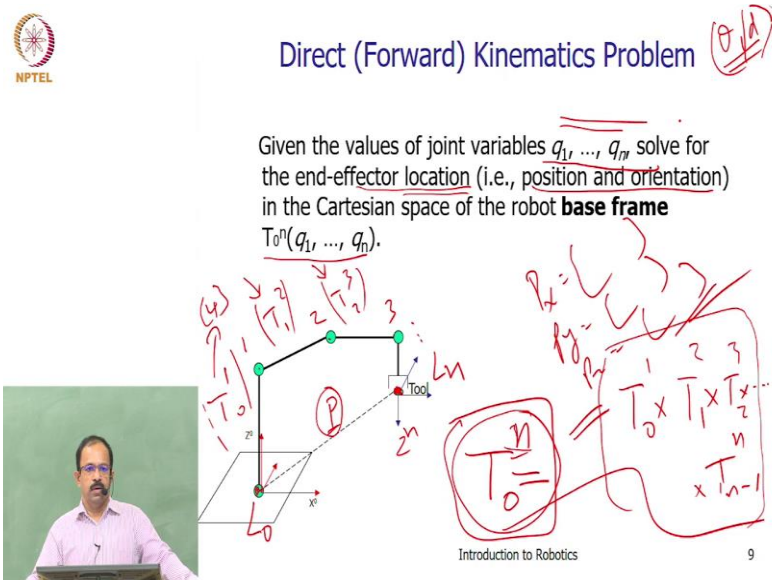

So, now let us see how do we get the direct kinematics or the forward kinematics of the manipulator. So, as I mentioned the point of interest is this point that is the tool tip. We are interested in knowing what is happening to this coordinate frame. Because we assume that the coordinate frame attached to the tooltip is this one and whatever happens to tooltip the coordinate frame will be rotating or translating. So, we will be able to get this point and the orientation of that particular frame by getting a relationship.

So, we are interested in knowing this vector P. So, that is how far this tooltip is from the base frame that is the position of this tooltip. So, the position vector is of interest to us and the orientation of this frame with respect to the base frame. So, once we have this, we have the complete definition of the tooltip with respect to the base frame. We want to know the position vector and the orientation of the tool frame, then we have the complete information of the tool with respect to the base frame.

So, we are interested in knowing this information, what is Px, Py, Pz that is this tooltip with respect to the base frame. So, that is the interest and then what is the orientation of this frame with respect to this frame. So, assume that this is Zn, this is Zn. So, we are interested in knowing how this Z0, Zn and X0, Xn and Y0 and Yn are aligned. So, if they are aligned you will be getting on [1 0 0; 0 1 0; 0 0 1] if they are aligned, otherwise you will be getting this as a normal sliding and approach vectors corresponding to the orientation of the tooltip.

So, this is finally what we are interested in. We want to know what is the position of the tool and its orientation with respect to the base frame, whatever may be the joint position. As a function of this joint variables we want to get this Px, Py and Pz. This is our what we are looking for. So, how do we get this, it is by looking at that joint variables q1 to qn. So, I define the joint variables as q in this case, because it can be theta or d any one of this can be a variable.

So, I can take as a q1. So, we have n variables, because an n degree of freedom robot, have got n variables. So, given the values of joint variables, so the end-effector location, that is the position and orientation in the Cartesian space of the robot base frame. So, effectively we are telling that we want to get the transformation n to 0, what is this transformation is of our interest.

Here, this is the nth frame, this is your Ln and this is your L0. So we are interested to know what is the transformation of nth frame to 0th frame. So, this is the transformation we are interested and once we know this transformation, we can easily find out what is the position because the this one gives you the position and this gives you the orientation. Now, the only catch is that, in between you have many points and there are many variables.

So, we need to find out how n to 0, how can we calculate n to 0 by using the individual transformation. So, now, we have 1, 2, 3, etc to n. We want to get Tn 0 and we can easily find out what is T1 0, what is T2 1, what is T3 2 can be obtained from individual transformation.

So, if we know the individual transformation, we can use this individual transformation to get Tn 0 by simply multiplying the transformation matrices and finding out what is the relationship. So, we know T1 0, so I will write it as T1 0 multiplied by T2 1 multiplied by T3 2 n minus 1 gives me Tn 0, that is the only thing we need to know.

$$T_0^n(q) = T_0^1(q_1)T_1^2(q_2)T_2^3(q_3)\ldots\ldots. T_{n-1}^n(q)$$

So, if it a n degree of freedom joint and if you can assign coordinate frame, find out all of these parameters, you can find out the new the transformation between adjacent coordinate frames and using this individual transformation, you can find out the final transformation T into 0 using this is a relationship 1, 2, 3, etc.

So this is the, and once you have this n 0, you will be having the forward relationship. That actually this matrix will tell you what is the relationship of this coordinate frame to this frame and we know that each one of this is a function of the joint variable q. So, this would be a function of not only joint variables, all the four parameters, this will be four parameters, this will be having the four parameters like this.

So, this function finally this will be an expression Tn 0 will be an expression, which has got all these parameters in that expression and out of this only one of this will be variable. So, you will be having 6 variables and you will be having all of this as constant. Finally, you will be getting Px as a function of all these parameters, you will get Py as a function of all these, Pz as a function of all this. This is the way how you begin forward relationship, the position and orientation of the tooltip as a function of the joint parameters and the link parameters for the DH parameters of the kinematics.

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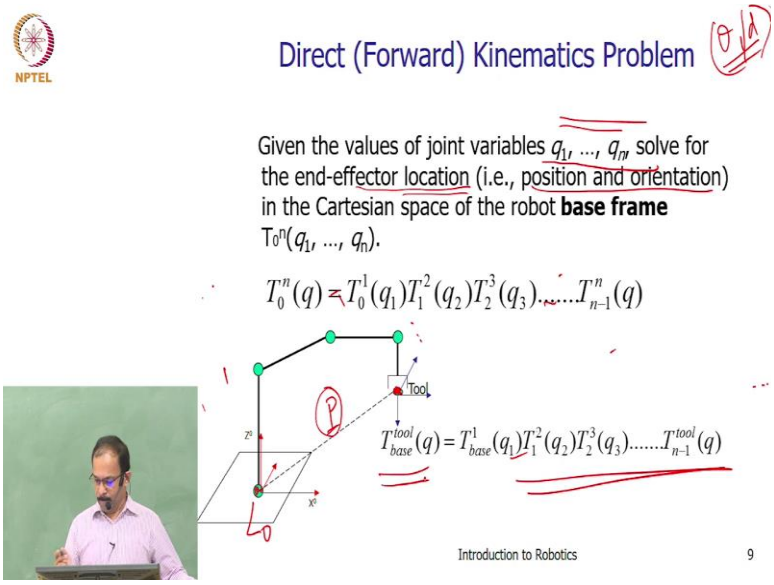

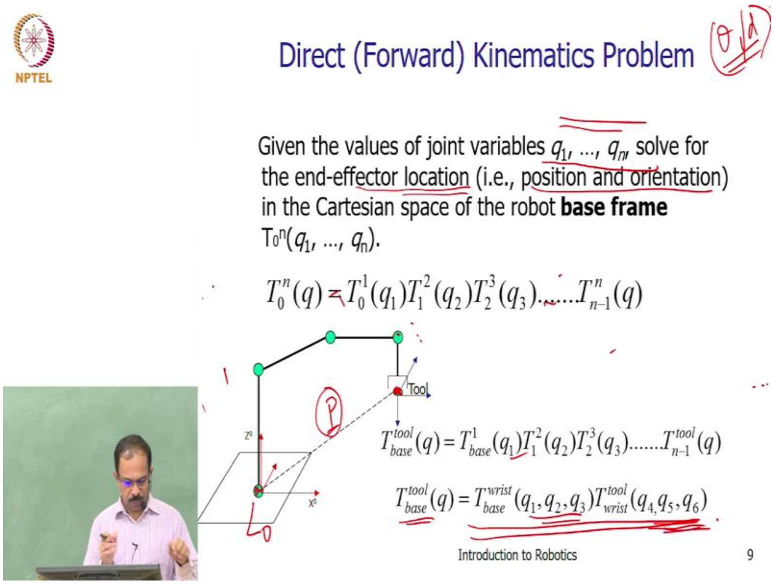

So, Tn q will be 1 0, 2 0 etc. So, now we are using the tool to base. So, this is tool and this is base, so 1 to base 2 to 1, 3 to 2, etc, n minus 1 will be the forward relationship for the manipulator. That is if you know all the DH parameters, you will be able to find the relation between the 2 lambda base as the 4 by 4 matrix representations. Any questions?

$$T_{base}^{tool}(q) = T_{base}^{1}(q_1)T_1^2(q_2)T_2^3(q_3)\ldots\ldots T_{n-1}^{tool}(q)$$

$$T_{base}^{tool}(q) = T_{base}^{wrist}(q_1, q_2, q_3)T_{wrist}^{tool}(q_4, q_5, q_6)$$

So, since I mentioned that we can have the first three degrees of freedom for positioning and the next three degrees of freedom for orientation, you can actually represent this also in this way, between wrist and base and the tool and wrist. Only for convenience because the positioning is done by the first 3 joints and the orientation is done by the next three joints, we can represent this as two transformations, wrist to base and the tool to wrist, only for convenience, otherwise, it is the same.

Only they have up to three we are bifurcating and then making it as two. So, tool to the wrist suppose this is the wrist point, then this will be three and then the first the other three will be deciding the wrist position. So, the wrist position is decided by the first three joints and orientation of the tool is decided by the next three joints.

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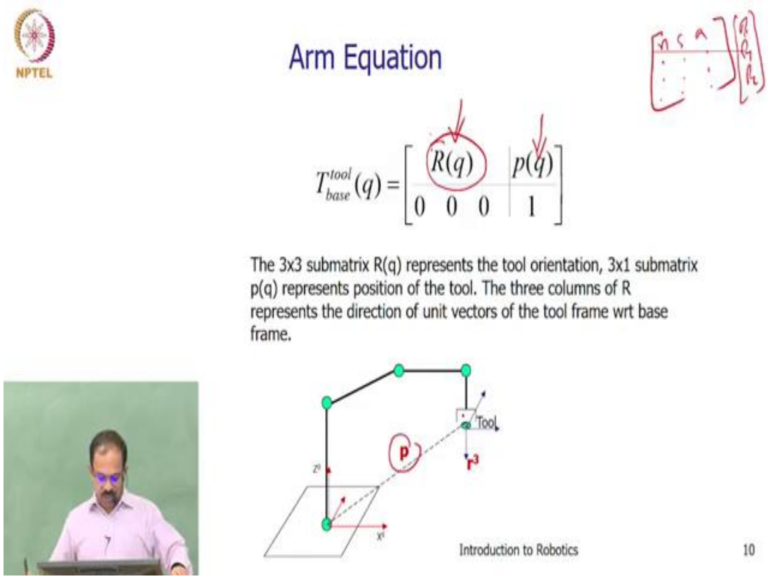
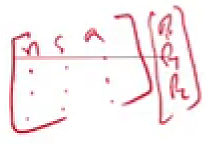


So, finally the equation will be like this, the tool to base you will be getting it as say 4 by 4 matrix with this as the position vector and this is the orientation matrix. So, your P will be this and your orientation matrix will be like this. So, that will actually having this three vectors, this three vectors will be the approach, normal, sliding and approach vectors. So, this would be the normal, sliding and approach vectors. You will be seeing this as three vectors and that will be one first will be the normal vector, sliding vector and approach vector and you will be having this three positions Px, Py and Pz.



So, that gives you see the P and then gives you the orientation of the tool frame with respect to the base frame. So, this is what we will be getting at the end of this transformation we are doing individual transformation between coordinate frames and finally finding out the transformation from base to the tool. We will take one or two examples to make sure that you understand this.

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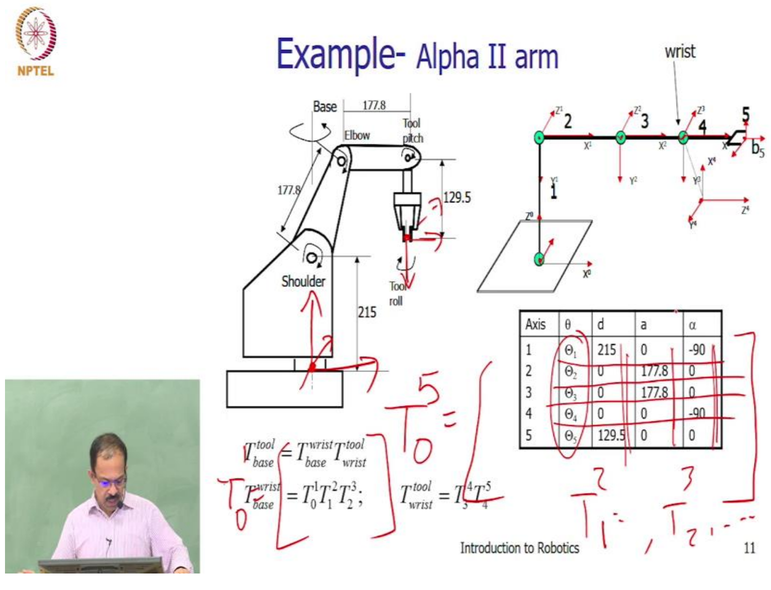

So, let us take an example of Alpha two Arm which we already discussed in the class. How to get the DH parameter? So, we will not go into that stage now, we will see how to get the transformation matrix and how do we represent the position of the tool. So we are interested in this tool position and orientation. So, we will be assigning a coordinate frame here as you can see there and then you will be having coordinate frame here and a coordinate frame here.

So, this are the two coordinates frame will be assigned. We want to find out what is the relation between this coordinate frame and this coordinate frame as a function of all these joint parameters. Now we have different joint and link parameters, we have multiple parameters, we need to represent the relationship between this two coordinate frame as a function of all these parameters. Also, that is the forward relationship. So, the first step in forward relationship is assigning the coordinate frame, we saw how to do this.

So, you assign first coordinate frame here, next one here, then next one here, here and the next origin will be the same place, next coordinate frame and finally your fifth coordinate frame will be at the tooltip. We are interested in finding out what is T5 0. So, this is what we are interested in and that should be as a function of all these parameters, this is what we need to develop as a forward relationship.

So, we assign the coordinate frame and then get the DH parameters based on what we discuss in our previous class. So, this is the parameters that we will get the table of DH parameters that we have. So, theta seems the all are joint variables I mean all are rotary joints we make this as variable and write the equation or the relationship 5 0 as a function of theta and since these are all constant, we will put the RDC to actually to the equations, the direct relationship.

T5 0 can be obtained by first finding out what is T 1 to 0. So, we find out what is T 1 to 0 and then we write what is T 2 to 0 then T 2 to 1 then T3 to 2 etc. So, since we have this parameters come we can easily write down the relationship T1 2 to 2 to 1, 3 to 2 etc and then you multiply this and get the final relationship. So, once you have these DH parameters.

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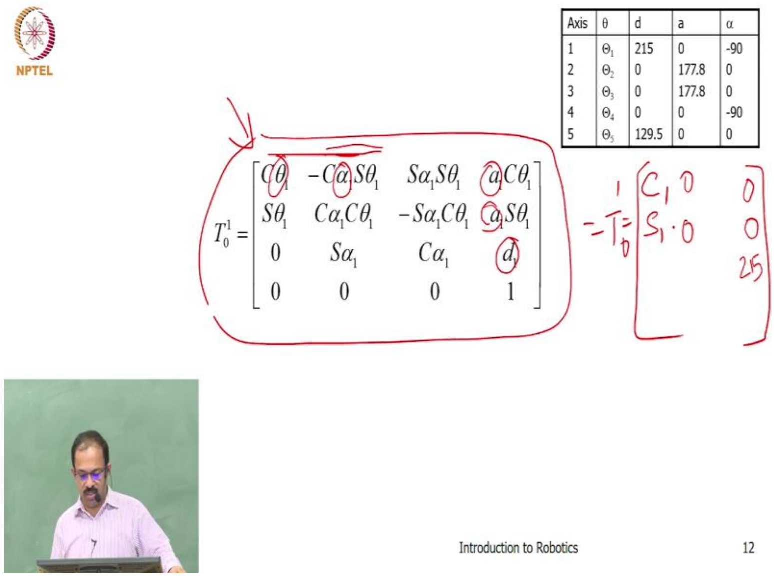
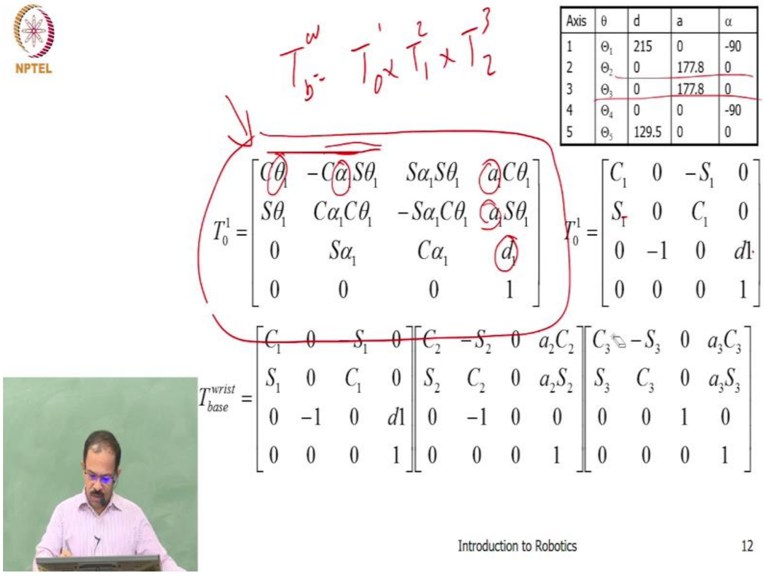
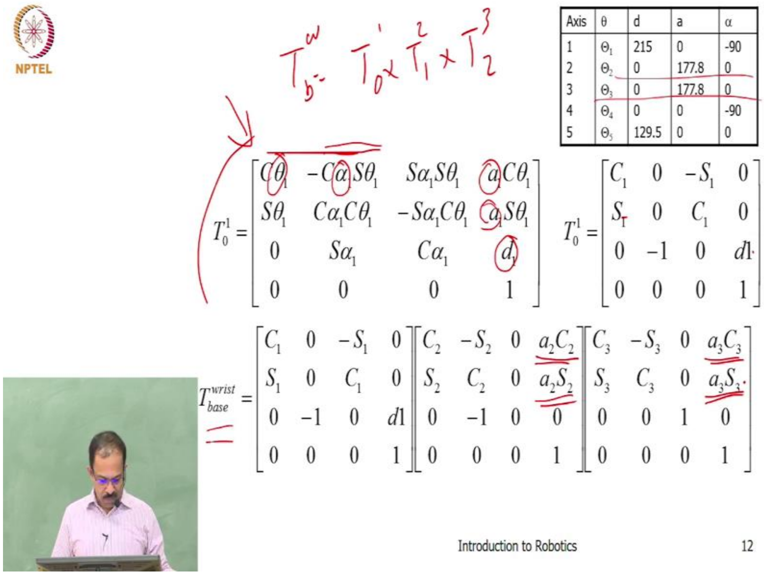


Axis	θ	d	a	α
1	Θ_1	215	0	-90
2	Θ_2	0	177.8	0
3	Θ_3	0	177.8	0
4	Θ_4	0	0	-90
5	Θ_5	129.5	0	0

$$T_0^1 = \begin{bmatrix} C\theta_1 & -C\alpha_1 S\theta_1 & S\alpha_1 S\theta_1 & a_1 C\theta_1 \\ S\theta_1 & C\alpha_1 C\theta_1 & -S\alpha_1 C\theta_1 & a_1 S\theta_1 \\ 0 & S\alpha_1 & C\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





The first thing is to go for 1 to 0. So, 1 to 0 can be obtained by substituting this values of theta 1, alpha 1, a1 and d1 by substituting here. This is your Arm matrix or transformation matrix 4 by 4 transformation matrix between 2 adjacent frames. So, this is what I showed you in the previous slides. Now substitute this values and then get what is the T1 to 0. So, what will be T1 to 0 assuming theta 1 not known, so theta is a variable. So, of cos theta 1 can be written as C1, then sine theta 1 is s1, then this is a1.

$$T_{base}^{wrist} = T_0^1 T_1^2 T_2^3 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1C_2 & -C_1S_2 & -S_1 & a_2C_1C_2 \\ S_1C_2 & -S_1S_2 & C_1 & a_2S_1C_2 \\ -S_2 & -C_2 & 0 & d_1 - a_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1C_{23} & -C_1S_{23} & -S_1 & C_1(a_2C_2 + a_3C_{23}) \\ S_1C_{23} & -S_1S_{23} & C_1 & S_1(a_2C_2 + a_3C_{23}) \\ -S_{23} & -C_{23} & 0 & d_1 - a_2S_2 - a_3S_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, a1 is given here 0 and theta 1 is not known, so this also will be 0, a1 is 0, so this will be 0 d1 is there, so you can write it is 215 or you can write this d1 also. Finally, you can substitute, so it will be 215 and then what will be this one, alpha 1 is minus 90. So, cos minus 90 is, what is cos 90, 0. So this will be 0, this will be 0 and sine minus 90, that is sine 90, sine 90 is 1 sine minus 90 minus 1 yeah. So, you will be getting it as the matrix here, minus S1.

So you will be getting it as, okay we have written it as d1 itself, you can substitute the d1 the numerical value can be given or you can write this d1. So, this will be the T1 0 matrix. Same way you can get T2 1, you can get T3 2 also. So T2 1 and T3 2 also can be obtained by simply substituting these values. You do not need to do anything other than that, just substitute these values and then get the matrix 2 to 1 and 3 to 2.

So, finally the wrist to base. So, if I bifurcate this into wrist and base, wrist and tool. So, wrist to base is T. So, if I write this as wrist to base, it will be 1 to 0, 2 to 1 and T3 2 that is going to be the wrist to base transformation. So, multiply these three matrices that is going to be the wrist to base relationship. That is the position of the wrist with respect to the base and the position of the and the orientation of the wrist frame with respect to the base can be obtained by this transformation matrix that is wrist to base.

So you need to multiply this, this is maybe somewhere you make mistakes. But nowadays with all this symbolic computation, you can do it in a computer and then easily get it. So, you can see here this is a2C2 and a2S2, this a3C3 and a3S3 for the positions and if you go back to the pictures and then try to understand you will be able to understand why it is coming as a2C2 and a2S2, because we are trying to go up and down in the same plane. So, your X will be a2C2 and Y will be a2S2. So, we will get this as this relationship.

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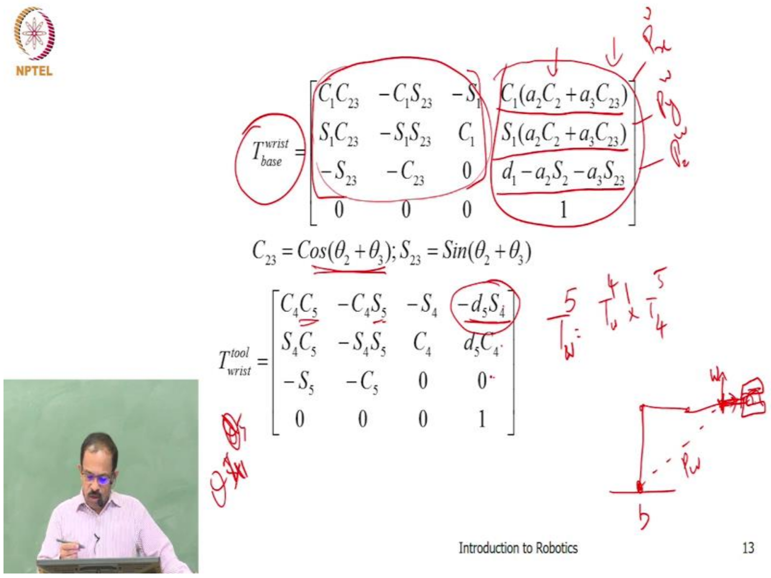

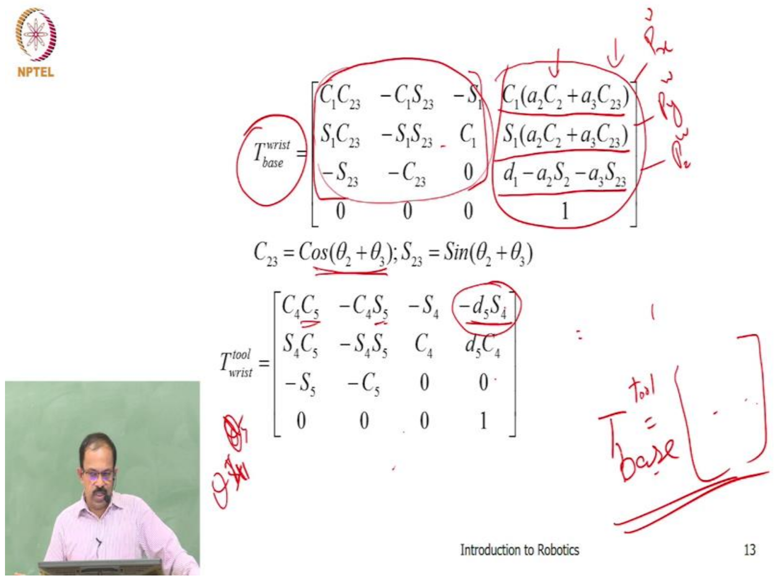

So, once you multiply this, you will be getting the wrist to base as a 4 by 4 matrix and you will see this one expression, just got cos theta 2 and theta 3 and theta 3. So, C23 stands for cos theta 2 plus theta 3. So, this is what actually S23 C23, this is cos theta 2 plus theta 3 is given as cos 2 3 nearly sine theta 2 plus sign theta 3 is given as S2 3. So, this wrist to base, so if you go back to that diagram, it was something like this one join here, one join here, one.

This was the wrist point. If this is the wrist and this is the base and then you have this two degree of freedom here, the roll and the pitch. So, the wrist point here you can see this is the P position of the wrist and the orientation of the wrist is basically the frame attached to that.

So, this says that here the position of the wrist is a2C2 plus a3 C2 3 minus multiplied by C1 that is the Pxw and this is Pyw and this is PzW, that is the position of wrists in x y z with respect to the base frame is given by this and the orientation of the wrist with respect to the base frame is given by this matrix.

So, this is the position vector of the wrist point, this is the orientation of the wrist frame with respect to base. Same way you can get from tool to the wrist also by multiplying 4 to 3 and 5 to 4, we have only 2 degrees of freedom for the wrist. So it will be, this can be obtained as T5 to 3, so i could adjust this so if 4 to 3, 4 to 3 and the 5 to 4. So, that will be the tool to wrist. So, 5 is tool, we will be getting this two matrices, multiplying you will be getting and you can see it is a function of d5 and S4 similarly C5 and S5.

So, it is a function of theta 5 and d theta 4 and theta 5, function of theta 4 and theta 5 and d5 is constant. So, variable will be theta 4 theta 5 and d is a constant. We will see C4 C5 minus C4 S5, S4 C5 minus S4S5 this will be the position of the tooltip with respect to base frame, with respect to wrist frame and this is the X position of the tooltip with respect to wrist frame, this is 5 position with respect to wrist frame and Z position is here, because rotation with respect to Z axis of that positon that is 0.

$$T_{\text{wrist}}^{\text{tool}} = T_3^4 T_4^5$$

$$= \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_4C_5 & -C_4S_5 & -S_4 & -d_5S_4 \\ S_4C_5 & -S_4S_5 & C_4 & d_5C_4 \\ -S_5 & -C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, now we got this two transformations that is wrist to base and then tool to wrist. Finally, we are interested in knowing what is the transformation from tool to base? So, this is what we are interested in. Now, multiply these two matrices, multiply this two matrices, you will be getting the tool to base transformations, which gives you the complete forward relationship of the manipulator and when you do this, again it will be a the expressions will be slightly longer in this case.

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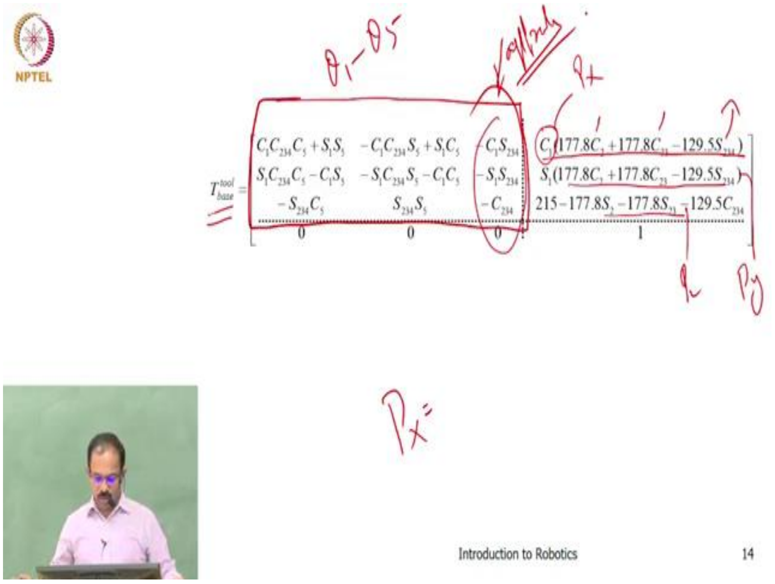

So, you will be getting this as tool to base as this matrix relationship. So, this is the final transformation matrix between the tool frame and the base frame, you will see that the Px is given by this. The position of the tool with respect to the base Px is given by this, this is Py and this is Pz.

So, Px is equal to cos theta 1 multiplied by 177.8 C 2 plus 177.8 C23 minus 129.5 S 234 will be the Px. Now we can see that the position of the tooltip Px is a function of the theta 1. So as a first joint changes, its Px changes, as theta two changes, it changes, theta 3 changes, position changes, theta 4 changes and it is not a function of theta 5, because theta 5 is the roll axis, the roll axis is not going to change the position of the tool with respect to the base frame.

$$T_{base}^{tool} = \left[\begin{array}{ccc:c} C_1C_{234}C_5 + S_1S_5 & -C_1C_{234}S_5 + S_1C_5 & -C_1S_{234} & C_1(177.8C_2 + 177.8C_{23} - 129.5S_{234}) \\ S_1C_{234}C_5 - C_1S_5 & -S_1C_{234}S_5 - C_1C_5 & -S_1S_{234} & S_1(177.8C_2 + 177.8C_{23} - 129.5S_{234}) \\ -S_{234}C_5 & S_{234}S_5 & -C_{234} & 215 - 177.8S_2 - 177.8S_{23} - 129.5C_{234} \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

That is why Px is not a function of theta 5, but it is a function of theta 1, theta 2, theta 3, and theta 4 plus are the DH other parameters, fixed parameters. Similarly, here you can see Py and this is also Pz. This is the orientation of the tool frame with respect to the base and here you will see it changes with respect to all the joint angles. So, theta 1 to theta 5, it is a function theta 1 to theta 5. So, any changes in any one of joint angles will change the orientation of the tool with respect to the base frame.

Pardon.

Column.

This one, yeah, so this one right? Yeah. So why is it so? So what is this vector? This is an approach vector. So approach vector is, so I am holding this, this is the approach vector and any rotation of this, so it is not going to change. So, it is going to be the same one. So, the 5 is not going to affect along that axis the approach vector is not going to change. This vector will remain same even if I rotate like this. But if I rotate with respect to this then the vector is changing. So, that is why approach vector is not a function of theta 5.

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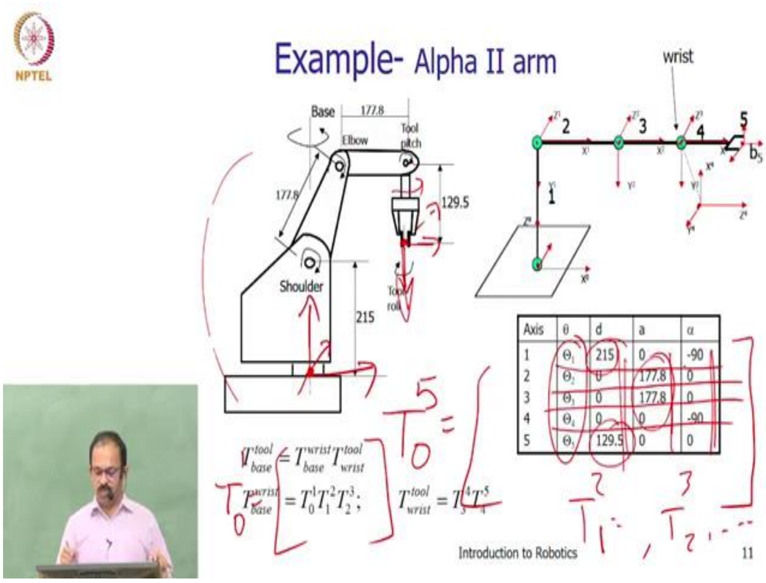

So, you can actually go back to the diagram, yeah, now look at this manipulator. So, this is the approach vector and even if you rotate with respect to this, the vector remains the same vector is not going to change and similarly, you can see the position of this is a function of theta 1, theta 2, theta 3 and theta 4, theta 5 is not going to affect the position, because theta 5 is again roll.

So, the position will not get changed, by simply be rolling this having that roll rotation, roll motion. So, that is why it is the position is independent of theta 5 and orientation of course gets affected. Now you see all these parameters are actually during the relationship, 129.5, 177, alpha has been taken already into account.

Axis	θ	d	a	α
1	Θ_1	215	0	-90
2	Θ_2	0	177.8	0
3	Θ_3	0	177.8	0
4	Θ_4	0	0	-90
5	Θ_5	129.5	0	0

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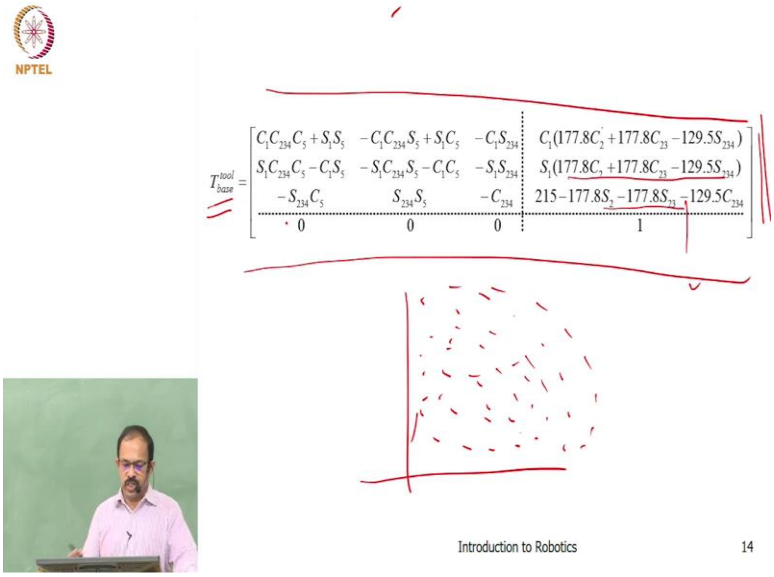

So, finally you have the relationship. Now what is the benefit of having this relationship? Now you do not need to worry about the orientation of the robot or anything. Now you have a complete mathematical representation of the position and orientation of the tooltip. For any value of theta, you can actually find what is the position and orientation of the tool. Whatever maybe the values if they are within the limits of the rotation, you would be able to find out the position.

Now if we have a manipulator, we have a robot and we want to find out its, where are the what are the points it can reach, you simply substitute the values of the theta 1, theta 2, theta 3, theta 4 whole range you will be able to see all the points where actually it can reach, that it actually becomes workspace of the manipulator. So for any value of theta, you will be able to get this relationship, I mean the position and orientation of the tool can be obtained for any value of theta.

So, this is the forward kinematic relationship for this particular manipulator. So, for any given manipulator, you will be able to find a relationship like this. Once you have designed the manipulator, then this will be fixed, only the theta or the d can change, all others will remain same.

So, you will be able to represent the position and orientation of the tooltip as a function of the joint variables as a theta or d and get the position and orientation for any value of this joint variables and that is known as the forward kinematic relationship. Because you know theta or d, you find out the position and orientation and then find out the relationship. Any questions? Fine.

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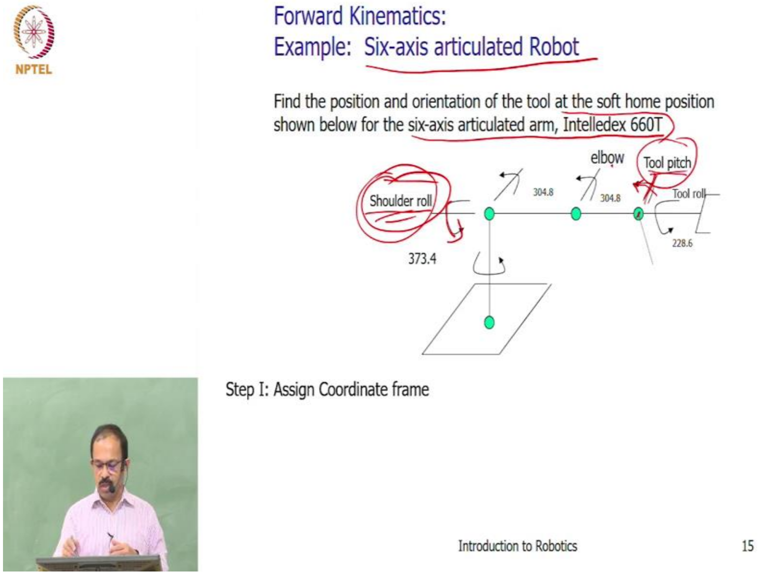
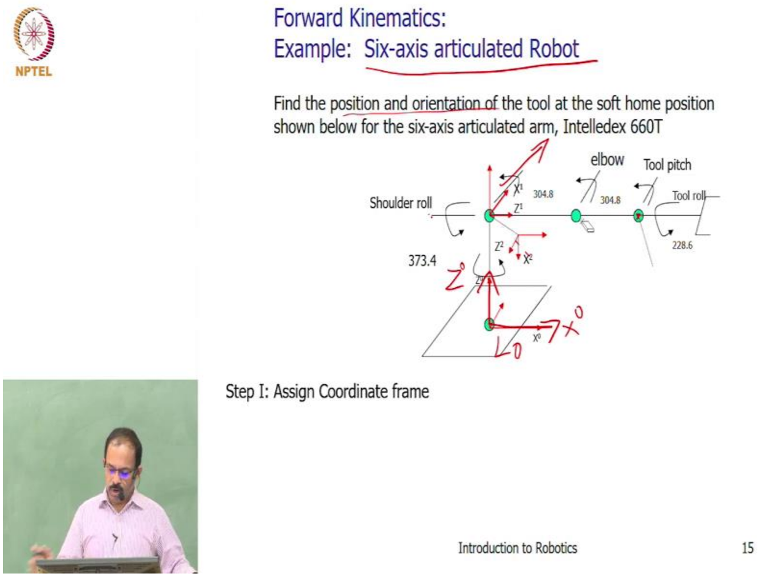
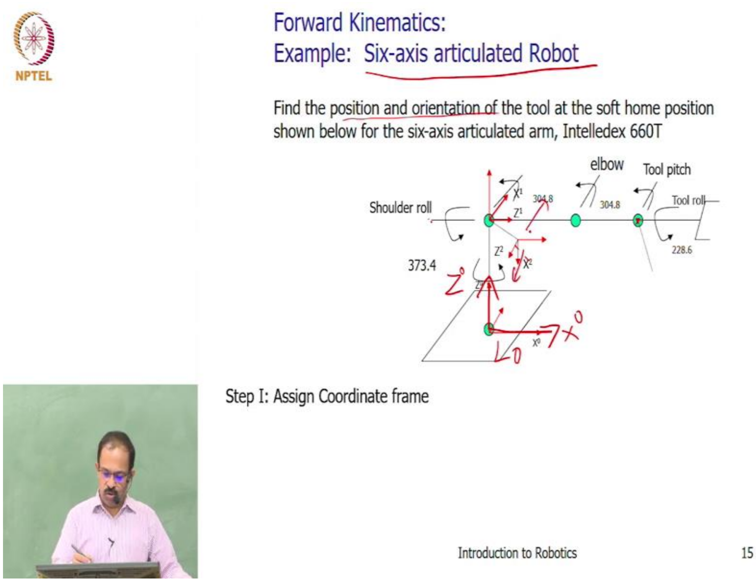
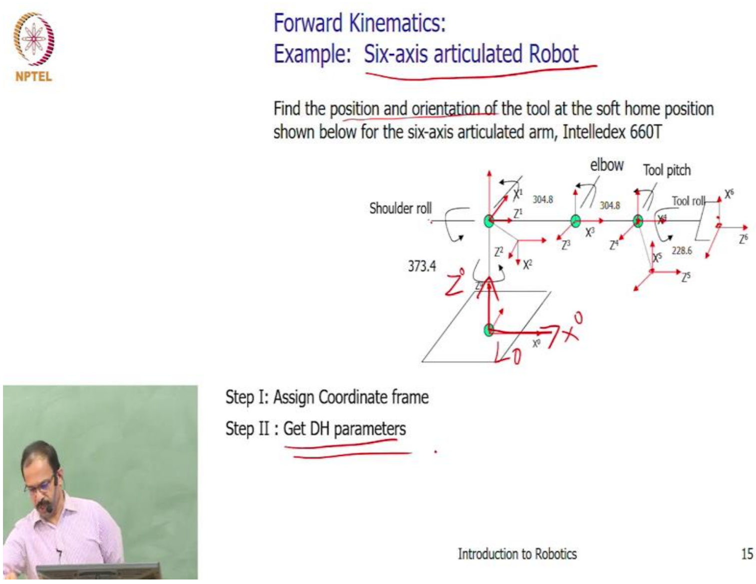








So, if you have understood this, please try to solve this, I will help you. So, please look at this six-axis robot. You have to find the position you have to find the position, orientation of the tool at the soft home position shown below for the six-axis articulated Arm Intelledex 660T. So, this is a commercial robot Arm known as Intelledex 660T. It has got six-axis. Only difference is that you have a roll here, a shoulder roll here compared to other robots and then you have a tool roll also here and you will see that there is a tool pitch here, but there is not tool yaw in this case.

You do not see a separate tool yaw axis here. But instead you will have a shoulder roll in this case. So, it has got six-axis. So, do you think it has all the 6 degrees of freedom or they are, is it a redundant manipulator or manipulator with the less number of joints than required.

Because you do not see a yaw joint here, yaw axis. So, you will be able to position the tool at any arbitrary yaw? Yes or no? You understood my question or not first say?

So, whether you consider this as a holonomic one or a non-holonomic one. So, one advantage here is that we can actually use this shoulder roll, the shoulder roll can be used to convert this axis to a yaw axis. So, you if this is the current elbow or this is the elbow you have a pitch. Now, this is the pitch axis but if I want a yaw, I just rotate it like this, and then do this. Now, I got yaw axis. So, this can actually act as a pitch or yaw axis by using a shoulder roll.

So, basically, you are having twist roll axis and you will be given a pitch axis, so you will still be able to control the yaw because you have an additional roll here, which can position it using the, can control yaw using this two joints. So, effectively just got all the 6 degrees of freedom, so it can actually be it is actually ergonomic robot.

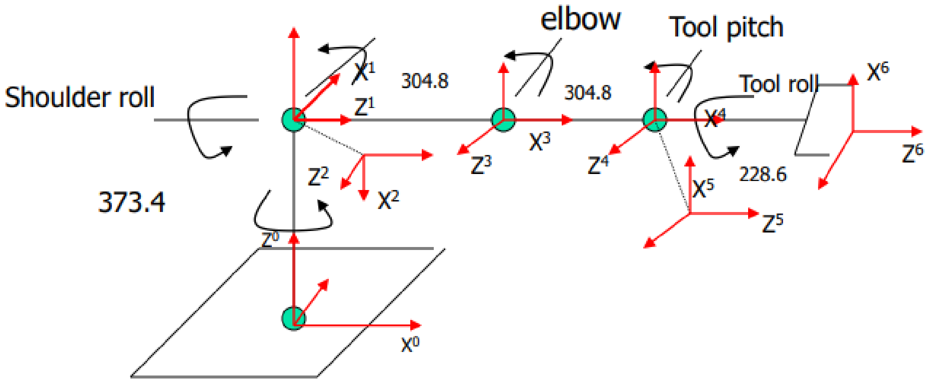
Yeah, so that is the question I am going to ask you. So, let us see how to assign the coordinate frames here. First question, the first thing to do is to assign coordinate frame. So we know that how to do this. So, we will start with this. So, this will be Z0, X0 and I call this as L0. Now, you have a shoulder roll and a shoulder, we call this as a shoulder pitch, this is shoulder roll, this is shoulder pitch and this is elbow pitch and then tool pitch and then tool roll, that is the way how it is defined.

So, we need to look at each, you can assign this as the first one or this as the first one depending on how you assign this. Either you take this as the second axis or this as the second axis, then that shown as the third axis. Now we have to see, the same principle you should follow. So your axis will be like this your Z will be like this and this will be the Z.

So, you will be getting a frame here, the origin will be there and once you have the Z axis, then you assign the X axis and Y axis based on the principle, it should be orthogonal to this and do go with the same procedure or follow the same rules, you will be getting the coordinate frames.

So, I am assigning this as the Z1 axis and you have the X1 and Y1. Because X1 should be orthogonal to Z0 and Z1, therefore you take X1 in this direction and then you assign Y1 and the next joint, what will be the next joint. The origin, you have to see first the origin, so the axis will be this one, that will be the Z axis and this is the Z axis. So, they intersect at the same point and therefore we will be having it at the same origin, it will be at the same origin and then go with the same rule.

So, you will be getting it as Z2, I have taken this in this direction. As I told you I can take in this direction or this direction, so any direction you can take. So, this is Z2 X2 and follow the procedure, you will get Z3, Z4 and then finally, your Z5 and Z6, Z5 and of course you have to assign here also, yeah. So, this is the way how you assign coordinate frame. So, I have not done anything new here, whatever we discussed in the previous classes, I just did it without going into the details, how did you get each one of this.



So, as you know Z2 this axis, this axis and this axis, they are parallel, the joints are parallel. Therefore, you get Z2, Z3 and Z4 in the same direction and then since the origin is to the same point, Z5 is in this direction, the tool roll is in this direction and therefore Z5 and ZX, Z4 intersect here at the same point. So, we take the same origin here and then assign the other axis and finally the tooltip will be here Z6. So, that is the way how you get the, time to stop.

So, this is the thing. So, now get DH parameters, so I am giving this to you. So, please find out the DH parameters and bring the DH parameters tomorrow, when we will see how to develop the forward kinematic relationship for this. So, once you get DH parameters, then there is nothing to do other than the matrix multiplication. So, get the forward kinematics relationship for this robot. So, see you tomorrow morning. Thank you.