Fundamentals of Automotive Systems
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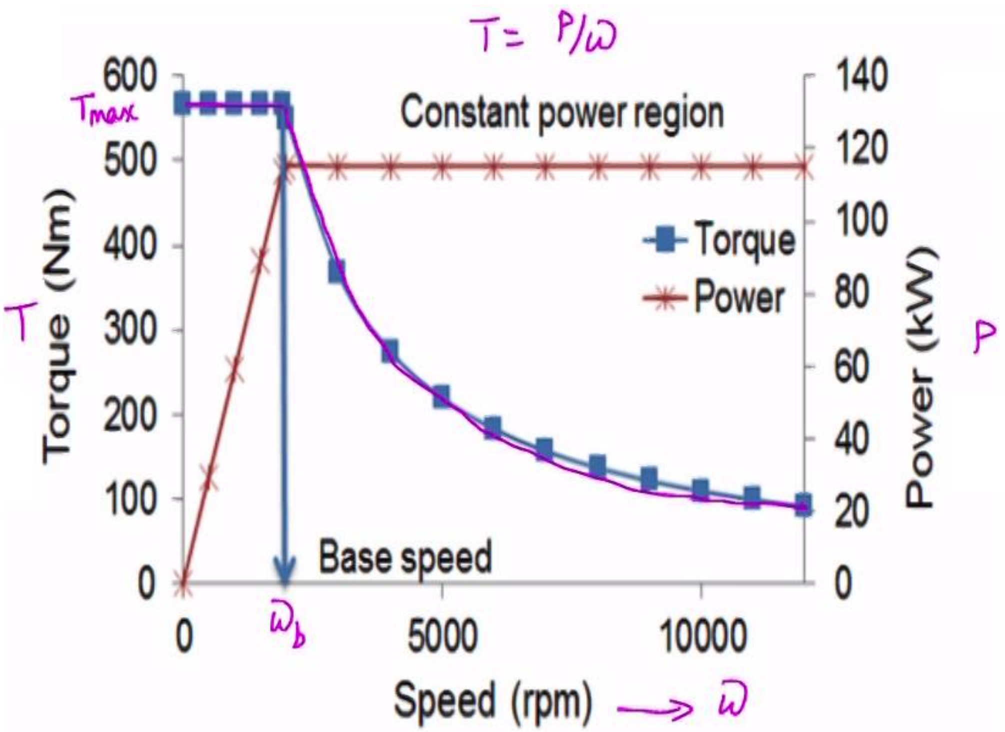
Introduction to Electric and Hybrid Powertrain - Part 02



MOTOR CHARACTERSTICS

So let us look at few characteristics of electric motor powertrain which have made them also attractive from the perspective of powertrain application ok. So if you look at the broad characteristic of an electric motor and please note that you know like an IC engine powertrain the energy system should also provide you know like in energy for let say you know the air conditioning system and it should drive the AC car AC and then it should also be providing power to the let say power steering system.

If you have an electric power steering the energy source should provide the energy to drive all those systems right what we called as auxiliarly devices. So if we look at a typical electric motor we can visualize the torque speed and power speed characteristics of an electric motor in this way. So what happens in an electric motor is that like the torque delivered by the electric motor remains by and large constant tell what is called as a base speed.

So let us called this as a ω_b. So in this case in this graph we are plotting the speed of the motor omega angular speed this is the torque and the power. So we can see that the motor delivers here let say a maximum torque T max till what is called as a base speed. And the power increases in this region and then like it is becomes a constant beyond the base speed. So at the base after the base speed the power remains a constant and then what happens to torque T equals T times omega. So or T becomes P by omega right.

So if the power is constant we can immediately see that the torque is going to go like a hyperbola right beyond the base speed. So this is a very nice characteristic of an electric motor right is not it? Because when we discuss the requirements from a powertrains perspective as regards to the characteristics of prime mover we wanted an ideal traction hyperbola. So an electric motor beyond the base speeds seems to have an ideal traction hyperbola ok. So that is point number one okay.

So we will see how this is going to affect our, what to say design of the electric powertrain. So ideally from this building on this discussion we will see that if we are able to keep the base speed as small as possible right. Then the characteristics are going to go like a hype the torque speed characteristics are going to go like a hyperbola over an extended speed range which is advantageous for us.

But the question becomes are their motors you know like which will provide a very narrow constant torque region ok. So and a much larger constant power region so those are some questions ok which we are going to revisit shortly. So if we look at the electric motor characteristics so we have what is called as the base speed which we shall denote by omega b. So it is the speed till which the motor delivers an almost constant torque ok. So that is the definition of base speed.

Base speed ω_b: Speed till which the motor delivers an almost constant torque.

Speed ratio (x): $x := \frac{\omega_{max}}{\omega_b}$

Then we have what is called as speed ratio? Speed ratio x is defined as the maximum speed of the motor to the base speed of the motor ok. So that is this definition of speed ratio obviously the value of speed ratio is greater than 1 right. So it is the maximum motor speed divided by the base speed. So, a few points to note as far as the motor characteristics are concerned. Let us say we consider a fixed power rating ok.

So essentially what we are going to denote as the P_{max} is given to us ok. So consequently we can immediately observe that for a given P max and increase in x of course given P max and omega max right. So an increase in x what will happen with an increase in x? If omega max is fixed with increase in x omega b will reduce. If omega b reduces means the point at which the point after which that power become constant is shifted to the left in this curve right.

So essentially if we are reducing x this point is going to be shifted to the left. So an increase in x ensures a longer constant power region and consequently a higher motor maximum torque why? Because if we are writing P equals T omega this base speed what will happen? We will have P_{max} to be equal T max times omega b right. If I am are reducing omega b for the same P max T max is going to increase.

So if we have a higher maximum motor torque what will happen this will imply better acceleration and gradeability characteristics is it not? Because the gradeability; and acceleration are decided by the initial torque that we get out of the prime mover. So essentially, we are going to get better acceleration and gradeability performance ok. So that is the advantage but there is an depending of type of a motor there is an upper limit on the speed ratio of the motor.

We cannot keep on increasing the speed ratio yes, we can increase it numerically but practically such motor should be available right to be used in vehicle. So there is an upper limit on the value of x ok. Another feature which we would be observing due to this hyperbolic or torque speed characteristics from an electric motor is that if we look at typical electric vehicle, we may even have a single speed transmission right. So there may be only one gear ratio or sometimes there will be two gear ratios right.

So the choice of the gear number of gear ratio is also decided by the corresponding motor torque speed characteristic viz a viz the requirement from the vehicle performance perspective. But the main point to note is that the number of gear is certainly reduced because the electric motor has a hyperbolic torque speed characteristic be beyond the base speed. So that is the advantage for us for a powertrains perspective. So we do not require as many gears as we what to say required with an internal combustion engine driven powertrain. So that is an important advantage of an electric powertrain.

So the choice of a single gear or multi gear transmission depends on the torque omega characteristics of the motor ok that is important. If a motor has a high value of x that results in a longer constant power region because if we decrease the value of x omega b is going to be sorry if we increase the value of x omega b is going to be reduced right. Omega b reducing means the base speed reducing means we are going to have a longer constant power region ok then a single transmission may be sufficient.

So that is one important point to note ok. So now let us consider a single speed transmission. You know like and then we will just write down some quick write down some expression quickly from what we have already learned before from powertrain perspective so that we get a picture of how we can get the initial powertrain ratings you know from the perspective of vehicle performance.

$$F_{max} = \frac{T_{max} N_t \eta_t}{r_w}, \quad N_t \rightarrow transmission\ ratio, \quad \eta_t \rightarrow transmission\ effciency$$

So let us first look at the what to say various requirements of vehicle as per as powertrain is concerned like maximum speed gradeability and acceleration right. Before we go there just a one more point to note please note that the relationship if I am considering a single speed gear box ok with an electric vehicle the relationship between T_{max} which is the maximum motor torque and the corresponding maximum traction force let us say we call it as F_{max} is F_{max} is going to be equal to T_{max} times N_t eta t by rw ok.

Where N_t is the transmission ratio ok. Eta t as we know is the transmission efficiency ok. So these are expression that we are already familiar with. So the transmission is going to transmission ratio is multiplied with the output of the motor then multiplied by an efficiency term and divided by the wheel radius right to get the longitudinal force at the longitudinal force that is available from the powertrain to drive the vehicle ok.

So now let us look at various performance requirements from the vehicle perspective and see how we can obtain the ratings for the motor right based on what we have learnt. So maximum vehicle speed so recall that the maximum vehicle speed maximum vehicle longitudinal speed, V max and the prime mover speed in this case the prime mover is the electric motor right so and the maximum speed maximum motor speed.

$$v_{max} = \frac{\omega_{max}}{N_t} r_w (1 - \lambda)$$

So I would not say recall because we initially wrote down for an engine so we did not write for a motor so let me write the statement so maximum motor speed omega max. So the statement is the maximum vehicle longitudinal speed and the maximum motor speed are related through a following equation which is very similar to what we wrote down when we did the analysis for the conventional IC engine powertrain right.

So what is V max? Let us say omega max is the maximum motor speed right. Omega max divided by Nt will give me the maximum wheel's rotational speed right corresponding to omega max. Then if we multiply by rw that is going to give the corresponding circumferential speed at the wheel then we go and multiply by 1 minus lambda where lambda is the wheels slip ratio that will give us V. So we can observe that this something which we already derived when we discussed transmission right.

So the concept is the same we are just applying for an electric motor ok. So this is the maximum longitudinal speed the relationship between the maximum longitudinal speed and the corresponding motor maximum speed right. So now the corresponding power and torque requirements are the following. So what are the corresponding power and torque requirements so if we look at the torque required to essentially satisfy this V max what do you think are the vehicle loads that the powertrain has to overcome we are cruising at longitudinal maximum longitudinal speed right.

The corresponding power and torque requirements are

$$T_{reqd}@v_{max} = \frac{\left[f_r\, W + \frac{1}{2} S_a C_d A_f v_{max^2}\right] r_\omega}{N_t \eta_t}$$

$$P_{reqd}@v_{max} = \frac{\left[f_r\, W + \frac{1}{2} S_a C_d A_f v_{max^2}\right] v_{max}}{\eta_t}$$

So the main on a flat road assuming a flat road so the main resistance is the vehicle powertrain has to have to overcome are the rolling resistance and the aerodynamic drag right. So that is going to be half Rho a CD V sorry Af V max squared right this will be the force or the load that the vehicle powertrain should overcome multiplied by the wheel radius will be the torque at the wheels then we divide by the transmission ratio and the transmission efficiency.

So this will tell us what will be the corresponding motor torque requirement. So how did we get this we just got it by here right. So essentially from this equation we can see that T max is going to be equal to F max times rw divided by Nt Eta t that is it okay. So what we did we just substituted for F max while going at maximum speed you know F rw and the aerodynamic drag are the forces that we need to overcome. So that is going to be the rating of the motor ok.

So then what will we get for power? Power required for V max will be this thing the force that we need to overcome right. This is the power rating of the motor divided by sorry multiplied by V max that will be the power required at the wheels then we divide by the transmission efficiency right. So that will give us the corresponding motor rating you know which would essentially be required to drive the vehicle at this maximum speed because what is this F times V.

F times V is the power right. So in the numeric instead of writing torque times omega I have just written F times V. So and then divided by the efficiency term and that will give us the power output ok. So that is as far as maximum vehicle speed is concern.

So then the second one is gradeability. So as we know gradeability is indicative of the maximum slope that the vehicle can climb upon right gradeability is defined as tan theta S max. So now similarly we can essentially write down the torque motor torque required for satisfying gradeability right. What will that be? That will be great resistance plus rolling resistance plus aerodynamic drag which will be this ok. So these are the forces that the vehicle powertrain needs to overcome.

Gradeability:

$$T_{req}@G_r = \frac{\left[W sin\theta_{max} + f_r W + \frac{1}{2} S_a C_d A_f v_{grade^2}\right] r_\omega}{N_t \eta_t}$$

$$P_{req}@G_r = \frac{\left[W sin\theta_{max} + f_r W + \frac{1}{2} S_a C_d A_f v_{grade^2}\right] v_{grade}}{\eta_t}$$

So how do we get the torque motor torque we multiplied by the wheel radius divide by the transmission ratio and the transmission efficiency. Then the corresponding power required would be what just the force that we wrote on the above right the great resistance plus the aerodynamic drag plus the rolling resistance right. Then once again we multiply by the speed at which the vehicle is going on the grade divided by the transmission efficiency ok. So this is how we get the expression for gradeability ok.

So then the last requirement that we will look at is the acceleration performance. So in an electric powertrain we will see that one more advantage is that like we will be able to get an explicit relationship between the torque and power requirements to satisfy the acceleration performance ok. So that is an advantage we will see how we get a simple expression for this let say we desired to go from a speed of zero meters per second to V_f in a time interval of t_f seconds let say ok.

$$\int_{0}^{t_f} dt = m \int_{0}^{v_f} \frac{dv}{F(t)}$$

$$t_f = m\left[\int_{0}^{0_{v_b}} \frac{dv}{\left(\frac{P_{max}}{v_b}\eta_t\right)} + \int_{v_b}^{v_t} \frac{dv}{\left(\frac{P_{max}}{v}\eta_t\right)}\right] = \frac{m v_b^2}{P_{max}\eta_t} + \frac{m}{P_{max}\eta_t}\left[\frac{v_f^2 - v_b^2}{2}\right]$$

$$\Rightarrow \boxed{P_{max}@\ \boldsymbol{acc} = \frac{m}{2\eta_t t_f}\left[v_b^2 + v_f^2\right]}$$

So that is our requirements right so for acceleration. So let us consider only inertia for time being of course when we are acceleration let say we accelerate on a flat road we inertia will be dominant terms that is why we considering inertia. We will see what happens if we add the other terms like rolling resistance and aerodynamic drag shortly ok. So let us consider inertia by utilizing the Newton's cycle second law of motion we can obtain this following equation.

So where we are just integrating over the acceleration time to figure out what will be the torque and force requirement ok because the force requirement F of t you know keeps on changing right and also please note that as a vehicle speed changes the output from the prime mover, also will change because the torque will remain constant till base speed after that it will fall hyperbolically right. So that is why I am writing F as a function of time. So if we were to integrate this equation so we will get t_f on the left hand side then we will have m.

This we are going to split into two regions. So we will see that this force or torque we can see that it is going to remain constant in the base speed region then it is going to fall hyperbolically and so there it will become a function of speed right. So using that fact so we are going to see that this so this is not tf this is V_f sorry ok I am going to integrate from zero to V_f on the right hand side right. So we will split into two region zero to V_b and V_b to V_f. Of course, I am assuming that the final speed is more than the base speed which is the reasonable assumption to make ok.

So integral zero to V_b so then it will become dv divided by So here the force output that we will get is going to be P_{max} V_b times eta t because how did we get this at the base speed P_{max} equals T_{max} times omega b or I can say F times V divided eta T alright. So I just essentially make use of that equation to get this expression. Then from V_b to V_f we will have dv by P_{max} by V. Now we can see that this is the function of time right. So let me right it as P_{max} by V times eta t. So this will be two paths ok.

So if we integrate we are going to get m times V_b square divided by P_{max} times eta t + m / P_{max} eta t V_f square - V_b square by two ok. So if we simplify this equation we will observe that we are going to get P_{max} equals m divided by 2 eta t tf V_b square plus V_t square. So we observe that we have got a simple expression that tells us what will be the motor rating to satisfy a given acceleration performance ok. So let me write it as P_{max} to satisfy this acceleration requirement ok.

So the last part which I am going to essentially point out to you is that in this expression for P_{max} we have not considered rolling resistance and aerodynamic drag. So if you want to get a first cut expression to include those how do we do that?

$$P_{addl} = \frac{1}{t_f} \int_0^t \left[f_r W + \frac{1}{2} S_a C_p A_f v^2 \right] v \, dt$$

Assuming $v = v_f \sqrt{\frac{t}{t_f}}$

So if aerodynamic drag and rolling resistance are considered the corresponding additional power is the following additional power requirement is the following. So P let me put as P additional that is in addition to this P max what we derived here right so that is why I am calling as additional. So this is going to be 1 by t_f zero to t $f_r W$ $f_r W$ is the rolling resistance half Rho a C_d A_f V square that is going to be the aerodynamic drag.

So what is within the square bracket is the net force that needs to be overcome resistance need to be overcome. Times V will give us the instantaneous power then we integrate over the entire region of acceleration and we average it over the time. So this, more look like a time averaged quantity alright so this how we will get it. Of course, here how we will see that we need to substitute for V alright because we are integrating over time but V is also a variable right. When we are accelerating, we are going to go from zero to V_f.

So to get a first cut relationship you know like what we do is that like we assume that the speed evolves with time in this manner that is the speed is what to say V is going to Vf times square root of “t” times by t_f ok which is the reasonable assumption to make looking at typical vehicle times P profiles during acceleration ok.

$$\Rightarrow P_{addl} = \frac{1}{t_f}\int_0^t \left[f_r W\ v_f \sqrt{\frac{t}{t_p}} + \frac{1}{2} S_a C_D A_f v_f^3 \left(\frac{t}{t_f}\right)^{3/2} \right] dt$$

$$\Rightarrow \boxed{P_{addl} = \frac{2}{3} f_r W v_f + \frac{1}{3} S_a C_D A_f v_f^3}$$

If we do this then this implies that the additional power requirement is going to be equal to 1 by t_f integral zero to t f_rW V_f square root of t by t_f plus 1 by 2 Rho a CD A_f V_f cube t by t_f to the power 3 by 2 d_t wait a minute I think this parenthesis should come here ok. So this is the additional power that is required. So if we do the integration we will observe that this P additional is going to be equal to 2 by 3 f_rW V_f plus 1 by 5 Rho a C_d A_f V_f cube ok. So this is the additional power that needs to be provided by the powertrain in addition to overcome the inertia right.

If we consider rolling resistance and aerodynamic drag so what do these simple expressions are provide us with. They provide us some tools you know like to calculate the torque and the power requirements for satisfying these various vehicle performance requirements. So the impact of these derivation is that if we are given some basic vehicle performance requirements we could use this experiment to get corresponding motor power rating.

Obviously depending on the type of vehicle and the type of operating conditions so these numbers and are going to have a relative variation right. See for example if we are going to have a high-performance car the acceleration power may be significant ok. So we have to calculate these rating and make a decision on what our final powertrain will be. So that we will satisfy all the performance requirement ok so that is the idea ok.

So I will stop my discussion on this electric powertrain with this. I just wanted to give a broad overview of what an electric powertrain is? What are its various realization and the characteristics perspective of powertrain and some simple analysis to supplement what we learnt in our conventional transmission analysis you know that was the objective of this particular lecture fine thank you.