Fundamentals of Automotive Systems

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Power Steering and Kinematic Steering Analysis – Part 02

Class of power steering system which are becoming very popular particular for passenger cars today is what is called as electric power steering okay. So as we can readily guess what the transition is right from hydraulic electrohydraulic then we are going to pure electric so as the name indicates an electric motor directly provides the steering assistant okay. So that is the feature of an electric power steering or an EPS okay so this is what is called as an on-demand assist system.

So compared to the other two configuration you know this is has better characteristics in the sense that you know like electric motor directly drives does the assist so the response characteristics are better right. So if you look at what is on-demand motor system the motor or the assist energy the assist is provided when the driver steers and of course the motor can provide the assist depending on the design at the various places it can be closer to the steering wheel you know like it can be towards the end of the steering column okay it can be on the rack okay.

So depending on the designs there are various configurations okay as far as electric power steering is concerned. So the assist is provided only when the driver steers okay and this also leads to lower fuel consumptions why because compared to hydraulic power steering now where the energy is consumed in operating the motor pump continuously right so this results in lower fuel consumption and consequently even reduce CO_2 emissions right because fuel is saved right so essentially these are positive benefits of electric power steering.

And let us say it is more amenable to active control right so essentially as we know you know electric motors can be are more tractable for real time control even we can control hydraulic systems also in real time but it is easier to what to say build these controlling and program these controllers with an electric motors right. So essentially the power assist characteristics can be oops okay can be readily controlled readily changed controlled by appropriately programming the motors ECU right.

So that is a big advantage for an electric power steering okay so another advantage is it can work even when the engines is stalled as long as the motor has energy input right so even when the engine is stalled right. But what is the limitation please note that the motor requires energy to be operated. Now if the size of the vehicle increases the steering assist is more then the motor rating increases then the amount of energy which needs to be provided to the motor also has to be more.

So in a typical electric power steering it will draw the energy from a battery and some which is interfaced with some intermediate power electronic circuits right. Now if we want a higher rating motor the power density of the energy source should be higher okay. So but if we go with the existing batteries you know what will happen is that the then the rating of the energy source required to operate the motor itself will become higher right. So things are improving but that is a limitation which we need to keep in mind right when we are trying to adopt electric power steering for heavier vehicles okay.

So, one limitation is that the energy density for the motor power source the power this is the assist motor right. The assist motor power source should be high enough to enable application in heavier vehicles okay so that is one limitation of electric power steering okay alright. So this essentially with this we have taken a broad overview of this type of steering systems and the configuration or components or operations and also power assist systems.

So the next what to say task we are going to do of course we have been doing some analysis as we are going along but we now we are going to analyze this steering response from a kinematics perspectives okay.

So let us do a simple first cut analysis essentially to analyze the kinematic response of the vehicle when it is steered okay. So for this purpose we will use what is called as a we just start the process we shall use what is called as a bicycle model okay for a car. Of course we are considering a 4 wheeled vehicle so in a bicycle model what happens is that the two front wheels are combined into one okay and the two rear wheels are combined into one okay.

So a 4 wheeled vehicle is viewed as a two wheeled vehicle and that is why it is called as a bicycle model. It is a reasonable model to start the analyze okay so let us look at simple kinematics and see how the how one could analysis you know the response of the vehicle when it is steered by the steering system that we are looked at.

So what is this bicycle model let me explain and we will also write down the assumption and we will derive the equations okay. So this is just a simple schematic so this kinematic what to say the simple diagram schematic to essentially look at this bicycle model is as follows okay. So our vehicle is this line segment AB so we have done is that like we have combined both front wheels into one both rear wheels into one and then we are looking at the vehicle from the top.

Kinematic Analysis

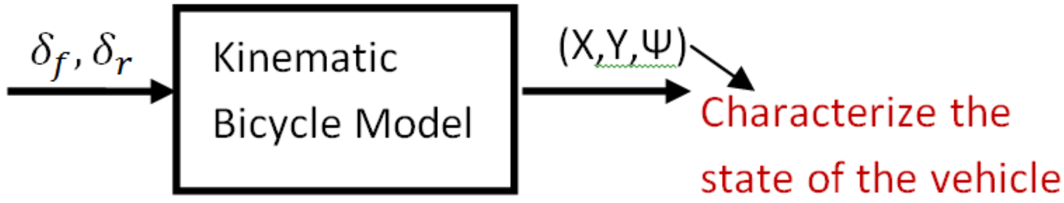
- Bicycle Model ⟶ the 2 front wheels are combined into one.
  the 2 rear wheels are combined into one.

- Kinematic Bicycle Model ⟶ reasonable at "low" vehicle speeds (≤ 5 m/s).
  Slip angle at the wheel is neglected.

So and then observing what happens when the vehicle is steered right so that is our motivation. So please note that when we are looking at this so called bicycle we cannot capture the roll of vehicle body because we have combined this one wheels into one and two front wheels into one and two rear wheels into one this model can only capture the yaw motion or the rotation about the vertical axis. But as we know when we take a turn the body also rolls the vehicle body rolls right so roll is nothing but rotation about the longitudinal axis that cannot be captured by this model but it is okay it is reasonable to start off with so what is this bicycle model?

$Y \rightarrow Heading/Yaw/Orientation$

$\beta \rightarrow Vehicle\ Sideslip\ Angle$



So this AB is the vehicle when looked from the top and as we know the wheel base is capital L and L_1 and L_2 vehicle indicate the location of longitudinal location of the CG from the front wheel and the rear wheel centers okay C indicates the center of gravity alright. So delta F and delta R are the steering angles of the front and the rear respectively okay without loss of generality let us assume that both front wheels and the rear wheels are steered if we do if you are steering only the front wheel delta R becomes 0 okay so that is it right so that is a simplification.

So we are deriving it for a general case right so if only front wheels are steered wherever we have delta R substitute delta R as zero that is it right so only for the front wheels are steered okay. Now this XY capital X and capital Y is an inertial frame of reference so the position of this center of gravity is given by a coordinate capital X and capital Y and this parameters PS is okay what is called as the heading angle or the yaw angle or the orientation angle of the vehicle okay.

So what is this psi? It is the angle made by the longitudinal axis of the vehicle with the capital X axis the horizontal axis okay that is the heading angle. Now once again what we do we draw a perpendicular lines to the wheel plane right and that they intersect at this instantaneous turning center of turn right. So that is the instantaneous turns okay so now if we draw a line segment to the center of gravity capital R is nothing but the turning radius okay.

So now velocity vector at the center of gravity is perpendicular to this line segment because it is rotating okay we are neglecting what is called what are called slip angles we will come to that shortly and then the velocity vector makes an angle beta all these angles are exaggerated by the way you know like this to convey the concept right. So this beta is what is called as the vehicle slide slip angle.

So it is the angle made by the velocity vector at the center of gravity with the longitudinal axis of the vehicle okay. So that is what is called as beta okay. So there are some interesting assumptions that are made when we look at the bicycle okay this bicycle kinematic model okay so what we call as the kinematic bicycle model is reasonable people are found out is that like it is accuracy is reasonable okay at low vehicle speeds.

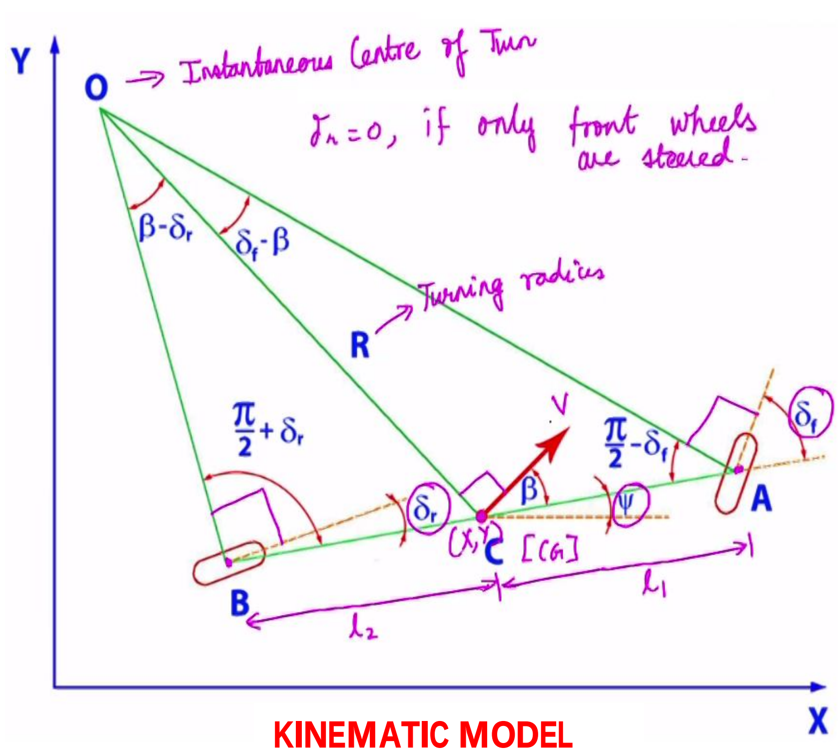
Let me put within what to say quotes so what is how much is low? Typically you know like within let us say 5 meters per second vehicle speed this is kinematic bicycle model is reasonable for road vehicles okay. So it has been found that it is accuracy is reasonable when you compare it with experimental results when the vehicle speed for road vehicles right is less than or equal to 5 meters per second okay so that is satisfies the reasonable model to use. And we neglect what is called as the slip angle at the wheel is neglected so what is this slip angle?

So typically when this when you go to a course on vehicle dynamics you will look at it further so let us say we consider this pneumatic tyre and let us say this is the central wheel plane and let us say this is the let us say this is the initial this is the longitudinal axis of the vehicle. So I am once again exaggerating so let us say this is the angle by which you steer the, what to say the wheel velocity vector at this wheel center will not be along the wheel plane okay.

It will be at an angle to the wheel plane okay this angle which it makes with the central wheel plane is what is called as the slip angle okay and it is very important okay in determining the vehicle dynamic response particularly in cornering response and even also longitudinal response okay. So we will see that this slip angle is very important it affects the longitudinal traction also more importantly the lateral traction okay which is available on the tyre road interface we are neglecting it here okay.

So that is an important assumption in this kinematic bicycle model so if we look at this bicycle model kinematic bicycle model from the big picture perspective. So if we draw it as a block diagram we would immediately observe that the input to this model or the inputs to the model or the steering angles at the front and the rear the outputs from the model are XY and zeta okay. Because why this XY and zeta will characterize the state of this vehicle right at each and every instant of time is it not?

Because let us say someone gives us the coordinates of the CG with respect to the coordinate system and the orientation we know where the vehicle is? Also how it is oriented at each and every instant of time that is why XY zeta will characterize the vehicle state at each and every instant of time okay.



KINEMATIC MODEL

So now we are going to derive the governing equations okay.

Governing Equations

$$\dot{X}(t) = V\cos[\varphi(t) + \beta(t)]$$

$$\dot{Y}(t) = V\sin[\varphi(t) + \beta(t)] \quad \rightarrow \text{(I)}$$

$$\dot{\varphi}(t) = \frac{V}{R}$$

So the governing equations can be very easily returned but then we will go further and look at that okay. So if we look at the governing equations we need evaluation equations for XY and zeta okay. So the evolution equation of X is given by X dot and that is nothing but the X component of the velocity vector at the CG what is that going to be assuming that the magnitude of the velocity is V that is going to be V Cos θ Si of t plus beta of t is not it?

Why Psi of t plus beta of t because we have to consider this angle right so that is what will give me the X component right. Then what will be Y dot similarly it will be just V Sin Psi of t plus beta of t right now what will be Psi dot you know like we are neglecting these slip angles we are considering the vehicle speed to be constant that is another important assumptions okay we consider V to be constant okay like the vehicle to go at a constant speed right.

So if it is going at a constant speed we are taking a steady state turn so this Psi dot is nothing but angular speed right so it is going to be V by R okay. So Psi dot is going to be essentially V by R in a transient maneuver it would not be okay. So this is basically going at a constant speed along the path of constant radius right so essentially it is going to be V by R right. So then these three equations form a set of governing equations but however we cannot solve them right now right because we do not know what is beta what is R right we need to get those values right because please note that the inputs are only delta F and delta R the outputs are Psi XY and zeta so for this we are going to use simple geometry that is what we are going to do right.

This is something which I am sure all of us can easily figure out so if this angle is delta R we already know why this Pi by 2 plus delta R right because this is the total angle. If this angle is Pi by 2 plus delta R how come this angle is beta minus delta R that is because this angle will be now Pi minus right Pi by 2 correct plus beta right. So then if you use the sum of the three angles of a triangle is sum to Pi radiance or 180 degrees you will immediately get that this angle is beta minus delta similarly if you look at this angle this is going to be delta of minus beta why?

Because we already know this is just Pi by 2 minus delta F that is pretty easy to figure it out this angle is once again Pi by 2 plus beta so this will be delta F minus beta okay. So we can easily get that this angle is delta R minus beta simple geometry okay nothing more than that. So one we have that let us quickly derive the governing equations.

$$Consider\ \Delta\ OAC: \frac{sin(\delta - \beta)}{l_1} = \frac{sin\left(\frac{\pi}{2} - \delta_f\right)}{R}$$

$$\Rightarrow sin\delta_f cos\beta - cos\delta_f sin\beta = \frac{l_1}{R} cos\delta_f$$

$$\Rightarrow \boxed{tan\delta_f\ cos\beta - sin\beta = \frac{l_1}{R}} \rightarrow (1)$$

So let us first consider triangle OAC I am sure all of us remember the Sin rule. So Sin of delta f minus beta divided by L_1 is going to be equal to Sin of Pi by 2 minus delta f divided by R right. Of course I am using the third ratio because that is not needed here right so that is what we get so immediately we can quickly apply trigonometric identity so this will become Sin delta f Cos beta minus Cos delta f Sin beta that is going to be equal to L_1 by R times Cos delta f.

$$Consider\ \Delta\ OBC: \frac{sin(\beta - \delta_r)}{l_2} = \frac{sin\left(\frac{\pi}{2} - \delta_r\right)}{R}$$

$$\Rightarrow sin\beta\ cos\delta_r - cos\beta\ sin\delta_r = \frac{l_2}{R}\ cos\delta_r$$

$$\Rightarrow \boxed{sin\beta - cos\beta\ tan\delta_r = \frac{l_2}{R}} \longrightarrow (2)$$

From here what do we get? We will get if we divide by Cos delta f on sorry Cos delta f on both sides we will get Tan delta f Cos beta minus Sin beta is going to be equal to L_1 by R okay. So let me call this as equation one alright okay. Next what we do we just go and consider triangle OBC. So if we consider triangle OBC and apply the Sin rule once again we are going to get Sin of beta minus delta r divided by L_2 is going to be equal to Sin of Pi by 2 plus delta r divided by capital R.

So this will give us Sin beta Cos delta r minus Cos beta Sin delta r that is going to be equal to L_2 by R Cos delta r. So what will this give us if we divide by what to say Cos delta r we going to get Sin beta minus Cos beta Tan delta r equals L_2 by R right.

So let us call this equation number two so we are almost through so just two more steps so now we can readily observe that if we add equation one and two what do we get? We get Tan delta f Cos beta right minus Sin beta cancels off Cos beta Tan delta r that is going to be equal to $L_1 + L_2$ by R and what is $L_1 + L_2$ by R ? It is wheel base R so this equation will give us an equation for the turning radius R. So L divided by Cos beta times Tan delta f minus Tan delta r.

① + ② : $\tan\delta_r \cos\beta - \cos\beta \tan\delta_r = \frac{l_1 + l_2}{R} = \frac{L}{R}$

$$\Rightarrow \boxed{R = \frac{L}{\cos\beta\left[\tan\delta_f - \tan\delta_r\right]}} \longrightarrow ③$$

① $* \, l_2$: $l_2 \tan\delta_f \cos\beta - l_2 \sin\beta = \frac{l_2 l_1}{R}$

② $* \, l_1$: $l_1 \sin\beta - l_1 \cos\beta \tan\delta_r = \frac{l_1 l_2}{R}$

$$\Rightarrow \cos\beta\left[l_2 \tan\delta_f + l_1 \tan\delta_f\right] - (l_1 + l_2)\sin\beta = 0$$

$$\Rightarrow \boxed{\tan\beta = \frac{l_2 \tan\delta_f + l_1 \tan\delta_r}{L}} \longrightarrow ④$$

Model: { ①,③,④ } ⟶ **can be solved to obtain the vehicle trajectory.**

So we have has we discussed initially we want to get equations for R and beta right if we want to solve equation once so we are not yet through because even you can see that the equation for R depends on beta right so how do we get beta you know one last step that is it. So for getting the equation for beta let us first multiply equation one with L_2 so what do we get? So equation one if I multiply by L_2 we are going to get L_2 tan delta f Cos beta minus L_2 Sin beta is going to be equal to L_2 L_1 by R.

Then we multiply equation two with L_1 right we get L_1 Sin beta minus L_1 Cos beta tan delta r to be equal to L_2 L_1 / R. Now you can immediately see that the right hand sides are the same so you subtract the two equations right what we will get? We will get if you subtract we will get Cos beta right I am just writing the final equation we will get Cos beta times l2 tan delta f plus L_1 tan delta r right minus L_1 plus L_2 Sin beta equals zero alright. And what is L_1 plus L_2? This is nothing but capital L so now we are almost through alright.

So if we simplify we are going to get an equation for beta tan beta is going to be consequently L_2 tan delta f plus L_1 tan delta r divided by capital L okay. So we are done so we have equations for R and beta so do we get beta at each and every instant of time yes we do right. Because delta f and delta r are the inputs L_1 L_2 L we know so we know beta once we know beta we know r and then we can substitute in the governing equation so the final model parse will be the set one equation three equation four okay and these equations can be solved to obtain the vehicle trajectory okay. So this is the kinematic bicycle fine okay. So we will stop here and then we will continue in the next class thank you.