Fundamentals of Automotive Systems
Prof. C. S. Shankar Ram
Department of Engineering Design
Indian Institute of Technology - Madras

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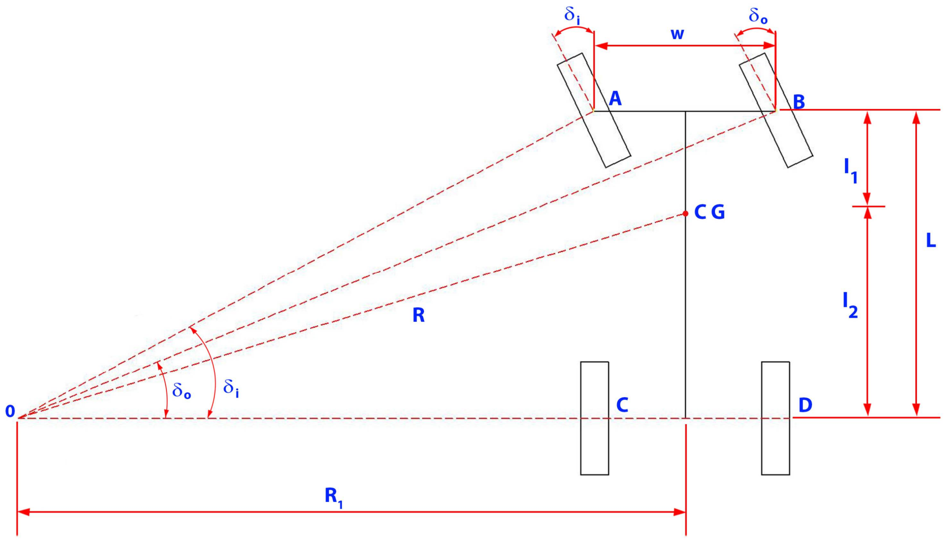
Steering System Part - 02

So now what happens when we displace these links in this Ackerman steering mechanism so that for we are going to look at, right. Then we will see how they are displaced. So we are going from the ground up, right. Then we will go till the steering wheel ok as we slowly look at the entire steering system. So let us consider what is typically called as the Ackerman Steering Condition. So let us look at this schematic.

So let us say that this is a simple schematic which essentially explains what we are going to discuss, ok, based on ok. So suppose let us say we had a mechanism like this and we want to achieve this, right. What do we want to achieve? We want to when the steering wheels are turned by using this mechanism we want to achieve what is shown in this figure, right. So the axes of the 2 steered wheels should intersect the extended rear axle centerline at the same point.

So that is our requirement, right. Suppose if we were to achieve there then what should be the constraint or the condition, right. So based on what has been proposed by Ackerman. So that is what called as Ackerman steering condition. So let us discuss this so let us say you know like we consider this what to say simple schematic where we have a vehicle of wheel base L so we already know that capital "L" is wheel base and our CG is at distance of L_1 and L_2 from the front and the rear wheel centre.



This W is what is called as a track it is not the vehicle width, right. Although people interchangeably sometime use it with width alright so it is what is called as a track of the vehicle right.

And we can see that without loss of generality let us assume that we are taking a turn to the left as per as this figure is concerned. So this delta “i" is the inner wheel steering angle and delta “o” is the outer wheel steering angle ok. So we draw lines which are perpendicular to the axis of the wheel plane. There is, along the axis of the wheels which are perpendicular of the wheel plane. Then let us say, those lines intersect at the same point with the line projected from the rear axle axis, right.

So let us say that is O as we know this is the instantaneous center of turn, right. And we draw a line segment from the instantaneous center of turn to the vehicle center of gravity we get the turning radius which is noted by capital R, right. Now we can immediately observe that this angle is also delta “I” why? Because the 2 line segments are mutually perpendicular to the other 2 line segments, right.

So essentially you know that the included angle between the 2 mutually perpendicular sets of line segments is going to be the same, right. So that is what for this angle is delta "I". Similarly the angle made by the line segments OB with OC is also going to be delta O pretty straight forward, right so that is comes from basic geometry, ok. So let us say we call this distance as R_1 what is this distance R_1? R_1 is the distance from O to the longitudinal axis of the vehicle the central longitudinal axis of the vehicle, ok.

So these are the various parameters. So now let us see what happens with the Ackerman steering condition. So let us say we consider the triangle OAC right.

Consider Δ OAC

$$tan\,(\delta_i) = \frac{AC}{OC} = \frac{L}{R_1 - \frac{W}{2}}$$

$$\Rightarrow cot(\delta_i) = \frac{R_1 - \frac{W}{2}}{L} \longrightarrow (1)$$

So if I consider the triangle OAC what will be tan of delta "I". It is going to be equal to L divided by of course to begin with it is going to be AC by OC, right. The AC is going to be wheel base L. OC is going to be as we can see it is going to be R_1 minus W by 2, right. So we subtracted W by 2 from R_1 we get OC, right.

So this will give us the cot delta "I" is going to be equal to R_1 minus W by 2 divided by L, right. So let us say we call this is equation 1. So similarly let us consider triangle OBD, ok.

Consider Δ OBD

$$tan(\delta_o) = \frac{BD}{OD} = \frac{L}{R_1 + \frac{W}{2}}$$

$$cot(\delta_o) = \frac{R_1 + \frac{W}{2}}{L}$$

$$\Rightarrow cot(\delta_o) = \frac{R_1 + \frac{W}{2}}{L} \longrightarrow (2)$$

So if we consider this triangle we can immediately observe the tan of delta O is going to be equal to BD by OD, right. BD is once again going to be L. OD is now R_1 plus W by 2. So this will give us cot delta O is going to be equal to R_1 plus W by 2 divided by L. Let us say we call this as equation number 2 alright.

So now let us say we subtract okay equation 2 and equation 1. So what do we get? We are going to get cot delta O minus cot delta "I" that is going to be equal to W by/ L, right. So that is what we are written here. So this condition, this equation is what is called as the Ackerman steering condition right. So this is the Ackerman steering condition right. So what does this condition give tell us? Suppose if you are given a vehicle of track W and a wheel base of L.

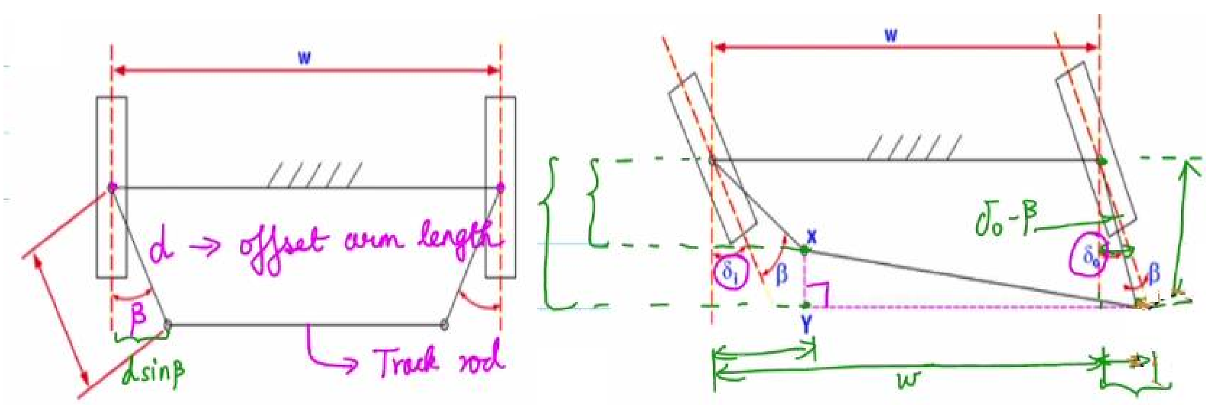
If we want to achieve you know like what was expected that is like the axis of the 2 steered wheels should intersect at the same point with the rear axle axis then we should have this equation to be satisfied. So what does this equation tell us you know like convey it gives us a relationship or a constraint between, delta not and delta i, right. So that is what is important. So the steering mechanism has been has to be designed in such a way that given delta “I” delta not should be related by this equation or given delta not delta “I” should be decided based on this equation.

So that is the significance of this equation. So in essence this provides a constraint on delta not and delta “I” given a fixed W and L right. So that is one way to look at this Ackerman steering condition right it is not? Okay so now if we continue and we add equation 1 and 2 so instead of subtracting 2 and 1 let us say we add 1 and 2. What are we going to get? We are going to get cot delta not plus cot delta I that is going to be equal to $2R_1$ by L.

② - ① : $$\boxed{cot(\delta_o) - cot(\delta_i) = \frac{W}{L}}$$

① + ② : $$cot(\delta_o) + cot(\delta_i) = \frac{W}{L} \Rightarrow R_1 = \frac{L[cot(\delta_i) + cot(\delta_o)]}{2}$$

So this implies that R_1 is going to be equal to L cot delta I plus cot delta not divided by 2l sorry 2 right. Correct that is about it so why is this important?



Thus the turning radius, see we can observe that the turning radius R. I can use this right triangle. R squared is going to L_2 squared plus R_1 square right so using the Pythagoras theorem ok. So thus the turning radius R is if we take R as the square root of L_2 squared plus R_1 squared this we have get it as L_2 squared plus L squared times cot delta I plus cot delta not whole squared divided by 4, right. So this will be the expression for the turning radius right.

So we can immediately see that the turning radius is obviously depends on how much the steered wheels are steered obviously right so that is it okay. So this is the Ackerman steering condition. So this is what we ideally want that is if you want to turn the steered wheels to achieve whatever we have already seen in the schematics right. Now question is that are there like how does one achieve it? Or in other words what happens with the actual mechanism that one has right.

So let us look at that so the question, the relevant question to ask is that how can one realize this condition right is it achievable perfectly every time or is there some trade off right. So for this let us look at a trapezoidal steering mechanism ok. Let us look at the trapezoidal steering mechanism and then like let us look at how well this is achievable. So let us consider a simple schematic this trapezoidal steering mechanism let me put at here and let expand it I think I have space alright.

So let us say as the name indicates you know like the steering mechanism is in the form of a trapezoidal, right. So let us say you know looking at this diagram. So let us see this is the solid axle to begin with right. So this is the track rod, ok and this angle this is the track vehicle track you know W. This angle is typically represented by beta ok. So that is the angle between the wheel plain the wheel central plain and the what to say the rod connecting the track rod and the to the king pin, right.

So this is typically denoted by d people called this as the offset arm length right. So this is essentially denoted by “d” as for as the trapezoidal steering geometric is concerned you know this is what is called as offset arm length. So if we look at this the trapezoidal steering mechanism the 2 main parameters are “d” and beta right. Those are the 2 important parameters right as for as the trapezoidal steering mechanism is concerned.

So now question is that how do we choose “d” and beta such that Ackerman steering condition is satisfied. So what do we want? We want to have a steering which will satisfy the Ackerman steering condition right. So let us look at that displaced steering linkage right. So let say I turn my steering wheel and let us say the schematic at the bottom is the what to say the displaced condition right.

And we can see that the inner wheel is turned by an angle delta “I” right and outer wheel is turned by an angle delta “o”. This angle beta between the central wheel plain and the tie rod right reminds the same ok that is the steering arm right. So it is going to pivot about the king pin or the ball joint but they are going to rotate. And so the relative angle between the central wheel plain and the steering arm the tie rod assembly is going to remain beta. That is what we can see in this figure.

So of course this is exaggerated, this is just to convey the point alright. So let us say you know like this is the displaced configuration. So now we see that we can trace out a right angle triangle XYZ alright in this bottom figure. So immediately we can observe and we can write down using the Pythagoras theorem that XY squared plus YZ squared is going to be equal to XZ square right so that all of us can immediately observe right from the triangle xyz.

Trapezoidal steering mechanism

$$XY^2 + YZ^2 = XZ^2$$

Now what can we say about the 3 lengths ok. So let us say first let us go from XZ. So what you think XZ is going to be? See XZ is going to be the length of this track rod that we can obtain from the above figure alright. And what is that going to be in terms of W d and beta. We can immediately observe that this is going to be W minus 2dsin beta, right because why? This is going to be d sin beta do you agree? So this entire length is W so you subtract 2 times d sin beta. So you get XZ.

So now what about XY? XY is going to be this distance, right. Is it not? Correct? Now we can project it into 3 parts. So XY is going to be this distance minus this distance. Do you agree? What is the bigger distance? You can immediately observe that I can rewrite it as d Cos delta not minus beta right which where do I get it from? It is from here it is this distance, right. So because this angle is going to be we are going to get d Cos delta not minus beta.

$$XY^2 + YZ^2 = XZ^2$$

$$XZ = W - 2dsin\beta$$

$$XY = d\,cos(\delta_o - \beta) - d\,cos(\delta_i - \beta).$$

$$YZ = W + d\,sin(\delta_o - \beta) - d\,sin(\delta_i - \beta)$$

Because delta naught is going to be this angle sorry you know this entire angle is going to be delta not. So this angle the smaller angle is going to be delta not minus beta, right. Do you agree? So and this length is anyway d so what is this distance?

$$\Rightarrow [d\,cos(\delta_o - \beta) - d\,cos(\delta_i + \beta)]^2 + [W + d\,sin(\delta_o + \beta) - d\,sin(\delta_i + \beta)]^2 = (W - 2dsin\beta)^2$$

This going to be what we wrote down right d Cos delta not minus beta now this distance smaller distance is going to be d Cos delta I plus beta, correct, do you agree? That is what.

Now what about YZ? YZ I can rewrite as 3 components. First is this component, right plus this component right minus this component correct. What is the first component? This is W, right, then what is this component? This is going to be d sin delta not minus beta, right. That is the opposite side right angle triangle, so that is going to be d sin delta not minus beta minus d sin delta I plus beta.

So this essentially gives us this condition for the trapezoidal steering mechanism. So I will quickly explain what this means so once I write down this expression. So we have d Cos delta not minus beta minus d Cos delta I plus beta whole squared plus W plus d sin delta not minus beta minus d sin delta I plus beta whole squared is equal to W minus 2d sin beta whole squared. So this is the expression that we get. So what is this expression gives us? It gives us a relationship between delta not delta I and d and beta and w and for the trapezoidal steering system ok.

So we will revisit this expression tomorrow and then we will discuss whether such a steering mechanism will indeed meet the Ackerman steering condition ok. So we will continue this discussion in the next class. Thank you.