Fundamentals of Automotive Systems
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Braking Analysis Part 02

So the next topic that we are going to look at in the braking analysis is the analysis of wheel lock up. So when we are discussing antilock brake systems we looked at the effect of front wheel lock and rear lock right if you take a single unit vehicle like a passenger car so if the front wheels are steered and front wheels lock up we lose the ability to steer. However, if the rear wheels lock and if there is significant lateral disturbance what happens? The vehicle may spin out of control so we lose directional stability.

Analysis of Wheel Lock-Up: We shall analyze the conditions under which the front and rear, wheels would lock.

In this regard we would consider:

- only braking force and rolling resistance,
- that the BFD to be K_{bf} : K_{br}

Thus, the governing equations become

$$\left(\frac{W}{g}\right)a = \underbrace{F_{bf} + F_{br}}_{F_b} + \underbrace{R_r}_{= f_{rw}} = F_b + f_r W$$

The expressions for W_f and W_r become

$$W_f = \frac{W}{L}l_2 + \left(\frac{W}{g}\right)a\frac{h}{L} = \frac{W}{L}\left[l_2 + \left(\frac{a}{g}\right)h\right]$$

$$W_r = \frac{W}{L}l_1 - \left(\frac{W}{g}\right)a\frac{h}{L} = \frac{W}{L}\left[l_1 - \left(\frac{a}{g}\right)h\right]$$

So for that reason we essentially discussed that the sequence of locking becomes important right. Of course, ideally I am repeating myself in an ideal scenario wheel lock up should never happen. But if it were to happen the sequence of wheel lock becomes important because we may want the front wheel to lock before the rear wheels since the front wheel locks at least can be readily more detectable by an experienced driver than a rear wheel lock and the driver can correct it right.

So that is where the analysis of wheel lock up becomes extremely critical. So what is this analysis going to deal with? Okay we shall what to say analyze the conditions under which the front wheels and rear wheels are locked. So the motivation behind this analysis because the sequence of the wheel lock matters right so that is why that is what we want to essentially look at. So in this regard we are going to make some basic assumptions.

So what we are going to do is the following we shall consider only braking force and rolling resistance ok. We are going to neglect the other resistances like grade resistance and aerodynamic drag and a drawbar pull and so on right. One can include them but then like for the sake of simplicity you know like we consider only a braking force in fact if you look at braking the magnitude of the braking force is much much above all other the resistances right.

But let say you know like we consider only braking force and rolling resistance in this particular analysis. And we shall consider that the BFD which is given to us is K_{bf} to K_{br} right. So we are already given a vehicle with a brake system and designed and installed in it which gives us a brake force distribution of K_{bf} to K_{br}. Let say you know we have a 60:40 BFD that means K_{bf} is 0.6 K_{br} is 0.4 that is been given to us.

So then what do we do with this? So under these assumptions I am just going to write the summary of equations you can go back and check these equations as an exercise because we just derived those equations right. So we what to say consider the same free body diagram with under these assumptions right in the presence of only braking and rolling resistance we neglect the other resistance. Thus the governing equations become the following right.

We get will W by g "a" to be equal to F_{bf} plus F_{br} plus R_r this is nothing but F_b plus f_rW we already know about this right. So why because this is the total braking force Fb this nothing but Fr times W right. That is the rolling resistance force ok. Now the expressions for W_f and W_r can be once again calculated by taking moments and taking Cos S theta is to be almost 1 right and making the same assumptions right.

So the expression for W_f and W_r become in this particular case W_f will be W by L L_2 sorry let me write it carefully will have W by L L_2 plus W by g a h by L which can be it is just the same equation written under this condition this is going to become W by L L_2 plus a by g times h ok so that is what it alright. So this is for the front how did we get this once again from the same expressions right what we derive right so nothing more than that ok.

Given the BFD of K_{bf} : K_{br}, the brake forces on the front and rear wheels are

$$F_{bf} = K_{bf}F_b = K_{bf}\left[\left(\frac{W}{g}\right)a - f_rW\right] = K_{bf}W\left[\left(\frac{a}{g}\right) - f_r\right],$$

$$F_{br} = K_{br}F_b = (1 - K_{bf})\left[\left(\frac{W}{g}\right)a - f_rW\right] = (1 - K_{bf})W\left[\left(\frac{a}{g}\right) - f_r\right]$$

Now the front wheels will tend to lock when $F_{bf} \geq \mu_p W_f$. In the limit,

$$F_{bf} = \mu_p W_f$$

$$\Rightarrow \left(\frac{a}{g}\right)_f = \frac{\mu_p l_2 + K_{bf} f_r L}{K_{bf} L - \mu_p h}$$

→ Longitudinal deceleration of the vehicle beyond which front wheels would tend to lock.

So W_r is going to be WL_1 by L minus W by g a h by L so this is going to be W by L L_1 minus a by g times h. So this is for the normal load on the rear. So once again we can see the effect of the dynamic load transfer right. See by the way 1 important point are that we should observe you know particularly with respect to dynamic load transfer.

Please note that the dynamic load transfer is dependent on the CG height right. Because what is the dynamic load transfer in this case? It is F_b plus the magnitude its F_r W times h by L right. F_r W is going to be small when compared to the braking force ok. So the CG height h influences it you can immediately see that the wheel base L also plays a role and so is the braking force or level of braking right. So it depends on whether we are doing mild braking or panic braking where we slam the brake pedal and so on right.

It depends affects the dynamic load transfer ok during braking. So if we come down we will see that this effect is felt here by means of deceleration still you see that the height of the CG and wheel base plays a role. But the level of braking force is reflected through the deceleration a by g the normalized longitudinal deceleration ok. So there we wrote it as F_b here we are writing it as a by g that is it ok so the equivalent representation right.

So now given the brake force or given the BFD of K_{bf} is to K_{br} the brake forces on the front and rear wheels are the following ok. So what will happen in the front you will get F_{bf} to be equal to K_{bf} times F_b how do we get this? Because that is the definition of K_{bf} right. K_{bf} S F_{bf} by F_b so I am just writing F_{bf} S K_{bf} times F_b. So now what we do is that we substitute for F_b where do I substitute for F_b? From this equation so I will get K_{bf} W by g a minus frW.

How did I get this equation from here you know I use this equation right? So I substitute further that net braking force. So this is nothing but K_{bf} W a by g minus f_r right ok. Similarly F_{br} is going to equal to K_{br} minus F_b, K_{br} is nothing but 1 minus K_{bf} right because K_{bf} and K_{br} is sum to 1. So I can write K_{br} to be 1 minus K_{bf} times F_b the expression is still the same we have W by g a minus f_rW. So here I will get 1 minus K_{bf} W times a by g minus f_r ok. So this will be the expressions for the brake force on the front and the rear right.

Similarly, the rear wheels will tend to lock when $F_{br} \geq \mu_p W_r$. In the limit,

$$F_{br} \geq \mu_p W_r$$

$$(1 - K_{bf})\cancel{W}\left[\left(\frac{a}{g}\right) - f_r\right] = \mu_p \frac{\cancel{W}}{L}\left[l_1 - \left(\frac{a}{g}\right)h\right]$$

$$\Rightarrow \boxed{\left(\frac{a}{g}\right)_r = \frac{\mu_p l_1 + (1 - K_{bf}) f_r L}{(1 - K_{bf})L + \mu_p h}}$$

Longitudinal deceleration of the vehicle beyond which rear wheels would tend to lock.

So now the front wheels will tend to lock so if you discuss our discussion on the traction at the tyre road interface the maximum longitudinal traction is Mu_p time W_r that tyre road interface right. So anytime the applied braking force exceeds that value of Mu times Mu_p times W we are going to have problems right. So that is what we discussed. So the front wheels will tend to lock when F_{bf} becomes greater than or equal to Mu_p time W_f right is not it?

Wf is the load of normal load at the front tyre road interface Mu_p is the corresponding peak friction coefficient Mu_p times W_f is the maximum traction force available at that tyre road interface. So if the front brake force exceeds it's limit there may be a tendency to lock because the slip may go beyond the value corresponding to peak Mu_p and then it may go towards lock. So if you recall the friction ellipse right.

So if you are exceeding this we are going outside the ellipse right beyond the boundary of the ellipse right. So that is what is going to happen. So in the limit what happens we can have Mu_p to be equal to sorry F_{bf} just to calculate the expression F_{bf} to be equal to Mu_p times W_f. This will just help us in calculating what is the deceleration after which front wheels will tend to lock ok? We are going to observe that shortly.

So let us look at the limiting case so F_{bf} is going to be equal to K_{bf} W_a by g minus f_r right. How did we get this from the above expression and then Mu_p times W_f so if we go up we already have the expression for W_f right? So, that is going to be W by L L_2 plus a by g h. So here we can see that W and W cancel off. So now we can see that there is an "a " by g on the left-hand side "a" by g on the right hand side we just do some simple algebra and then find an expression for "a" by g.

So what we will get is the following. So if I take "a" by g common I am going to get K_{bf} L minus Mu_p times h that is going to be equal to Mu_p L_2 plus K_{bf} f_r L right I am just rearranging the terms. So then what we will have is that like we will have "a" by g equal to Mu_p L_2 plus K_{bf} f_r L by K_{bf} L minus Mu_p h alright. So this we will put a subscript F and what is this expression give me? It is the longitudinal deceleration this is the limiting longitudinal deceleration of the vehicle right beyond which front wheels would tend to lock ok.

So that is the meaning of this particular expression ok the physical interpretation of this is that this “a” by g subscript F is the vehicle longitudinal deceleration beyond which the front wheels will tend to lock ok. So that is why I wrote subscript F in a different color because this is not what to say longitudinal deceleration of the front wheels no this is the longitude “a” is that longitudinal deceleration of the vehicle alright. So this gives us a limiting value beyond which the front wheel will tend to lock

So we can do a similar analysis for the rear wheels right. So similarly the rear wheels will tend to lock when Fbr is greater than or equal to Mu p times Wr. So in the limit what happens Fbr becomes equal to Mu p times Wr alright. So we apply the same process we have 1 minus K_{bf} W “a” by g minus f_r that is our F_{br} and on the right hand side we will get Mu_p W by L L_1 minus “a” by g h. So W and W once again cancel you can see that there is an “a” by g on the left and the right hand side.

So this once again you know like if we rearrange and simplify what we are going to get is that we are going to get an expression for “a” by g that I that intermediate step I leave it as a simple exercise right. So this expression will now be Mu_p L_1 plus 1minus K_{bf} f_r L by 1 minus K_{bf} L plus Mu_p h ok. So this let us put a subscript R. So this is the once again the longitudinal deceleration of the vehicle beyond which rear wheels would tend to lock ok.

So that is the meaning of this expression right. So now let us say you know like “a” by g subscript F is let us say 0.7 “a” by g subscript r is 0.8. So during the braking process which wheel do you think will lock first front wheel right because the vehicle deceleration has to cross 0.7 only then it will go to 0.8 right.

- The front wheels would tend to lock before the rear wheels if

$$\left(\frac{a}{g}\right)_f < \left(\frac{a}{g}\right)_r$$

- The rear wheels would tend to lock before the front wheels if

$$\left(\frac{a}{g}\right)_r < \left(\frac{a}{g}\right)_f$$

So consequently we can immediately observe that the front wheels would tend to lock before the rear wheels if "a" by g f is less than "a" by g r ok so that is the first point Similarly the rear wheels would tend to lock before the front wheels if it is other way around "a" by g r is less than "a" by g f ok. So we can see that by doing this analysis you know like we would be able to get an idea as to how you know like one can determine the sequence of wheel lock under different operation conditions right. So that is the main impact of this analysis.

Thus this analysis helps us in understanding the sequence of wheel lock up under different operating condition because why because you can see that it is a function of K_{bf} brake force distribution is a function of Mu_p , it is function of CG and so on right so under various operating conditions ok. So that is the at least impact of the analysis ok.

$$\eta_b = \frac{min\left[\left(\frac{a}{g}\right)_f, \left(\frac{a}{g}\right)_r\right]}{(\mu_p + f_r)}$$

So from this 1 parameter that is defined based on this understanding is what is called as braking efficiency ok. So what is that definition of braking efficiency it is the ratio so if you look at it so if continuing upon the previous discussion even if you look at the normalized deceleration that you can expect you know like if you are looking at this expression right. So if you look at W by g a ok and you look at the limiting case F_b is going to be Mu_p time W plus f_r W right. So this is the maximum right in the limit.

So theoretically what we are going to get is that the "a" by g max is going to be equal to Mu_p plus f_r do you agree because if I cancel off W right. So that is if you look at this governing equation neglecting all the other resistances. So we can observe that I will get the maximum deceleration when F_b is maximum when is F_b maximum? The maximum brake force is going to be Mu_p time W right. That is he maximum limit of the braking force right based on the interface tyre road interface conditions right.

So now we can immediately see that the maximum normalized vehicle longitudinal deceleration from this what to say simple analysis is going to be Mu_p plus F_r please remember that ok. So that is what we are going to use in this definition of braking efficiency. So it is the ratio of the maximum deceleration of course in terms of "a"by g ok normalized by g ok achieved prior to lock up of either front or rear wheels to the sum of Mu_p and f_r.

So what is the expression for this braking efficiency? So eta b I can rewrite as minimum of a by g f, "a" by g r right because I need to take the minimum right it is not? Because as per the definition it is the vehicle deceleration achievable prior to lock up of any wheel on the vehicle so I need to take the minimum of "a" by g of and "a" by gr and divided by Mu_p plus f_r right. So when I have very effective brake force distribution you will see that "a" by g f and "a" by g r will be almost equal and this braking efficiency will almost into 100%.

So if you have perfect brake force distribution right between the front and the rear you will achieve the best possible braking but unfortunately as we just discussed brake force distribution is dependent on so many conditions right. The vehicle CG location the tyre road friction levels and so on. So this quantity also keeps on changing alright. So that is why you know like active brake control becomes pretty interesting to do alright. So I think this is a good place to stop for today's class ok. So we will continue in the next class thank you.