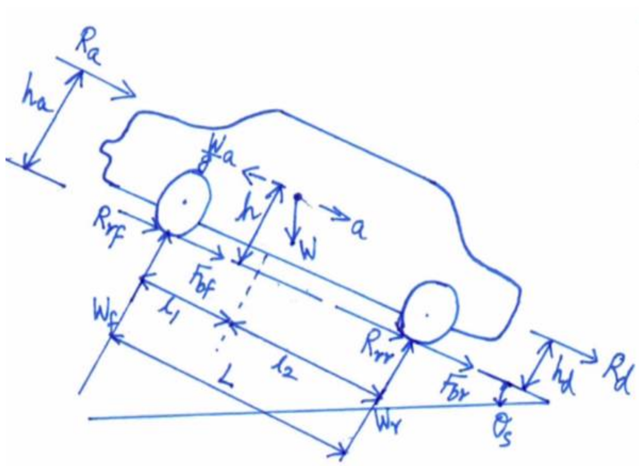
Fundamentals of Automotive Systems
Prof. C.S. Shankar Ram
Department of Engineering Design
Indian Institute Technology - Madras

Module No # 11
Lecture No # 55
Braking Analysis Part – 01

Okay so greetings so welcome to this class so in today's class we are going to look at analysis of braking response okay. So we have been looking at the brake system it is operation and so on and we looked at the antilock brake systems and what is the concept how they work in a broad sense right. So today we come to the analysis part so if you look at the braking process itself what analysis can be used to characterize the vehicle braking process right.

So that is something we will look at so we will we are going to consider a free body diagram which you will see that it is pretty similar to what we consider during analysis of the powertrain. So if we recall the analysis that we did for calculating the gear ratios or getting the basic equations during the drive motion right when we did powertrain analysis we considered a similar free body diagram.



FREE BODY DIAGRAM

So if you recall we consider that the vehicle was moving up a grade of angle theta S alright and the vehicle's weight is W. So and the wheel base is capital L the CG is located at a distance of L_1 from the front wheel center and L_2 from the rear wheel center W_f and W_r are the normal loads at the front and the rear wheels respectively. Ra is the aerodynamically drag R_d is drawbar pull R_{rf} and R_{rr} indicate the rolling resistance at the front and the rear wheels respectively okay.

So all these are quantitates we have already looked at now what are the main differences between this free body diagram and the one that we looked at when we did powertrain analysis. Please note that in this case we are applying a braking force rather than a drive force. So we can immediately observe that there are there is a braking force F_{bf} acting at the front tyre road interface and F_{br} that is acting at the rear tyre road interface right.

So those are the braking forces that act on the wheels now "a" for us is the declaration so that is why it is been marked in this direction direction opposite to that of the motion we consider "a" to be the deceleration and consequently the D'Alembert's force which is nothing but M times a which we mark at the C.G. and at a you know in a direction opposite to that of the deceleration that is going to W by g times "a" right that is the D'Alembert's force okay.

So and what to say the weight of the vehicle is W and that is going to be resolved into 2 component W Cos theta S perpendicular to the road and W Sin theta S along the longitudinal direction right so those are the various forces acting on the vehicles. So the main differences are the presence of the braking force rather than a drive force at the tyre road interface and the deceleration is now the other way around right and the vehicle is being decelerated and consequently the D'Alembert's force W by g times "a" is in this direction okay.

F_{bf}, F_{br} $\longrightarrow$ Braking force at the front and rear wheels respectively.

Balancing the forces along longitudinal direction.

$$\left(\frac{W}{g}\right)a = \underbrace{F_{bf} + F_{br}}_{F_b} + \underbrace{R_{rf} + R_{rr}}_{R_r = f_r W} + R_a + R_d \pm W Sin\theta_s$$

$$\Rightarrow F_b + f_r W = \left(\frac{W}{g}\right)a - R_a - R_d \pm W sin_{\theta_s} \quad (1)$$

So those are the main changes so let us consider the free body diagram of the vehicle during braking. So as we just discussed F_{bf} and F_{br} indicate the braking force of the front and rear wheels respectively. Now if we balance the forces right along the longitudinal direction so balancing the forces along the longitudinal direction we obtain in this particular case W by g times "a" that is nothing but M times a that is going to be equal to F_{bf} plus F_{br} plus R_{rf} plus R_{rr} which are the rolling resistances plus R_a plus R_d plus or minus W Sin theta s.

So if you recall when we did powertrain analysis aerodynamic drag draw bar pull grade resistance and also rolling resistance were called loads because those had to be overcome by drive force in this case please not that these are resistive forces anyway right they aid in the deceleration process right so that is one important difference. So F_{bf} plus F_{br} is the total braking force F_b which we are denoting F subscript B this we already know is the total rolling resistance R subscript r which we can rewrite as F_r time W F_r being the rolling resistance coefficient so we are aware of these aspects right.

So consequently what will happens is that F_b plus F_{rw} that is going to be equal to W by ga minus R_a minus R_d minus plus W Sin theta S I am just rearranging the terms right so this let us call this as some equation 1 right okay. So now what is the next step that we would do you know like let as say we call this points of contact you know like A and B respectively that is where the center of the front tyre road contact as A and the rear one has B.

So we take moment about these 2 points okay so taking moments about point B or balancing the moments about point B right we obtain the following so we can immediately see that W_f times L is a clock wise torque and then we have R_a h_a is a another clock wise torque plus R_d h_d is another clock wise torque plus or minus W Sin theta is times h is another clock wise torque right these are torques which are acting in the clockwise direction about when we take moments about point B then we balance the them with the counter clockwise torque so that is those are going to be W by g "a" times h then we will get W Cos theta S L_2 right.

So this is what we will get so we assume once again theta s to be small such that Cos theta s is almost one we assume this value of h_a, h_d to be almost equal to h then what do we get this results in W_f being equal to WL_2 by l if we simplify and take terms to the same side we are going to get W by g a minus R_a plus R_d minus plus W Sin theta S times h by L okay. So just rearranging the terms right so we have already done a similar analysis before so this pretty straight forward right.

Taking moments about point B:

$$W_f L + R_a h_a + R_d h_d \pm W sin\theta_s h = \left(\frac{W}{g}\right) ah + W cos\theta_s l_2$$

Assume $\cos\vartheta \approx 1,\ h_a \approx h_d \approx h$:

$$W_f = \frac{W l_2}{L} + \underbrace{\left[\left(\frac{W}{g}\right) a - R_a - R_d \mp W \sin\theta_s\right]\left(\frac{h}{L}\right)}_{= F_b + f_r W, \text{ from } (1)}$$

$$\Rightarrow \boxed{W_f = \underbrace{\frac{W l_2}{L}}_{\text{Static Load}} + \underbrace{[F_b + f_r W]\left(\frac{h}{L}\right)}_{\text{Dynamic load transferred during braking}}} \rightarrow (2)$$

So what is the term within the square bracket from equation 1 this is nothing but F_b plus f_{rw} right okay so this implies that W_f is going to be equal to WL_2 by L plus F_b plus F_r W times h by L. So this is equation 2 okay so this is the load on the front wheel. So once again if we recall the analysis that we did before please note that the static load on the front wheel is going to be W L_2 by L right so because if assume that the vehicle is parked on a flat road it is stationary so then W_f is going to be W L_2 by L right.

So this component is now the longitudinal dynamic load transfer during braking, okay that is the component of the longitudinal dynamic load transfer right. So during the process of braking so during braking please note that you know we are going to have transfer from rear to front as we will observe shortly even from the equation so we can see that the load on the front wheels is going to increase so that is what we can observe from here. Now similarly taking moments about point A and simplifying.

So this I am going to leave it as "a" an exercise because we have already done this process right for the other wheel right. So if you do the same process and make the

Similiarly taking moments about point A, and simplifying

$$W_r = \underbrace{\frac{W l_1}{L}}_{\text{Static Load}} - \underbrace{[F_b + f_r W]\frac{h}{L}}_{\text{Dynamic load transferred during braking}} \quad \rightarrow (3)$$

same assumptions Cos theta S is almost 1 h_a, h_d almost being equal to h and so on. So and then like use equation 1 so what we will ultimately get is that like we will get an equation for W_r which is going to be W L_1 by L minus F_b plus F_r W times h by L okay. Let us call this as equation 3. So this I will leave it you as an exercise right so pretty straight forward alright so you will get this.

So immediately we can observe that Wl1/L is the static load and F_b plus F_r W times h by L is the dynamic load transfer right during braking from the rear wheel right. Please note that during acceleration there is a dynamic load transfer along the longitudinal direction from the front to the rear. Now during braking it is the other way around there is a dynamic load transfer along the longitudinal direction from the rear wheels to the front wheels.

So we can see that the same quantity is being subtracted from the rear and added to the front so that is what is happening during braking okay. So this is something which we can observe. Now we move forward and then like we look at what will be the maximum braking force that one can obtain at the front and the rear wheel right. So when we did powertrain analysis typically we reasoned out that in a typical road vehicle either the front wheels or the rear wheels are driven so we looked at front wheel drives and rear wheel drives and derive expressions for the corresponding tractive forces right.

But in a typical road vehicle all wheels are braked okay so frictions brakes are installed on all the 4 wheels in the 4 wheel vehicle. So we have to look at what is called as brake force distribution okay between the front and the rear. So let us look at that important concept okay.

Let μ_p be the peak friction coefficient at the tyre-road interface.

The maximum total brake force is

$$F_{r\,max} = \mu_p W$$

The maximum brake force from the front wheels is

$$F_{bf,max} = \mu_p W_f = \mu_p \left[\frac{W l_2}{L} + \big(\underbrace{F_{b,max}}_{=\mu_p W} + f_r W\big)\frac{h}{L}\right]$$

$$\Rightarrow F_{bf,max} = \frac{\mu_p W}{L}\left[l_2 + h\left(\mu_p + f_r\right)\right]$$

So to analyze that let Mu_p be the peak friction coefficient okay at the tyre road interface let us say this is same for all the 4 wheels you know without the loss of generality right now right. Let us assume that it is a same right so let Mu, be the peak friction coefficient at the tyre road interface then what happens is that the maximum brake force the total brake force so F_b is so it is going to be F_b max that is going to be equal to Mu_p times W.

So this is the maximum brake force that we can expect right because the weight of the vehicle is W so Mu times W gives me a numerical upper bound on the maximum brake force that we can get we can apply on that vehicle right. So that is what is sustainable at the tyre road interface right totally. So now the question is that how do we distribute this between the front and the rear. So let us say once again taking round number just for the sake for argument let us say this maximum brake force F_b max comes to be 100 Newtons.

Now the question is that like out of this 100 Newtons how many Newton's should I ask the front wheel brake to deliver and how much should the rear brake deliver you know that is what is called as brake force distribution. So let us analyze that so the maximum brake force from the front wheels is F_{bf} max that is going to be equal to Mu_p times W_f then we substitute the expression for W_f that is going to be equal to WL_2 by L + F_b max + F_rW times h by L right.

So the only difference I am making is that rather than F_b I mean using F_b max how did I get this equation from equation 2 for W_f right. So we just derived it right so if you look at equation 2 above for F_b I am just subtitling F_b max because we are interested in the maximum braking force right. So this is F_b max as we already just saw it is going to be Mu p times W right so from the above equation.

So this will give as F_{bf} max will be if we simplify this we just take W out and then L out we are going to get L_2 plus h times Mu_p plus F_r. So that is what we will get so this is the maximum brake force that can be obtained from the front wheels right. Similarly the maximum braking force of course this should be maximum brake force from the front wheels right.

The maximum brake force from the rear wheels is

$$F_{br,max} = \mu_p W_r = \mu_p \left[\frac{W l_1}{L} - \underbrace{\left(F_{b,max}\right.}_{= \mu_p W} + \left. f_r W\right) \frac{h}{L} \right]$$

$$\Rightarrow F_{br} = \frac{\mu_p W}{L} \left[l_1 - h\left(\mu_p + f_r\right) \right]$$

So similarly the maximum brake force from the rear wheels is F_{br} max is going to be equal to Mu_p times W_r that is W_r that is going to be equal to just the same process it is going to be Mu_p W L_1 by L minus F_b max plus frW times h by L. So once again this F_b max is nothing but Mu_p times W if we substitute we get the final expressions this implies that F_{br} max it is going to be equal to Mu_p times W by L times L_1 minus h times Mu_p plus F_r okay so this is the maximum brake force at the rear wheels okay.

IDEAL BRAKE FORCE DISTRIBUTION [BFD]: It is that distribution between the front and rear wheels of the total brake force that results in the maximum brake force on all wheels at the same time.

$$\frac{F_{bf,max}}{F_{br,max}} = \frac{l_2 + h(\mu_p + f_r)}{l_1 - h(\mu_p + f_r)} = \frac{K_{bf}}{K_{br}}$$

$$K_{bf} = \frac{F_{bf}}{F_b}, \quad K_{br} = \frac{F_{br}}{F_b}$$

$$\Rightarrow K_{bf} + K_{br} = 1$$

Eg: 70% - 30% Front – Rear BFD

$\Rightarrow \boldsymbol{K_{bf} = 0.7, \quad K_{br} = 0.3}$

So then we define what is called as a ideal brake force distribution okay the term ideal brake force distribution is typically abbreviated as B_{fd} okay. B_{fd} stands for brake force distribution okay so what is this ideal brake force distribution? So it is that distribution, of the total brake force right that distribution of course between the front and rear wheels of the total brake force that results in the maximum brake force on all wheels at the same time okay.

So this is what is called as ideal brake force distribution okay so it is that distribution of that brake force total braking force between the front and the rear that will result in the maximum brake force at all wheels at the same time so that means that in essence we are utilizing the traction that is available at each tyre road interface to the best extent possible that means that we are going to get the highest stable declaration right because if you are essentially using the maximum friction which is available at a particular tyre road interface I am getting the best out of my brake system right when I am braking for a given tyre road interface right.

So that means that is the maximum that is indicative of the maximum stable braking force that I can achieve so consequently that will give me the maximum stable deceleration right. So that is what is called as the ideal brake force distribution so that is obtained by we just divide the 2 expressions. So essentially what we do is that we essentially have F_{bf} max by F_{br} max if we divide it what will happens is that we will get L_2 plus h times Mu_p plus fr divided by L_1 minus h times Mu p + fr okay.

So this is the ideal brake force distribution so some people will express it as ratio of 2 parameters K_{bf} and K_{br}. So K_{bf} is nothing but F_{bf} by fb it is just a proportion of the brake force on the front and the rear okay K_{br} is once again a proportion what is a proportion of the total brake force on the rear right. So this implies that K_{bf} plus K_{br} it sums to 1 okay. So typical for a given vehicle you know like if you are given L_1, L_2, h Mu_p and fr you can calculate that ratio of K_{bf} and K_{br} then how do you get the individual values you use the second equation which is K_{bf} plus K_{br} equals 1 right because from this equation you can only get that ratio of K_{bf} and K_{br}.

You use the second equation K_{bf} plus K_{br} equals to 1 to calculate the individual value of K_{bf} and K_{br}. So what does this mean if someone says we are having a 70, 30 front rear Bfd okay this is an example okay they are essentially implying that the value of K_{bf} is 0.7 okay K_{br} is 0.3 okay so that is the implication. So if someone says you know like the B_{fd} is 70, 30 on the front and the rear so that means the value of K_{bf} is 0.7 the value of K_{br} is 0.3 so that is how we should interpret okay.

So immediately we can observe that note that the ideal BFD depends on the CG location because you can observe that in the equation we have L_1,L_2, h so that is going to affect the ideal brake force distribution it is dependent on the maximum friction coefficient Mu_p and to a lesser extent rolling distance coefficient okay. So because relatively this value is going to be smaller and so it is impact on this aspect is going to be lower okay the impact of rolling resistance coefficient on this what to say calculation of ideal B_{fd} is going to be relatively small compared to the first 2 aspects.

So why are this important? First thing let us say we even take a passenger car let us say the mass distribution in the passenger car remains more or less the same. So let us say we for the sake of argument we say that the CG location remains pretty much the same. So immediately you observe that as a designer as a vehicle designer I may design my brake system for the fully loaded car you know travelling on a nice dry road surface right.

So I will assume the best possible value of Mu_p okay and I let us say I calculate the brake force distribution as let us say some 70, 30 front and rear. Suppose let us say it rains the roads becomes wet the value of Mu p reduces and I use the same expression for the same car right the number will be maybe 65, 35. Suppose it become Icy the Mu_p drops even further then the ideal brake force distribution may become 60, 40. So I am sure you get the point right so essentially this ideal brake force distribution keeps on changing but we have only have one brake system right.

So then what do we do right so that is where active braking control becomes important right because even if I want the same brake force the way I distribute the brake force between the front and the rear is dependent on the vehicle operating conditions okay that is one aspect. Second aspect is that let us say we take for example a truck let us say that weighs around like let us say 16 tons and it is carrying sand to construction site it dumps the sand and comes back it may weigh around 4 and half tons.

So from 16 tons it is going to 4 and half tons so then what is going to happens is it the value of L_1,L_2 h will change drastically right for the empty truck as opposed to the fully laden truck. So even though Mu_p where the same now the K_{bf} , K_{br} for the unladen truck will be different right fine but once again we have the same brake hardware right so that is where you know like this control of braking becomes a very critical right. So with different operating conditions right so we have to aware that there are interesting problems you know like in regulating the brake force distribution.