Fundamentals of Automotive Systems
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Lecture - 39
Transmission Matching

Okay so greetings so welcome to todays class. So just a quick recap of where we stopped in the last class. So we were looking at you know how to choose the gear ratios of a multi-speed gearbox. And we discussed that typically the gear ratios are decided by various performance criteria given a prime over characteristics that is the engine characteristics. So one of the main requirements that is used to choose the gear ratio is what is called as Gradeability.

So that is indicative of the steeper slope that a vehicle can climb on at a given constant speed. So Gradeability is defined as tan of theta s max but theta s max is this angle of the steepest slope right. So that the vehicle can climb or is expected to climb. And when the vehicle is climbing on this slope the significant resistant is the resistance or the load that the vehicle needs to overcome is a grade resistance. W sin theta “s” max since it is going at a relatively lower speeds. So we are neglecting aerodynamic drag however we can include rolling resistance.

So there is something else called starting Gradeability where we expect the vehicle to just start from a slope. Then inertia also comes into play, right? So we will be looking at continuous Gradeability currently and we saw that typically you know when we want to go on a grade. We wish to engage the first gear and we want to operate the engine at a point near to where it delivers the maximum torque because we need maximum thrust essentially to overcome the gradient.

So using that idea we essentially balanced out what the engine will provide which is on the left hand side of this equation to what are the loads that need to be overcome which are on the right hand side of this equation. Okay so we have matched the two.

So that is the first requirement and that is where we stopped. And then like I defined something called as a wheel longitudinal slip ratio which is during traction is given by this formula. So typically it is represented as Lambda some people will call it a kappa (02:49) or s and so on. Okay so during traction or forward there is during providing a thrust to the vehicle, right? So the wheel slip ratio is defined as r radius or the wheel times angular speed of the wheel minus the longitudinal speed of the vehicle divided by the product of r_w and omega "w" okay.

So typically when a pneumatic tyre contacts a tyre road interface it sticks and slips. So essentially, we do not have a pure rolling motion. Okay so that means "v" is not equal to "r" omega you know in the case of a pneumatic tyre. So the wheel slip ratio quantifies that phenomena. And during traction it is essentially defined in this manner. And as we discussed for a purely rolling wheel v will be equal to r omega.

So this lamda will become zero in the limit. And the other extreme case is when a driven wheel is stuck in a pothole full of let us say mud then we applied traction on the wheel. We will keep on spinning at the same place, right? So "r" omega will be finite however "v" omega will be close to zero. So the other limit will be 1 right for what is called as a spinning wheel. So if we use this equation, we get the longitudinal speed as r_w w omega w times 1minus lamda. So this is where we stopped so let us continue from here.

$$Now, if\ the\ \ i^{th}\ gear\ is\ engaged\ with\ the\ engine\ speed\ being\ \omega_{e,}\ then$$

$$v = \ r_\omega \omega_\omega (1-\lambda) = \ r_\omega \frac{\omega_e}{N_{ti} N_d}(1-\lambda)$$

Okay now if the i_{th} gear is engaged okay so let us say we have n speed gearbox in general and we are engaging the i_{th} gear with the engine speed being omega e at some engine speed omega "e" then we can use this above equation to get the corresponding longitudinal speed as r_w right? Omega w times 1 minus Lamda. Now the question is that how will how is Omega w related to Omega "e" right? Omega "e" is going to be sorry when we go w is going to be Omega e if you recall the definition of a gear ratios right from the engine output, we engage the flywheel and the clutch together and that goes to a primary gearbox whose gear ratio is going to be in N_{ti} right?

So because like we are engaging the i_{th} gear, so we divide by N_{ti}. So this is going to be the speed of the output shaft or the primary gearbox. Then we have a final drive. So whose gear ratio is Nd, so we divide by N_d and this is going to be the speed of the wheels angular speed of the wheels. So that is going to be equation. So this equation would be used later by us. Okay so this equation relates the longitudinal speed of the vehicle to a given engine speed. Okay when a particular gear is being chosen.

Now if the maximum vehicle longitudinal speed let us say we call it a some v max it has to be achieved when the engine speed is let us say we call it as omega max right? So this is close to the speed at which the engine delivers the maximum power. So we have already looked at the power speed and torque speed characteristics of a typical IC engine. So we do not get constant power output from an IC engine. We have the maximum power at a particular speed, right?

So Omega max is close to the speed at which the engine delivers the maximum power. Okay so let us say that is omega max and the highest gear with the gear ratio N_{tn} being engaged. Okay then so typically you know like when we are going at a maximum speed let us say on a highway right? We are cruising at the maximum speed on a highway. what we typically tend to do we tend to go to the highest gear right?

And we want to operate the engine close to where we get the maximum power right? Because when we are going at maximum speed let us say on a highway there is going to be hardly any inertia load. The main loads are going to be aerodynamic drag and rolling resistance and grade resistance is not going to be significant. Yes, if the highway is that there is very negligible grade resistance.

Even if there are small ups and downs that are going to be small amounts of grade resistance not to the extent of what we encountered in the requirement of Gradeability okay. So consequently you know like we engage the highest gear and we use this equation to get a second equation. Okay so then what happens we just use this equation “v” max is going to be equal to r_w omega max divided by N_{tn} in the times 1 minus lamda.

So let us call this equation as equation two. Okay so we have got equation two by considering the requirement of maximum vehicle speed okay. And essentially making use of the fact that when we want the vehicle to go at its maximum speed and we are going to engage the highest gear or the n_{th} gear in the n speed gearbox. Alright, so now we can unitedly observe that we have two equations.

One from the Gradeability requirement where we want the vehicle to be operated and the first gear is engaged and one a maximum speed requirement where we essentially engage the nth gear right? There are two equations but then how many unknowns are there? We do not know N_{t1} we have to find N_{tn} we have to find N_d. So essentially, we have two equations and three unknowns' right? So we are not done yet. So we need to figure out how to address this problem right? And get the first cut values of these gear ratios.

If the maximum vehicle speed (v_{max}) has to be achieved when the engine speed is ω_{max} [close to the speed at which the engine delivers the maximum power] and the highest gear (N_{tn}) being engaged, then

$$v_{max} = \frac{r_\omega \omega_{max}}{N_{tn} N_d}(1 - \lambda) \quad \longrightarrow \quad 2$$

$N_{ti},\ N_{tn}, N_d$

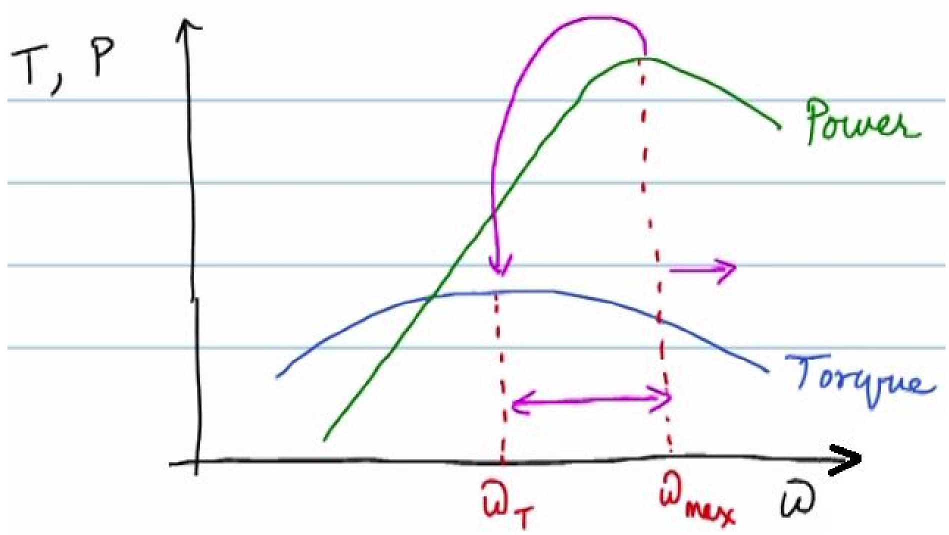
So for addressing that let us first consider a four speed gearbox. We will consider to other variants shortly. Okay so to begin with let us consider a four speed gearbox. So then we need to determine the values of N_{t1}, N_{t2}, N_{t3}, N_{t4} and N_d okay. So we need the values of these parameters, right? Four primary gear ratios and one final drive ratio. Now what happens is that typically it is just a convention the fourth gear is taken as a direct drive as we observed even when we are discussing the transmission layout right?

In a typical gearbox in the fourth gear you know like the input shaft is directly connected to the output shaft or there is a mechanism by which the speed of the input shaft and the output shaft may equal. Okay? So this implies that N_{t4} is typically equal to one, right? If this were the case, then are we able to solve this problem? Yes, right If N_{t4} were one then we can use equation two to obtain the value of N_d, right?

Because we are given the maximum speed specification, right? We are also given the engine speed at which we want the maximum speed maximum vehicle speed. So we can use equation two to calculate N_d because now small "n" will be. So if we consider four speed gearbox that means small "n" is four, right? So N_{tn} will become N_{t4} in equation two so N_{t4} is one so we can find N_d.

So once we find N_d then we can use the equation one to find N_{t1}. All right so then equation one can be used to calculate the value of N_{t1} okay. So that is how we use equations one and two in case of a four speed gearbox please note that we are doing basic calculations to get the first cut gear ratios know so okay. So now what have we done? We have we know the value of N_{t1}, we know the value of N_{t4} we know the value of N_d

The question now becomes what happens to the intermediate gear ratios right? So question that we need to ask ourselves is the following how to determine the values of the intermediate gear ratios. Okay so because we do not know N_{t2} and N_{t3} yet right? How do we do that?



In order do that we revisit the engine characteristics. So let us just revisit the engine characteristics. So let us just revisit the characteristics of a typical IC engine the torque speed and the power speed characteristics. Let us say torque speed is something like this is our torque and power speed is something like this. Okay so let us say the point where we get the maximum torque. Let us say we call it as some omega T the point where we get the maximum power now close to that we call it as omega max. Okay.

So now imagine us driving a car okay so let us say we start from rest. Of course, we engage the clutch we press the throttle and then like we want to synchronize the motion of the engine crankshaft and the wheels right through the transmission right through the drive train components. So now let us say we start somewhere close to the point where we get the maximum torque. Now what is going to happen as the speed of the vehicle increases, we are in the first gear right?

The speed of the engine what to say increases right? And then after some speed where the engine power reaches its maximum if we still continue to operate the power train in the same gear, we will see that the engine is going to drag right? Why? Because if we exceed and go to the right of this region the torque also decreases the engine power output also decreases. So the engine is not delivering enough energy to meet the demand from the driver. Okay.

Based on the loads that are acting on the vehicle. So then what we do we change to the second gear and by and large during this gear change the vehicle speed more or less remains constant. Yes, it may fall down slightly because the power train is briefly disengaged but we can neglect that variation. Right? So let us say we essentially come to the second gear what is going to happen? The engine speed is going to fall because the vehicle speed is still the same.

So if you go back to this equation, right? So if the vehicle speed is still the same and the value of N_{t2} is going to be lower than value of N_{t1}. Right when we changed the gear so we can immediately see that omega e will drop. Now the question is that where do we want omega e to go? Right by design. So obviously I would want omega e to go from here to close around about here because from the point where we are delivering the maximum torque sorry maximum power to the point that I get the maximum torque maximum torque means it will enable me to get the maximum thrust right? Because we are starting from thrust and accelerating.

So I would want to go to the speed engine speed where I get the maximum torque right? And then we do this once again. So we keep on accelerating the vehicle speed keeps on increasing. The second gear we close we reach close to omega max. Then we switch from second to third gear. This is what it is called up shifting right. That is going from a lower gear to a higher gear. When we up shift from the second gear to the third gear what do you want?

Same thing, we want to come back from omega max to close about to omega T. So essentially, we want to operate the engine while operating the vehicle in this speed range and that this physical motivation or requirement is going to help us in figuring out the intermediate gear ratio. So consider that the gear is up shifted from i minus one to i without loss of generality in a "n" speed gearbox. Okay so the basic idea is to make the power train operate in such a manner that the engine is operated between omega T and omega max that is the basic idea.

$$\frac{r_\omega \omega_{max}}{N_{t(i-1)} N_d}(1-\lambda) \approx \frac{r_\omega \omega_r}{N_{ti} N_d}(1-\lambda)$$

$$\Rightarrow \boxed{\frac{N_{ti}}{N_{t(i-1)}} = \frac{\omega_T}{\omega_{max}}} \qquad i = 2, \dots\dots.., n$$

$\Rightarrow$ The ratio of any two successive gear ratios is the same.

$\Rightarrow$ The gear ratios are in geometric progression.

Okay hence when the gear is shifted from i minus one to i at particular vehicle speed we have what we are going to do is that like we are going to make use of this equation. Okay once again we are going to come back to the equation relating speed and the vehicle longitudinal speed and the engine rotational speed. Okay so what is going to happen? The speed is going to be almost the same. Now what will be the vehicle speed when we are at this point in the i minus one gear that is going to be equal to r_w omega mx by N_ti minus one.

Because we are in the i minus one here times N_d times 1 minus lamda, right? We are making an assumption that the slip ratio more or less remains constant need not be but to make a first grade calculation we assume that to be the case then that has to be approximately equal to, right? Because the speed can drop a little bit. But we are going to neglect that variation. So that is going to be equal to r_w omega T by N_ti N_d one minus lamda.

So I hope everyone can understand the motivation behind this equation. So what does this equation give us? So it gives us if we simplify this equation it essentially tells us that N_ti by N_ti minus one is going to be equal to omega T/omega max. So almost going to be close to okay approximately equal to omega T by omega max. So we are typically given an engine, right? We know these rated speeds.

That is a speed at which we get the rated at torque and the speed at which we get the rated power. So omega T and omega max are known to us. So what does this give as for any given "i" in the gearbox the ratio of two successive gear ratios are the ratio of two successful gear ratios will be the same. Okay? So this implies that the ratio of two successive gear ratios is the same. So any two right that is important right?

So you take let us say "i" goes from one to or two to N minus1 the way we have used the index. Okay in this case you know like the way we have used the index you know like i goes from two to N in an N speed gearbox. So that we go from first gear to n_{th} gear, right? So we see that the ratio of any two successive gear ratios in the multi-speed gearbox is the same. Okay using this argument then what do we call such a sequence of numbers to be? So that implies that the gear ratios are in geometric progression. Okay so the set of gear ratios are essentially taken to be in geometric progression.
So what does that give us? Hence, we take any two pairs of successful successive gear ratios. So N_{t2} by N_{t1} equal to N_{t3} by N_{t2} equal to N_{t4} by N_{t3} that is going to be equal to omega T by omega max.

And that is going to be let us call that as a K subscript g some parameters K subscript g right.

Hence,

$$\frac{N_{t2}}{N_{t1}} = \frac{N_{t3}}{N_{t2}} = \frac{N_{t4}}{N_{t3}} = \frac{\omega_T}{\omega_{max}} = K_g$$

$$\Rightarrow \frac{N_{t2}}{N_{t1}} \times \frac{N_{t3}}{N_{t2}} \times \frac{N_{t4}}{N_{t3}} = K_g^3$$

$$\Rightarrow K_g = \sqrt[3]{\frac{N_{t4}}{N_{t1}}}$$

→ *K_g can be calculated using the values of N_{t4} and N_{t1}*

Okay so this will immediately tell us that. Now if I multiply N_{t2} by N_{t1} with N_{t3} by N_{t2} with N_{t4} by N_{t3} what will I get? K subscript g to the power three.

Now what is going to happen we can immediately observe that N_{t2}, N_{t2}, N_{t3} ,N_{t3} cancels. So what are we going to be left with? K subscript "g" the parameter is going to be equal to the cube root of N_{t4} by N_{t1} okay that is the implication. So we can immediately see that K subscript g can be calculated using the values of N_{t4} and N_{t1}. Do we know those values? Of course we do, right? Because in the example that we are considering a four speed gearbox. N_{t4} is anyway 1 have we determined N_{t1} of course, yes right?

We use the what to say equation for the maximum torque to determine N_{t1} after we use the equation of our maximum speed to get N_d. So we know the values of K subscript g. So this implies the values of N_{t2} and N_{t3} can be calculated right. Once we know the ratio what does N_{t2} it is K subscript g times N_{t1} what is N_{t3} it is K subscript "g" time N_{t2} that is it right can be calculated. Okay so this is the idea, right?

So if you have if one has a four speed gearbox the idea is to essentially take the fourth gear ratio to be one okay then use a maximum speed requirement to calculate the value of N subscript d. Then what we do is like we use the math equation for maximum torque to calculate the first gear ratio. Then we consider a case where the gear ratios are in a set of gear ratios are in geometric progression. Then we essentially calculate the intermediate gear ratios by calculating this parameter K subscript g that is it.