Fundamentals of Automotive Systems
Prof. C. S. Shankar Ram
Department of Engineering Design
Indian Institute of Technology - Madras

Lecture – 38
Powertrain Analysis 2 - Part 02

All right, so to continue the next question that we are going to ask ourselves and answer is the following; having what to say, looked at what are all the forces that act on the vehicle and what is the output delivered by the powertrain for a given prime mover torque and selected gear ratios, the question becomes if we are given a set of vehicle performance requirements and a base prime mover an engine, how do we select the gear ratios.

In other words, how do we match the capabilities of the powertrain with what is required by the vehicle designer as far as vehicle performance is concerned. This exercise is typically referred to as what is called as transmission matching by some, okay. So, what is the question that we are posing; the question is how to obtain the gear ratios of a multi speed gearbox and the final drive to meet vehicle performance requirements of course, given a base engine, right.

So, given a ratings of an engine, you know how do we essentially go about and calculate these values okay, in other words we would also want to essentially figure out how do we convert you know, like vehicle performance requirements like gradeability, maximum speed requirements etc., into some useful mathematical relationships that can be utilized to obtain first cut values of these gear ratios, okay.

So that is a question we are asking ourselves, so in this regard, let us consider a "n" speed gearbox okay, without loss of generality, so what is meant by n speed gearbox? There are n gears okay, in that gearbox so, let us consider a n speed gearbox with the corresponding gear ratios being N_{t1}, N_{t2}, all the way till N_{tn}, okay so, we need to determine the values of N_{1t} t through N_{tn}, okay.

Further, we also need to calculate the gear ratio of the final drive which is denoted by N_d , so in other words in the process of transmission matching, we need to calculate the values of

N_{t1}, N_{t2}, all the way till N_{tn} and this parameter N_d . The question is that like, how do we go about doing this, so let us discuss that.

Gradeability: [Continuous] Indicative of the maximum grade that the vehicle can climb at a given speed. If $\theta_{s,max}$ is the maximum slope angle, then gradeability is defined as

$$G := tan(\theta_{s,max})$$

$$Significamt\ Loads \rightarrow Grade\ Resistance, Rolling\ Resistance \quad (at\ constant\ speed)$$

So, the first vehicle performance requirement that we are going to look at is what is called as gradeability and to be very specific you know like we are going to discuss what is called as continuous gradeability okay, like we will see what is the difference you know, like between this and another form of gradeability shortly okay. So, what is gradeability? So, gradeability is indicative of the maximum grade that the vehicle can claim at a given speed okay, so that is the notion of gradeability.

So, if theta s max is the maximum slope angle, then gradeability is defined as G; parameter G is denotes gradeability, G is the tangent of theta "s" max okay, so that is the notion of gradeability, okay. So, tan theta "s" max is gradeability, okay fine, so typically when we want to go on a grade right, a vehicle goes on a grade, we would want to essentially look for a choice of a gear ratio such that the vehicle travels at a constant speed, okay.

If that were the case we are looking at what is called as continuous gradeability okay, so the adjective continuous essentially considers a scenario where my vehicle is going on a grade but at a constant speed. Now, if a vehicle is going at a constant speed on a grade, what do you think are the loads that the powertrain needs to overcome? Yesterday, we identified five loads right, inertia, what to say aerodynamic drag, grade resistance, rolling resistance and drawbar pull, right.

So, in our analysis for continuous gradeability, what do you think are the loads which will play a dominant role? So, significant loads would be the following, so would you think would inertia play a big role, no right because the vehicle is going at a constant speed on a slope right, so inertia will not play a big role. The one term which is going to play an important role in gradeability is grade resistance.

Because W sin theta “s”max is going to dominate all the other forces that the power train has to overcome, so rolling resistance would be there okay, aerodynamic drag typically as I mentioned you know like for most vehicles you know like, it becomes reasonably important at mid and high speeds typically, when you are going on a grade, you design for low speeds right.

So, consequently it is a reasonable assumption to neglect aerodynamic drag for this particular calculation, if you want you can include, you will see that some people will include it some people may not okay, so it is a matter of choice, okay. So, as a first cut analysis in this exercise we are not going to include and we will also neglect drawbar pull.

We are not; we are assuming that the vehicle is not basically towing any other entity okay, so those are the assumptions. So, the significant loads that the vehicle,the powertrain should overcome during climbing a grade at a constant speed or these, right so, this is at a constant speed. So, there is something else called starting gradeability.

$$(Starting\ Gradeability\ \rightarrow Grade\ Resistance, Rolling\ Resistance, Inertia)$$

Thus, the tractive force required to overcome the significant loads is

$$\rightarrow F = W sin(\theta_{s,max}) + f_r W$$

So, what is starting gradeability? Starting gradeability this as a aside typically, what happens is that like the people evaluate what is the ability of a powertrain to just start moving the vehicle, when the vehicle is initially stationary on a grade, so imagine that the vehicle is kept stationary on a grade okay. The question that we are asking when we are dealing with starting gradeability is that, what is the capability of the powertrain to just start moving the vehicle up the grade from a stationary position.

Then, in addition to the grade resistance and rolling resistance, inertia also will play a role right because of course, it will play a small role but nonetheless, there will be a role for inertia to play because we are starting from rest, no matter that we are starting from 0 speed and then we are going to a low speed maybe over a longer time interval, still there is going to be some average acceleration, right which is going to introduce that M_a component right in the equations.

So, inertia will get added and the powertrain then has to overcome these three loads okay, so we are going to consider continuous gradeability, this is just as an aside because you will see that you know some what to say, designs may require us to deal with starting gradeability, then the only difference from the analysis that we are doing is that we need to add that M_a term that is what.

And as I told you some people will add that what to say, aerodynamic drag term okay, so it depends on the scenario, okay. So, for us for the first cut analysis that we are going to do, we are going to deal with grade resistance and rolling resistance okay, so that is what we are going to consider. Thus the tractive force so, in this course if I talk about gradeability, by default I am talking about continuous gradeability, okay, let me clarify.

So, the tractive force require to overcome the significant loads is W equals sorry, sorry not W, F which is a tractive force, this will be grade resistances sine theta sorry, W sine theta s max, okay that is the grade resistance, then rolling resistance is f_r times W, so these are the two loads or forces that the vehicle should sorry, the powertrain should overcome right, for continuous gradeability.

Now, typically what happens is that like we expect to engage the first gear when we are going on a maximum or the highest grade possible you know for a particular vehicle design. So, imagine that we are going on a mountainous terrain or a flyover by and large what we would do is that like the design is such that we want to get the maximum possible thrust you know from the prime mover.

And the gear should be such that the magnification of the torque which is delivered by the prime mover is also the highest, okay because anyway we are dealing with low speeds, so if you even recall the ideal traction hyperbola right, when you are going at low speeds on grades you know like we want to essentially have a high force, right because we want to overcome these resistances okay, which are significant at when we are going on a grade.

So, now the way this requirement has matched to the gearbox is that like we would engage the first gear, when we are going on this grade, okay. So, let us recall what we derived you know as the output of the powertrain in the previous class, we are going to use that expression, okay. So, let the engine provide a maximum torque of T_e max okay, so considering that the vehicle climbs the grade with the first gear being engaged.

We obtain the following; so, if you recall the expression for F from yesterday's class, let me go up so, if you recall we derived this expression right, so at any engine torque and any chosen gear, this is the expression for F considering the inertia of the powertrain components. Now, imagine that you know like the vehicle is going at a constant speed on the grade, so the vehicle acceleration is close to zero.

So, what do you think happens to the second set of terms on the right hand side of this equation, they will vanish, right because a is almost zero, so what are we going to get; so the tractive force will be equal to the maximum engine torque multiplied by the corresponding gear ratio multiplied by the transmission efficiency divided by the tyre radius okay, so or the wheel radius, right.

Let the engine provide a maximum torque of $T_{e,max}$. Considering that the vehicle climbs the grade with 1st gear being engaged, we obtain

$$\frac{T_{e,max} N_{t1} N_d \eta_t}{r_\omega} = W sin(\theta_{s,max}) + f_r W \quad \longrightarrow \boxed{1}$$

To find N_{t1}, N_d

So, that is what we are going to get so, we obtain F to be equal to T_e max, N_{td} I am splitting as N_t times N_d , N_{t1} being the gear ratio of the first gear, right then, we have the transmission efficiency divided by the wheel radius, this has to be matched with this requirement, this is the requirement from the vehicle, right, is it not, so this is the output from the powertrain when we are going at; when we are operating at a point where we are getting the maximum torque, okay.

So, if you are getting the maximum torque as Te max and we have engaged the first gear, so T_e max times N_{t1}times N_d times eta “t” divided by r_w is the corresponding output from the powertrain, okay that force output from the powertrain that is going to be matched to what is required by the vehicle. What is required by the vehicle? It is W sine theta “s” max plus f_r W, so this is one equation that we have, so let us call this is equation one, okay.

So, we can see that we have matched what the powertrain would provide when engaged in the first gear and the engine is operated at its maximum torque point, right to what is required by the vehicle, when it is climbing on a grade, this is the expectation that is how we are matching both the things, right okay. Now, in this expression if you look at this equation what are all the unknowns here, right now?

Right now, if we look at this expression, there are two unknowns, right because all the other; all the parameters on the right hand side would be given to us, T max is known, transmission efficiency and r_w are known, so we need to find N_{t1} and N_d okay but we have only one equation till now, right so, we are not yet done okay, so we are going to see how do we what to say go further on and address this issue, okay.

But this is the first equation this is a starting point, now we are going to get one more equation from the specification of maximum speed, okay. Gradeability is one requirement where we are going to engage the first gear now, when my vehicle is going at the maximum speed which is the gear, which I would typically engage imagine, that you know I start my car from rest, I keep on up shifting from the first gear to the second and so on.

When I reach the maximum speed right, I would obviously engage the n_{th} gear or the highest gear, so that is the other extreme so, the maximum gear; maximum speed choice will give me an equation that would essentially involve the n_{th} gear ratio, so that is an exercise which we are going to do now. So, what is that expression, before we derive that expression let me introduce this concept of what is called as wheel slip ratio, okay.

The wheel slip ratio during traction is given by

$$\lambda = \frac{r_\omega \omega_\omega - v}{r_\omega \omega_\omega}, \rightarrow \lambda = 0, for\ a\ purely\ rolling\ wheel$$

$$= 1, for\ a\ spinning\ wheel$$

We are going to come back to this in more detail when we essentially do braking but for traction, the wheel slip ratio or what is called as a wheel longitudinal slip ratio during traction that is when the powertrain is driving the vehicle okay, is given by; it is defined as so, different people will use different symbols for this wheel slip ratio; small i, small s, lamda, Kappa and so on okay, there are different symbols.

Where v is the vehicle longitudinal speed.

Hence, $v = r_\omega \omega_\omega (1 - \lambda)$

We are going to choose lamda okay, during traction it is defined as r_w times omega w which is essentially the tangential speed of the wheel, r times omega right, minus the vehicle longitudinal speed, v divided by r_w omega w, okay where v is the vehicle longitudinal speed. Hence, from this expression, we obtain that the vehicle longitudinal speed is going to be equal to r_w omega w times 1 minus lamda.

That means, that if the vehicle is rotating at an angular speed of omega w, sorry wheel, sorry the wheel is rotating at an angular speed of omega w at a; with the slip ratio being lamda okay at the tyre road interface, then the corresponding longitudinal speed is given to be this. Now, what is this lamda? Essentially, if you consider a typical pneumatic tyre, you know which is used in road vehicle that deforms.

And if you look at the contact between the tyre and the road, you have a contact patch or a contact area, when the tyre goes into that, it deforms it sticks to the road and the later path slips and the vehicle essentially is propelled. So, it is not a free rolling motion okay, there is a slip which is happening at the tyre road interface and this parameter lamda essentially quantifies that slip.

So, we can immediately see that lamda is equal to zero for a purely rolling wheel, right, if the wheel is exhibiting a pure roll motion what is lamda; zero, now what do you think is lambda is for what is called as a spinning wheel that means, imagine that you know a wheel is a driven wheel is caught in a pothole full of mud and you press the accelerator pedal you know like the wheel keeps on spinning, the vehicle hardly moves right.

So, what happens; this lamda tends to almost 1, so that is how, that is why during traction, the definition of lamda is chosen in this manner, so that the value of lamda remains between zero and one okay that is the motivation behind the choice, okay. So, that is the slip ratio, we will also come back to this when we are dealing with braking and we will see how it is defined during braking and what is its impact and so on okay.

So, I will stop here so, in the next class we will come back and continue from here we will look at the maximum speed specification how that gives us an equation for the highest gear and then we also need to figure out, we will also reason out some mechanism by which we determine the intermediate gear ratios, okay so that is something which we will complete in the next class. Thank you.