Fundamentals of Automotive Systems
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Lecture – 37
Powertrain Analysis 2 - Part 01

Okay, greetings, so welcome to today's class, a quick recap what we were doing yesterday was to consider the forces that were acting on the vehicle, we looked at the physics behind these various force terms and we essentially wrote down the governing equations.

And the place where we stopped was to get the expressions for the loads on the front wheels and the rear wheels respectively, okay so, this is where we stop. So, let us continue from here so, if you look at these 2 expressions, if we look at the first term on the right hand side of these 2 expressions, we can immediately observe that these indicate the static load on the front and the rear wheel respectively.

$$\Rightarrow W_f = \frac{W\ell_2}{L} - \frac{h}{L}\underbrace{\left[\frac{W}{g}a + R_a + R_d + W sin\theta_s\right]}_{A}$$

A $\longrightarrow$ $= F - R_a$

$$W_r = \frac{W\ell_1}{L} + \frac{h}{L}\left[\frac{W}{g}a + R_a + R_d + W sin\theta_s\right]$$

$$\Rightarrow W_f = \frac{W\ell_2}{L} - \frac{h}{L}[F - R_r],$$

$$W_r = \frac{W\ell_1}{L} + \frac{h}{L}[F - R_r].$$

Static Load

Dynamic Load Transfer along the longitudinal

That is let us say the vehicle is standing, what to say on a flat road, right at rest, then Wl_2 by l and Wl_1 by l will be the static load distribution between the front and the rear respectively right, so that is how we get the longitudinal location of the centre of mass also, okay. So, we can immediately observe that the first term on the right hand side of both equations indicate the static load, okay.

So, just to highlight that further so, suppose let us say, we consider this stationary car, I am just drawing a simple schematic okay, so this car okay, so let us say W is of course this is the forward; this is the direction of motion, right so, let us say this is the front, W_f and this is the rear W_r and this is l_1 and this is l_2, right and the total distance is capital L which is the wheelbase.

Then, we can immediately see that Wl_2 by L will be the load on the front, Wl_1 by L will be the load on the rear right, so that is why the first term on the right hand side of the expressions that we derive indicate the static load on the front and the rear wheel respectively, okay. Now, what is the second term? So, we can see that the magnitude of the second term is the same.

It is "h" by l times F minus the rolling resistance but what can we observe; that term is being subtracted from the load on the front wheel and added to the load on the rear wheel and that is what we can physically observe, right. Let us say we are sitting in a vehicle and the vehicle suddenly starts to accelerate, what happens to all of us; all of us tend to go backward.

So, the second term indicates, what is the so called dynamic longitudinal load transfer that happens due to acceleration okay so, this is the indicative of the dynamic load transfer along the longitudinal direction, okay. So, during acceleration, this load transfer happens from the front to the rear, so that is why we see that a quantity is subtracted from W_f and the same quantity is added to W_r r, okay, so that is the second term on the right hand side of both equation.

So, we can see that the expression for W_f consists of two components; the static load and the dynamic load transfer. So, now what we are going to do is to essentially determine what will be the maximum tractive effort that is possible when we drive the front wheels and the rear wheels, okay. So, typically if you look at most vehicles either the front wheels or the rear wheels are driven, some vehicles you know all the wheels are driven okay.

So, let us look at front wheel drive and rear wheel drive, okay so, for a front wheel drive like in most passenger cars today, the maximum tractive effort; please note that the from yesterday's free body diagram, you know like the tractive effort of the front was called as F_f; tractive effort of the rear was called as F_r, for a front wheel drive, F_r is 0, okay so, only the front wheels are driven.

So, the maximum tractive effort is given by; so let us say we call the maximum tractive effort as F max that is going to be mu "p" times W_f. So, what is this mu "p" okay, we will discuss this in more detail when we go to braking.

Maximum Tractive Effort:

For a Front Wheel Drive [FWD], the maximum tractive effort is given by

$$F_{max} = \mu_p W_f = \mu_p \left[\frac{W\ell_2}{L} - \frac{h}{L}\left(F_{max} - R_r\right)\right]$$

R_r: $f_r W$

μ_p: Peak/Maximum Coefficient of friction/traction/adhesion at the tyre-road interface

$$\Rightarrow L\ F_{max} = \mu_p W \ell_2 - \mu_p h F_{max} + \mu_p h f_r W$$

$$\Rightarrow \boxed{F_{max} = \frac{\mu_p W[\ell_2 + f_r h]}{[L + \mu_p h]}}$$

FWD

But right now, this is the peak or maximum coefficient of friction you know like or traction or adhesion at the tyre road interface. So, this is a simple way of expressing the maximum traction available at a particular tyre road interface, what we do is that we take a parameter mu, okay and multiply it by the load on that particular tyre, the normal load at the particular tyre road interface, okay.

So, this coefficient mu is called as the friction coefficient or the traction coefficient or the adhesion coefficient at a particular tyre road interface, okay. So, I have put the subscript p to indicate that there is a peak value okay, so this mu p is the maximum value that you can get at a particular tyre road interface. The value of mu p changes with the type of road surface suppose, let us say we have a dry asphalt concrete road surface, the value of mu p would be pretty high.

But that would decrease as soon as let us say, it rains okay and the same asphalt concrete road surface becomes wet you know due to the rains, the value of mu p will drop and let us say if it snows, it will drop further, if the snow freezes as ice, it will drop even further. So, typically depending on the road surface conditions and the tyre conditions and the condition at the interface, this value of mu would change, not only that this value of mu also changes with what is called as a wheel slip ratio which we will come to when we are dealing with braking and anti-lock brake systems, okay.

So, right now we will just take it that given on the load at a particular tyre road interface; the normal load the maximum tractive effort that one could expect is the maximum traction coefficient or friction coefficient multiplied by their corresponding load, so that is what we are going to tell. So, if for a front wheel drive, since only the front wheels are driven so, we are taking F max to be mu p times W_f, okay, where W_f is the normal load on the front wheels.

So, now what we do is that like we just go and substitute for W_f, so that is going to be Wl_2 by l minus h by l multiplied by F minus R_r but now F will become F max, right because we are dealing with the limiting maximum force that is available, right so, we are just going to get F max here. So, from this expression we can immediately observe that there is F max on both sides, so just simplifying this expression we will get L times F max is going to be equal to mu p W times l_2 minus “h” times sorry, minus mu p h times F max.

And this rolling resistance as we recall we are going to write it as f_r times w, so this will give us plus mu p h f_r times W, right so this is what we will get, okay. So, if I simplify for F max, we can immediately observe that this expression reduces to mu p W times l_2 plus f_r h divided by l plus mu p “h”, so this is what we will get for the maximum traction that is available for a front wheel drive okay, so this is the expression for F max for a front wheel drive.

For a Rear Wheel Drive {RWD}, the maximum tractive effort is given by

$$F_{max} = \mu_p W_r = \mu_p \left[\frac{W\ell_1}{L} + \frac{h}{L}[F_{max} - \underbrace{R_r}_{f_r W}]\right]$$

$$\Rightarrow LF_{\max} = \mu_p W\ell_1 + \mu_p hF_{max} - \mu_p hf_r W$$

$$\Rightarrow F_{max} = \frac{\mu_p W[\ell_1 - f_r h]}{[L - \mu_p h]}$$

RWD

Let us do a similar exercise for a real wheel drive and then like we will compare them, so similarly for a rear wheel drive, what we are going to abbreviate as RWD, the maximum tractive effort is given by the following. So, F max now is going to be equal to mu p, which is a peak friction coefficient or traction coefficient times W_r , so let us go back and substitute for W_r.

So, what is W_r ; if you recall, it is going to be Wl_1 by l plus "h" by l times F max minus R_r, right so this is going to be Wl_1 by capital L plus h by L times F max minus R_r, so we do the same simplifications here, so R_r is going to be written as f_r times W, so what we will have is L times F max will be equal to mu p Wl_1 plus mu p h F max minus mu p h fr W. So, if we simplify this expression, we will get F max to be equal to mu p W times l_1 minus f_r h divided by L minus mu p.

So, this is the maximum tractive force for a rear wheel drive, this is the maximum tractive force for a front wheel drive, okay. So, now we can immediately observe the following, so having obtained what is the maximum limit of traction, you know like that is possible for a given set of wheels, okay either the front wheels or the rear wheels obviously, you know like we would want to drive the wheels where the maximum capacity is available, right.

So, how do we make a choice between a front wheel drive and a rear wheel drive, so now for the sake of what to say, comparison let us first consider the denominator term of the two expressions right, for the front wheel drive and the rear wheel drive. So, the term in the denominator of the front wheel drive F max is L plus mu p h, okay the term in the denominator of the rear wheel drive, F max is L minus mu p h.

So, which do you think is smaller; L minus mu p h, right so, the reciprocal is going to be greater in the expression for the rear wheel drive. Now, why are we getting this effect because of dynamic load transfer see, from where did this mu p h come; it came from here, right when we substituted for F max as mu p times W, you know like we got, we essentially had that mu p h and minus mu p h appearing in the denominator for the two drive configurations, right.

So, the dynamic load transfer during acceleration seems to be increasing the maximum tractive capacity available at the rear wheels, if you compare only the denominator term, okay we are going one by one okay, so that is the first observation. Now, let us look at the numerator; mu p times W is the same right, however we have l_2 plus f_rh and l_1 minus f_rh, for the time being, for the sake of argument let us neglect f_rh, okay, f_r times h is going to be smaller when compared to l_1 and l_2.

For the sake of argument, let us neglect it, while doing calculations, you will include them just for the qualitative, what to say a comparison, let us neglect the term f_rh and plus or minus f_rh in the two expressions. So, the front wheel drive expression has l_2 in the numerator and the rear wheel drive expression has l_1 in the numerator right, mu pW being common.

So, let us take a typical passenger car that we see on the road, we all of us have observed that it is a front wheel drive vehicle irrespective of the fact that the dynamic load transfer takes place from front to rear during acceleration, as a result having that L minus mu p h in the denominator would have helped but still we go for a front wheel drive. What do you think is a reason?

Because it is due to the way in which mass is distributed right, in the vehicle so, if you look at a typical passenger car, the engine, the transmission and almost all other components are towards the front, okay. So, what is going to happen is that the centre of mass of the vehicle or centre of gravity is going to be shifted to the front so, if the centre of gravity is shifted to the front which value is going to be higher, l_2 or l_1; l_2, right.

So, obviously we can observe that despite the fact that dynamic load transfer is going to transfer the load from front to rear, the fact that l_2 is higher, still makes it advantageous to drive the front wheels, right so that is an observation we could make and please remember this amount of dynamic load transfer also depends on h, in addition to mu p, right if you look at denominator.

So, the lower the value of “h”, you know like the lower is going to be the effect okay, of dynamic load transfer, so that is something which we need to what to say, observe also, okay. So, on the other hand let us say we take a bus or a truck or a lorry right, what happens in a truck let us say, you know we drive the rear wheels so, in a truck; in a typical truck even though the engine is in the front, typically right, there is a propeller shaft which takes the output from the transmission all the way along the length of the vehicle and drives the rear wheels.

Why would we do that? Because of course, we are going to get an advantage due to dynamic load transfer, if we drive the rear wheel however, in a truck, the main load comes from the goods or the material that the truck will carry and where is it carry; it shifted towards the rear so, you will see that for a fully laden truck right, the CG is going to be shifted to the rear.

So, which value will be higher; l_1 and where does l_1 appear, in the expression for the rear wheel drive, so it makes more sense to drive the rear wheels because of this fact and also like dynamic load transfer also helps of course, the actual capacity of the powertrain is different okay, this is from the perspective the vehicle's dynamic response, right. So, that is this, these two expressions gives us what is the capability or capacity available at the front and the rear wheels.

And this sort of analysis becomes very important particularly when we go for high performance vehicles that is let us say, you go for high performance vehicle let us say, even we take a race car, what happens in a race car? Typically, we want to put the engine powertrain everything closer to the rear and we drive the rear wheels, right and the even the driver is shifted to the rear due to various reasons.

But from this perspective what happens; although, “h” is very small right, the value of l_1 will be larger, so we drive the rear wheels because we want to have faster acceleration, higher acceleration, if you want to have higher acceleration yes, my engine transmission should develop that force or torque that thrust but the tyre road interface should also be able to sustain that force and that is what this expression gives me, right.

This gives us what is the capacity at a particular tyre road interface, either at the front or the rear wheels, so using this we can get a rough idea as to why the drive and the powertrain are realize the way they are right for different class of vehicles.