Fundamentals of Automotive Systems

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Lecture – 36
Powertrain Analysis - Part 02

Now, let us look at it from; look at what the vehicle requires you know like in order to drive it and meet some vehicle performance requirements, as we talked about initially, when we want to design a powertrain, we want the vehicle; we want the powertrain to satisfy some vehicle performance requirements such as maximum speed, maximum acceleration, gradeability right, a fuel economy, range and so on.

So, now the question arises, you know like how do we look at this from the vehicles perspective and then you know like try to match what the powertrain is giving with the expectation's of the vehicle, so let us look at it from that perspective.

So, let us do an analysis with respect to what are all the forces acting on the vehicle that need to be overcome by the powertrain and drive the vehicle, so let me introduce a very simple free body diagram of the vehicle okay, so just to illustrate and discuss what are all the various forces acting on it, okay. So, let us consider this free body diagram, so what we have is it like we have a car moving up a grade okay, so the direction of motion is like this.

So, the grade angle is some is denoted by theta s, okay so that is the grade angle of the slope on which the car is moving, okay. So, let us say the acceleration of the car is some "a" and this is the centre of gravity of the car, so W is the weight of the car, so this we take it as M times g; M is a mass multiplied by acceleration due to gravity. Now, we can see that I have marked essentially W by g times "a" which is M times "a".

M times "a" is nothing but the inertia right, inertia term which one needs to consider and I have marked it in a direction opposite to that of the deceleration typically, this is what is called as the D'Alembert's force, right. So, if you recall basic dynamics what we do in the D'Alembert's approach is that we introduce this inertia term at the vehicle centre of gravity and what we do is that like we convert this problem into an equivalent statics problem.

So that, like we can essentially sum all the forces acting along the direction of motion to be 0 and we could also take moments about any point, okay and equate yeah, so that is the process we are going to follow. So, let us say the point of contact at the front and the rear tyre road interface is denoted by “a” and “b” respectively, okay and Wf and Wr indicate the normal load at the front and the rear respectively, okay.

Forces acting on the vehicle

$W_f + W_r = W$

$\ell_1 + \ell_2 = L$

$F_f, F_r \rightarrow Tractive\ Forces$

$$Aerodynamic\ Drag, R_a = \frac{S_a}{2} C_D A_f v_{rel}^2$$

$R_{rf}\ , R_{rr}\ \rightarrow Rolling\ Resistance\ Force$

$Rolling\ Resistance\ force,\ R_r = R_{rf} + R_{rr} = f_r W$ ← Rolling Resistance Coefficient

So, W_f and W_r are the normal loads at the front and the rear respectively, so obviously W_f plus W_r should be equal to W right, so that is the weight of course, we are essentially considering this as our the; we are considering more forces okay, we are coming one by one. So, this capital L is what is called as the wheel base of the vehicle okay, so that is the distance between the front wheel centre and the rear wheel centre of the vehicle, okay.

And the centre of gravity is taken to be at a distance of L_1 from the front wheel centre and L_2 from the rear wheel centre, so consequently L_1 plus L_2 will be equal to capital L, okay and it is considered that the centre of gravity is at a height h from the ground okay, so the height of the centre of gravity from the ground is h, okay. Now, F_f and F_r are the tractive forces at the front and the tyre road interface respectively okay, F_f and F_r are the tractive forces.

So, the tractive forces come from the powertrain okay, of course they are also very closely related to the tyre road interface conditions okay, we will discuss them later on but if you have a for example, a front wheel drive, F_r will be 0, if you have a rear wheel drive, F_f will be 0, so depending on which wheels are driven by the powertrain we have to consider the corresponding tractive force.

If you have a 4 wheel drive both F_f and F_r will be non-zero, okay and so on okay, so without loss of generality, I am indicating both F_f and F_r. Now, what is this R_a; R_a is the aerodynamic drag which has been lumped on the vehicle in this manner okay, so as a vehicle moves you know like we are going to have air flowing or passed it over in and so we are going to have this aerodynamic resistance which essentially is quantified and lumped by this what to say, variable R_a, okay.

And typically, this is taken as rho "a" CD A_f V_{rel} square okay, so what is rho "a"; rho "a" is "a" density of air, CD is the drag coefficient, A_f is what is called as a frontal area of the vehicle, V_{rel} is the relative speed between the vehicle and the local wind speed okay so, that is how this aerodynamic drag is quantified. This aerodynamic drag is a result of what is called as pressure drag and viscous drag.

Pressure drag is due to the normal component acting on the surface right, viscous drag is due to the formation of boundary layers as air travels upon the surface of the vehicle, right so, both contribute to this aerodynamic drag, okay and that is lumped for our analysis at this point denoted by this R_a and the height of this aerodynamic drag from the ground is taken to be some H subscript "a", okay so that is the notation that we are following, okay.

We are lumping all the aerodynamic resistance forces as one quantity right, now what is this R_d; R_d is what is called as drawbar pull, so what is this drawbar pull? Suppose, I fit something to my vehicle let us say a small trailer, right to my vehicle and I want to pull that trailer along with my vehicle, so the trailer requires some force right to displace it, so that is what is denoted by R_d; drawbar pull.

And that is assumed to act at a height of H subscript d, of course if you do not fit anything to the vehicle, drawbar pull is 0, okay so, without lot of loss of generality, including the drawbar pull. Then, we have 2 more forces, what are called as rolling resistance forces at the front and the rear respectively okay, R_{rf} and R_{rr}. R_{rf} and R_{rr} are the so called rolling resistance forces.

So, what is this rolling resistance force? The rolling resistance force comes about because the pneumatic tyre, okay has hysteresis loss you know when it is rotating okay, its sticks to the road surface and then slips, okay and due to the deformation of the pneumatic tyre and due to its viscoelastic nature, there is a hysteresis loss and that gets realized as this so called rolling resistance.

So, what happens; so let us say I draw a very simple schematic, so this is the rotating tyre I am just exaggerating just to convey the concept, so let us say the tyre is rotating counter clockwise, okay and this is the tyre road contact patch or the contact area. So, what happens is that due to this hysteresis loss, the normal stress distribution at this contact patch is asymmetric, okay it is shifted in this manner.

So, if you take the resultant of this normal stress distribution of the tyre road contact patch, what will happen is that you will see that there is going to be an opposing resultant moment, right because the normal stress distribution is asymmetric, so this moment is what is called as a rolling resistance moment, okay which is denoted by M subscript r.

When we want to essentially account for it, when what to say we want to analyse the translation of the vehicle; translational motion of the vehicle, we introduce an equivalent force which is what is called as a rolling resistance force, okay which is denoted by R subscript r, okay. So, R subscript r is the total rolling resistance force which is the sum of this R_{rf} and R_{rr}, the additional subscript f and r denotes front and rear respectively.

And typically, this R subscript r is taken as fr times W okay, for first cut analysis, simple analysis, we assume that the rolling resistance force is directly proportional to the weight of the vehicle, okay and this coefficient fr is what is called as a rolling resistance coefficient okay, we shall consider to be constant for our first cut analysis and we shall what to say express, rolling resistance force as product of the rolling resistance coefficient and the weight of the vehicle, okay.

So, this is the background, so these are all the different forces, their physics and also the way they are expressed okay so, we are going to use these in our calculations.

$\textit{Rolling Resistance force}, \ R_r = R_{rf} + R_{rr} = f_r W$

$\pm W \sin \theta_s$

"Loads" → Inertia, Aerodynamic Drag, Rolling Resistance, Grade Resistance, Drawbar Pull

Balancing forces along the direction of motion yields

$$\underbrace{M}_{\frac{W}{g}}a = \underbrace{F_f + F_r}_{F} - R_a - R_g - R_d - \underbrace{R_{rf} - R_{rr}}_{R_r = f_r W}$$

$$\Rightarrow \boxed{F = \frac{W}{g}a + R_a + R_r + R_g + R_d} \longrightarrow \boxed{\text{A}}$$

Taking moments about point A yields

$$W_r L = W cos\theta_s \ell_1 + W sin\theta_s h + \frac{W}{g} ah + R_a h_a + R_d h_d$$

So, if we look at what are all the loads that act on the vehicle during traction, so these are some terms that we would encounter, so when we do this analysis, so what people mean by load on the vehicle are the resistive forces that the powertrain needs to overcome, right. So, the loads or the resistive forces are inertia because when we are accelerating, the inertial term becomes important, M times “a” right.

Then, aerodynamic drag; aerodynamic drag becomes very important particularly when we are cruising at mid and high range speeds okay, it may not be very significant at low speeds of course, it depends on the type of vehicle, right but it becomes more critical when we are cruising at constant speeds and the speeds are in mid to high ranges, okay so, there aerodynamic drag plays an important role.

Then, we have rolling resistance then, we have grade resistance, what is grade resistance; grade resistance is W sine theta s, right because there is going to be a component of the weight along the grade along the slope, right which has to be overcome by the powertrain when the car is or when the vehicle is trying to go up the grade, so that is what is called grade resistance.

Some people will put a plus or minus sign to it just to indicate that it is a resistance while going up but it aids motion will while coming down, so the plus sign indicates that it is a resistance while the vehicle is going up, the minus sign indicates that it is aiding motion when the vehicle is coming down the grade, okay and another load is what is called as drawbar pull, okay.

So, these are typical loads that act on the vehicle, so this is the setup okay, so now the process that we are going to follow is the following; we are first going to sum all the forces along the direction of motion, we are going to balance them right, then what we will do is that we will take moments about points "a" and "b", so that we get expressions for W_f and W_r okay, so that is the analysis we are going to do.

So, balancing forces along the direction of motion yields the following, right so, we have M times a to be equal to F_f plus F_r minus aerodynamic drag okay, R_a minus the grade resistance R subscript g minus the drawbar pull minus the rolling resistance at the front wheel and the rear wheel respectively, okay. So, these are all we are just balancing the forces alright, okay. So, typically in most analysis of road vehicles, we will find that you know like the mass of the vehicle is typically written as W by "g", okay.

And even the acceleration levels are normalized by "g", okay, people will talk about acceleration or deceleration in terms of "g" units okay, so that is a convention, so we will write M as W by"g", okay so, this F_f plus F_r is the net traction force F, okay and this R_{rf} plus R_{rr} we will take it to be the net rolling resistance R_r, okay. So, consequently what will we get we would get the net of course, R_r as we discussed earlier we will take it as f_r times W also, right.

So, using this above equation, we will get the net tractive force that is needed to overcome the loads will be W by "g" a plus aerodynamic drag plus the rolling resistance plus the grade resistance plus the drawbar pull okay, so this is one equation okay, so let us call it as some equation A, right, so it is pretty intuitive right. So, the net force, traction force that is provided by a power train should overcome inertia.

These are the 5 loads right; inertia, aerodynamic drag, rolling resistance, grade resistance and draw bar pull, whatever we just wrote down correct, so that is what we need the power train to overcome. Now, what we do; we will take moments about point A, let us do that exercise. So, now taking moments about point A yields, so what we; what do we get?

So, if we take moment about point A, please note that there is a component of W; W cos theta "s", right which will provide a clockwise moment and there are other forces which provide a counter clockwise moment, right so, we are just going to balance out all these moments, right. So, if you look at it when we take moments about point A; Wr times L is going to be a counter clockwise torque, right.

Do we have any other counter clockwise torque due to any other force, when we take moments about point A? No, right, the torque due to all other forces are clockwise, right because we are at point A, we are taking moments of all the forces about point A, please note that the rolling resistance force and the traction force will not be having any moment right.

Because they pass through that point correct, the moment arm is 0, so we can immediately see that Wr will have a counter clockwise torque, all the other forces we will get a clockwise torque, so let us write them down. So, one torque is going to be W cos theta "s" times l_1, right that is the torque due to this force, right, this component. Then what do we have; we have a torque due to this other component, right.

$$\Rightarrow W_r = \frac{1}{L}\left[W\ell_1 cos\theta_s + \frac{W}{g}ah + R_a h_a + R_d h_a \pm W sin\theta_s h\right] \longrightarrow \boxed{B}$$

Taking moments about point B yields

$$W_f = \frac{1}{L}\left[W\ell_2 cos\theta_s - \frac{W}{g}ah - R_a h_a - R_d h_a \mp W sin\theta_s h\right] \longrightarrow \boxed{C}$$

$Assume\ i)\ \cos\theta_s \approx 1, \quad ii)\ h_a \approx h_d \approx h$

W sine theta "s" times "h" h, then we have a torque due to the inertia term W by "g" "a" times h, then we have a torque due to the aerodynamic drag, then we have a torque due to the drawbar pull, right.

So, this implies that Wr which is the net load on the rear wheel, the normal force on the rear wheel will be one by L Wl_1 cos theta "s" plus W by g a times h plus R_a h_a plus R_d h_d plus W sine theta "s" times "h" h, right this is another equation, okay. So, let us call this as equation B, as it mentioned some people will write for this, they will write plus or minus because depending on whether the vehicle is climbing up the grade or down the grade, okay.

We are considering the case where the vehicle is climbing up the grade okay, so similarly if you take the moments about point B, I leave it to you as an exercise, taking moments about point B yields finally you know like if you simplify, you will get Wf to be equal to 1 by L Wl_2 cos theta s minus W by g a h minus $R_a h_a$ minus or this should be $R_d h_d$ right sorry, okay, $R_d h_d$ minus W sine theta "s" "h", okay so this is what we will get for the other one.

Please do this you know like it is pretty straightforward right, so let us call this as some equation C, as I mention once again some people you will see this expression as minus plus okay, just to indicate climbing up and coming down and grade, right, so that is a notation that you will see. So, now we are going to make an approximation okay, let us assume that the grade angle is small enough such that cos theta s is almost equal to 1, that is one assumption.

And we will also assume that the values of h_a , h_d are almost equal to h, okay, so these are 2 assumptions that we are going to make, okay just to simplify these expressions, okay so, let us make these 2 assumptions okay, so this is one assumption and this is a second assumption, okay. So, once we make these assumptions immediately, we can observe that Wf, what will happen to W_f?

It will be 1 by L and let me rewrite in this form, I can rewrite it as Wl_2 by l because cos theta is almost becomes 1 and then we pull h common because h_a, h_d and all are approximated as h, so we take h by L common and this becomes W by g a h plus R_a plus R_d plus W sine theta "s", correct I am sure all of; all can agree with me, right. So, what happened; cos theta "s" we are taking as 1, h_a and h_d you are taking as "h".

$$\Rightarrow W_f = \frac{W\ell_2}{L} - \frac{h}{L}\underbrace{\left[\frac{W}{g}a + R_a + R_d + W\sin\theta_s\right]}_{\boxed{A}\ \longrightarrow\ = F - R_a}$$

$$W_r = \frac{W\ell_1}{L} + \frac{h}{L}\left[\frac{W}{g}a + R_a + R_d + W\sin\theta_s\right]$$

So, it gets simplified in this manner, so similarly W_r r will become Wl_1 by L plus h by L W by g a h plus R_a plus R_d plus W sin theta "s" okay, okay, so yeah sorry, you know h is not there in both equations thanks yeah, so this is just, yeah we have taken h outside right, so h is no longer there, thanks yeah, so we get this equation. Now, we can immediately see that the term within the square parentheses in both these equations, we get from equation A, right.

So, if you go to equation A; W by g a plus R_a plus R_d plus the grade resistance R_g will be essentially equal; be equal to F minus R_r, right. So, how do I get this; from equation A, right, if you go to equation A, if you take R_r to the other side whatever is remaining on the right hand side is what we find in the square bracket, so this becomes F minus R_r okay, so this also becomes the F minus R_r, right.

So, consequently the expressions for the normal load become W_f is equal to Wl_2 by L minus h by L F minus R_r and the normal load on the rear W_r becomes Wl_1 by L plus h by L times F minus R_r, so this is all the normal loads on the front and the rear simplify to okay. So, in the next class what we are going to do is that like we are going to physically interpret these equations and what to say work with them further.

And ultimately, what we are going to figure out is to when we shall consider a front wheel drive and the rear wheel drive and then like how do we match the vehicle performance requirements with that of the powertrain to determine the first cut values of the gear ratios, so that is the analysis which we will follow up in the next class. So, I will stop here for today, thank you.