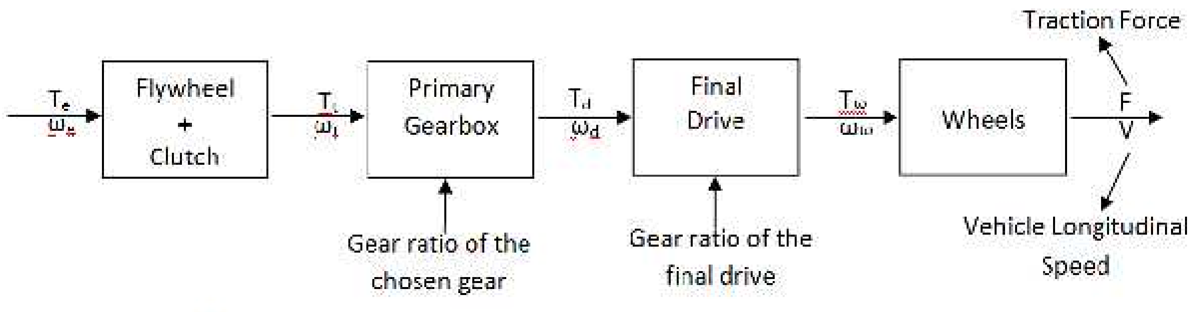
Fundamentals of Automotive Systems
Prof. C. S. Shankar Ram
Department of Engineering Design
Indian Institute of Technology - Madras

Lecture – 35
Powertrain Analysis - Part 01

Okay, greetings, so let us get started with today's class, so a quick recap of what we did in the previous class; we looked at the various components of a gearbox, we looked at the construction and also the operation of what is called as a synchromesh gearbox and we looked at the concepts of gear engagement you know like through what is called as a synchroniser right, so that is something which we learnt in the previous class.

So, the role of the synchronizer is to ensure that the gear engagement is smooth now, when we essentially or engaging an output gear to the output shaft, right that are initially rotating at different speeds. So, we also looked at how a multi rail selector is used to select a particular gear right, so this is where we stopped in the previous class.

So, in today's class I will get started with powertrain analysis, so having looked at how a manual transmission is realized and is operated, so let us look at how do we analyse such a power train and how do we select first cut values of various gear ratios for a multi speed gearbox depending on vehicle performance requirements, so that is an analysis that we are going to do.



We shall account for friction losses through the transmission efficiency

So, if you look at the flow of energy you know like in a power train that we are considering so, let us say the engine delivers a torque T at a speed omega e and that is transmitted to the flywheel, the clutch engages with the flywheel, right and let us say it transmits a torque; T subscript t at a speed omega t okay, of course when the clutch is rotating with the flywheel without slip, omega e and omega t will be the same, right.

And this is then transmitted to the primary gearbox, so N subscript t is the gear ratio of the primary gearbox you know like whatever gear that has been chosen, okay and the primary gearbox outputs a torque T subscript d at a speed of omega subscript d which goes to the final drive, the gear ratio of the final drive is denoted by N subscript d sorry, this is not the; this is the gearbox; gear ratio of the selected gear.

So, N subscript d is the gear ratio of the final drive, okay and the final drive transmits a torque T_w at a speed omega w to the wheels which is then realized as a force; traction force F you know at a longitudinal speed v, okay so, this is the traction force that is provided by the powertrain to the vehicle, this is the vehicle longitudinal speed. So, we can see that a typical powertrain transmits energy through these set of components, right.

So, this is the gear ratio of the chosen gear in the primary gear box now, what we are going to do is that like we are going to relate all the terms that appear in this block diagram to get an idea as to what will be the output from the powertrain you know for a particular chosen gear and a particular input engine torque, okay so that is our motive. So, in this analysis you know what we will do is that we shall lump and account for friction losses at the end, okay.

So, we will not deal with the friction loss at each and every component but we will look at the friction, we will account for friction losses in the entire powertrain as a transmission efficiency at the end, okay, friction losses through the transmission efficiency parameter, so that is something which we will look at later, all right. So, now we will come one component at a time, okay.

The torque input to the primary gearbox is

$$T_t = T_e - I_e \alpha_e$$

The torque output from the primary gearbox is

$$T_d = (T_t - I_t \alpha_t) N_t$$

The torque output from the final drive is

$$T_\omega = (T_d - I_d \alpha_d) N_d$$

The tractive force available at the tyre-road interface of the wheels is

$$F = \frac{(T_\omega - I_\omega \alpha_\omega)}{r_\omega}$$

So, the first equation that we are going to write is for the torque input to the primary gearbox, so the torque input to the primary gearbox is given by T subscript t which is nothing but the input torque T subscript e minus the inertia of the rotating components, right so, the engine and the clutch components. So, let us lump them as some Ie times alpha e okay, so where alpha e is the corresponding angular acceleration.

So, we are not considering the torque due to friction; viscous friction right now because as we just discussed you know, we will account for all the frictional losses at the end okay, so we are only going to consider what to say, the inertial term okay, when we are relating the input and the output torque of each block, okay. So, this is the torque input to the primary gearbox.

Now, next comes the torque output from the primary gearbox that is given by T subscript d which is what is going as input to the final drive and that we can write as the input T subscript t minus the inertia of the rotating components in the primary gearbox which is taken; lumped as It alpha t and we are essentially going to multiply it by the gear ratio N subscript t, okay.

Because there is an scaling taking place, right due to the gear ratio, so that is how we are going to write this equation for the torque output from the primary gearbox, so that is T_d , so now this torque is going to; go to the final drive, so the torque output from the final drive can be similarly written. So, the torque output from the final drive is now going to be the; that is represented as T_w.

Because the output from the final drive is denoted as T_w that is going to be the input torque minus the inertia multiplied by the corresponding gear ratio, right N_d , okay, so that is going to be T subscript d which is the input minus Id alpha d times N_d , okay. So, this is what is coming out of the final drive. Next from the final drive we go to the wheels, so this is what is going as input to the wheels.

So, the tractive force F available at the so called tyre road interface of the wheels from the powertrain, okay is F which is going to be equal to Tw minus the inertia of the rotating components of the wheel assembly, this is the torque which is coming off divided by the radius of the wheel assembly, okay that will give me a force, alright so, along the longitudinal direction of motion, okay.

So, this is going to be the force at the wheels, okay that can be used for driving the vehicle and this coming from the powertrain, okay we are also going to look at what is required from the vehicle's perspective shortly right, okay. So, now what we are going to do is that like when shortly, we are going to learn about what is called as wheel slip and wheel slip ratio.

Because in road vehicles, if you look at the tyre; the pneumatic tyre, what happens is that it sticks and slips at the tyre road interface, so it is not undergoing a pure rolling motion, so there is something called as wheel slip you know which becomes very important, okay and that has a very strong impact on the tractive force limit available at the tyre road interface. We shall look at it more closer particularly, when we are looking at braking analysis and anti-lock brake systems, okay.

So, right now we will; for this analysis, we will not consider wheel slip ratio, okay and then like we will write down simple equations okay.

Neglect slip at the tyre-road interface: $\alpha_\omega = \frac{a}{r_\omega}$ → Longitudinal vehicle acceleration (a)

$$\alpha_d = N_d \alpha_\omega, \quad \alpha_t = N_t \alpha_d = \underbrace{N_t N_d}_{=N_{td}} \alpha_\omega$$

Using all the above equations:

$$F = \frac{T_\omega}{r_\omega} - \frac{I_\omega}{r_\omega}\left(\frac{a}{r_\omega}\right) = \frac{T_d N_d}{r_\omega} - \frac{I_d \alpha_d N_d}{r_\omega} - I_\omega \frac{a}{r_\omega^2}$$

where $\alpha_d = N_d \alpha_\omega = N_d \frac{a}{r_\omega}$

$$= N_d \alpha_\omega = N_d \frac{a}{r_\omega}$$

$$= \frac{T_t \overbrace{N_t N_d}^{N_{td}}}{r_\omega} - \frac{I_t \alpha_t N_t N_d}{r_\omega} - I_d N_d^2 \frac{a}{r_\omega^2} - I_\omega \frac{a}{r_\omega^2}$$

$$= N_{td} \frac{a}{r_\omega}$$

$$= \frac{T_e N_{td}}{r_\omega} - \frac{I_e \alpha_e N_{td}}{r_\omega} - I_t N_{td}^2 \frac{a}{r_\omega^2} - I_d N_d^2 \frac{a}{r_\omega^2} - I_\omega \frac{a}{r_\omega^2}$$

So, we will neglect slip at the tyre road interface to obtain the angular acceleration of the wheel will be the longitudinal acceleration of the vehicle divided by the wheel radius, okay so, by neglecting the wheel slip, okay. So, this is nothing but longitudinal vehicle acceleration okay, so that is alpha w. Now, we go backwards okay, in the reverse sequence as far as the components are concerned.

So, if alpha w is the angular acceleration of the wheels which is at the output of the final drive, what do you think is going to be the angular acceleration at the input to the final drive? We essentially use the definition of gear ratio right, gear ratio is the ratio of the input speed to the output speed. By using that relationship what, how can I write alpha d; alpha d which is essentially the angular acceleration, the final drive that is going to be equal to N_d times alpha w.

Because N_d is the gear ratio of the final drive right, so similarly what will be the angular acceleration alpha t which is at the input to the primary gearbox that will be the corresponding gear ratio N_t multiplied by the angular acceleration of the output shaft, okay which is alpha d. So, alpha t will be equal to N_t times alpha d, this we can rewrite as N_t times alpha d is N_d alpha w, okay.

So, typically the product of N_t and N_d is represented as N_{td} , okay, so N subscript td is nothing but the product of the 2 gear ratios; N_t and N_d okay, so this is what we will get. Of course, when there is; when the clutch and the flywheel rotate as one unit, alpha t and alpha e will become the same, right because it is rotating without any slip right, when the clutch is fully engaged and the entire flywheel clutch assembly is rotating as one unit, okay, so that is something we know.

Now, we put together all these equations okay, so that is what we are doing, so the equations for the torques and equations for angular accelerations we are going to simplify, right. So, let us do that simplification so, using all the above equations the idea is to get this equation for F, so F is going to be equal to nothing but T_w by r_W minus I_W by r_W times alpha w, what is alpha w?

It is nothing but "a" divided by r_W, right we just observe, so what am I doing; I am just taking this equation and rewriting it and just substituting for alpha w, right. Now, we go up and then substitute for Tw, from where do I get Tw; from this equation right, so Tw is going to be equal to $T_d N_d$, right divided by r_w minus I_d alpha d N_d divided by r_w and I_w a divided by r_w square remains as it is, right so what we have already got in the previous term.

So, now I hope you get the idea right, so we keep on substituting for each variable and eliminating them. Now, what is alpha d; alpha d, we can essentially I am writing here alpha d is nothing but N_d times alpha w and that is nothing but N_d times "a" divided by r_w right, so that is a substitution we will make. Then, what do we do; we go and substitute for T_d . From which equation can I substitute for T_d?

Here, right we already have an equation for T_d , so let us go ahead and substitute for T_d , so this is going to be T_d is T subscript t N_t N_d , right divided by r_w minus; what do we get, we get "I_t" alpha t N_t N_d by r_w, that is the first term in the previous equation, this becomes I_d N_d squared a divided by r_w squared, right. So, this term once I substitute alpha d to be N_d "a" by r_w , we will get this equation.

And I_w by I_w times a r_w square remains as it is right, so that is what we get. Now, what we do; we essentially simplify further, right. So, we essentially simplify for alpha t, what is alpha t; alpha t is N_{td} td alpha w, from where did I get this; from here, right and we already know that alpha w is going to be "a" divided by r_w and this N_t times N_d is going to be another N_{td} , correct, so that is something which we can immediately figure out, okay.

And this N_t times N_d also going to be equal to N_{td} right okay, so now what we do is that like we have to substitute for T subscript t, so what is T subscript t? If you recall T subscript t is going to be equal to; if we go back, T subscript t is going to be equal to T_e minus I_e alpha e alright. So, let us substitute that so, this is going to be equal to Te N_{td} divided by r_w minus I_e alpha e N_{td} divided by r_w minus I_t.

What we are going to do is that we are going to rewrite this as N_{td} square a divided by r_w square minus $I_d N_d$ square a divided by $r_w$square minus I_w divided by; I_w multiply by “a” by r_w square right, so that is it. So, now we just need to eliminate alpha e, so when the flywheel, the clutch rotate as one unit, we just discussed alpha e is going to become equal to alpha e and what did we just simplify for alpha e is going to be N_{td} times a divided by r_w, so that is something which we have already done, right.

Transmission Efficiency

$$\Rightarrow F = \frac{T_e N_{td} \eta_t}{r_\omega} - |(I_e + I_t) N_{td}^2 + I_d N_d^2 + I_\omega| \frac{a}{r_\omega^2}$$

Note that, if $a = 0$, $F = \frac{T_e N_{td} \eta_t}{r_\omega}$

So, now if we put all these things together, so simplifying these we are going to get F to be equal to T_e N_{td} divided by r_w minus; let me collect all the terms together, I_e plus I_t times N_{td} square plus I_d N_d square plus I_w multiplied by “a” divided by r_w square, okay, so this is what I will get as a final expression. So, this is an expression that relates the force that comes out of the powertrain to the input engine torque and the gear ratios, okay and the inertia of the rotating components and the acceleration right of the vehicle, okay so that is what.

Now, here we will put an factor eta t, okay so, as I discussed you know we lump for the all the frictional losses right, as a transmission efficiency, so at this point we introduce a parameter eta t which is nothing but the transmission efficiency. So, we introduce it here and this is what you want, okay, so as an expression for characterizing the powertrain; a simple expression, right.

Please note that if the vehicle is cruising at a constant speed what will happen to “a”; note that if a equals 0, there is a vehicle is cruising at a constant speed, we will get, just get F equals Te N_{td} eta t divided by r_w, okay so this is what we will get, okay so, please remember this, we will use it later on okay, so this is from the perspective of the powertrain.