Fundamentals of Automatic Systems
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Module No # 03
Lecture No # 14
Engine Performance

So another important parameter that is critical for internal combustion is the fuel air ratio or the air fuel ratio. So as it indicates you know like you we have to essentially determine what is the ratio of fuel and air in a mixture of them right so that is what is specified by fuel to air ratio. And before we go into defining some parameters we are going to define what is called as a stoichiometric fuel air mixture I am sure we would have encountered this words sometime before in basic chemistry right.

$$HC \longrightarrow H_2O + CO_2$$

So stoichiometric mixture is a chemically correct mixture in this case what is stoichiometric mixture? It is a mixture of fuel and air that has just enough air for complete combustion of all fuel in the mixture is what is called as the fuel air mixture okay. So what does this mean by complete combustion? All the hydrocarbon should be completely oxidized to H_2O and CO_2 so that is a perfect fuel air mixture okay chemically correct fuel air mixture or what is called as a stoichiometric mixture okay.

So this is a chemically correct fuel air mixture now we can have more fuel in the mixture than the chemically correct mixture okay that implies what is called as a rich mixture. So a rich mixture is one which has more fuel in the fuel air mixture than the chemically correct mixture. So if you have less fuel we have what is called as a lean mixture okay so these are some terms and based on these we are going to define some parameters for the equivalence ratio Phi which is the actual fuel to air ratio to the stoichiometric fuel to air ratio okay so that is an definition of equivalence ratio.

Some people use another parameter called excess air ratio it is just a inverse of this so it is the actual air fuel ratio divided by the stoichiometric air fuel ratio. Okay so that is another definition so you just swap the ratios okay first case it was fuel to air now it is air to fuel. So why would

More fuel $\Longrightarrow$ Rich Mixture

Less fuel $\Longrightarrow$ Lean Mixture

$$\text{Equivalence Ratio, } \emptyset = \frac{Actual\ Fuel-Air\ Ratio}{Stoichiometric\ Fuel-Air\ Ratio}$$

$$\text{Excess Air Ratio, } \Lambda = \frac{Actual\ Air-Fuel\ Ratio}{Stoichiometric\ Fuel-Air\ Ratio}$$

some people prefer this because typically you know like even if we consider a typical fuel where let us say the stoichiometric fuel to air mixture should contain one part of fuel and 15 parts of air.

Fuel to air ratio will be 1 by 15 the air to fuel ratio will be 15 so some people find it convenient to say okay like 15 is to 1 or 14.2 is 1 right and so on right. Rather than saying 1 by 15 is to 1 right 1 by 15 goes into some decimal places and so on which is what to say which people and people find more convenient to state the air fuel ratio but both are the same so it is just a matter of convenience.

So the a rich mixture implies the equivalence ratio is greater than 1 and the excess air ratio is less than 1 right a lean mixture is 1 where the equivalence ratio is less than 1 and the excess air ratio is greater than 1 okay so that is the definitions of equivalence ratio and excess air ratio.

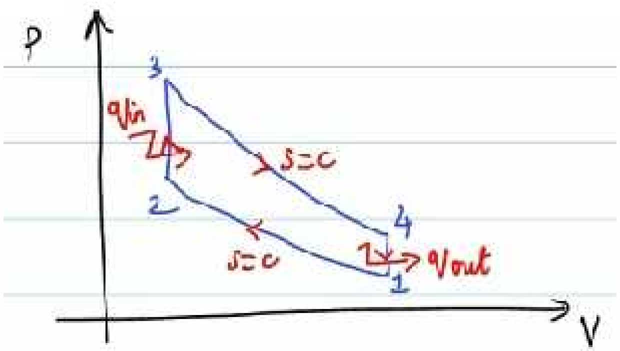
Rich Mixture $\Longrightarrow \emptyset > 1 \ \ \& \ \ \Lambda < 1$

Lean Mixture $\Longrightarrow \emptyset < 1 \ \ \& \ \ \Lambda > 1$

So one more parameter which we are going to encounter is the calorific value so we have already encountered calorific value when we defined the efficiency terms right so what is the calorific value. So the calorific value of a fuel is the thermal energy released per unit quantity of the fuel when it is burnt completely and the products of combustion are brought back to the initial temperature.

So that is what is the calorific value of the fuel so it is the indicative of how much what to say energy input heat chemical energy right the heat energy that you can get by burning the fuel right by combusting the fuel. So these are some parameters that are used for quantifying engine performance okay.

So the next thing I want to discuss in today's class is give a notion on the indicated mean effective pressure right. So let us derive an expression for the indicated mean effective pressure or IMEP for the air standard Otto cycle. So that like that we understand what are the parameters that affect or influence this mean effective pressure. So just to quickly recap the air standard Otto cycle the PV diagram looks something like this so this is state 1, 2, 3 and 4.



$$r = \frac{V_1}{V_2} = \frac{V_4}{V_3} \quad r = \frac{V_1}{V_2} = \frac{V_4}{V_3}$$

Let us evaluate the net work output from the Otto cycle.

So we always need to mark the flow of the processes is right and also some critical characteristics of each process okay. So we already know that the compression ratio r is going to be V_1 by V_2 and that is also going to be equal to V_4 by V_3 and pressure ratio which is given by RP is defined by P_3 by P_2 okay so that is something which we these are parameters that we already aware of. So now let us calculate or evaluate the net-work output from the Otto cycle.

So let us do that exercise so why are we evaluating the net-work output because that is the definition of indicated mean effective pressure right is nothing but the net-work output per unit cycle divided by the displacement volume.

So we can immediately see that the net-work output as two component the first component is from during the expansion process so in the expansion process the work which is delivered is nothing but integral from going from 3 to 4 P times dv right so that is the work output correct. Now how do I simplify this I use a process relationship because we know PV power gamma equals constant right that is an isentropic process relationship.

$$PV^{\gamma} = C \qquad P = CV^{-\gamma} \qquad C = PV^{\gamma}$$

$$3 \rightarrow 4: \int_3^4 PdV = \int_3^4 CV^{-\gamma}\, dV = C\frac{V^{-\gamma+1}}{-\gamma+1}\Big|_3^4 = \frac{PV}{-\gamma+1}\Big|_3^4 = \frac{P_4V_4 - P_3V_3}{-\gamma+1} = \frac{P_3V_3 - P_4V_4}{(\gamma-1)}$$

$$1 \rightarrow 2: \int_1^2 PdV = \frac{P_1V_1 - P_2V_2}{(\gamma-1)}$$

$$\Longrightarrow W_{net} = \frac{P_3V_3 - P_4V_4}{(\gamma-1)} + \frac{P_1V_1 - P_2V_2}{(\gamma-1)}$$

So this implies that P is some C_v power minus gamma right so I just substitute that here so if I substitute it here I am going to get this. So what will happen if I integrate I will get C_v power minus gamma plus 1 divided minus gamma plus 1 going from 3 to 4 right. Before we evaluate here once again I can substitute C equals PV power gamma right so what will I get? I will get PV divided by minus gamma plus 1 evaluated between 3 and 4 how did I get this?

Here I can substitute C equals PV power gamma right PV power gamma times V power minus gamma plus 1 will give you PV so that is what I did. So as a result we will get P_4V_4 minus P_3V_3 by gamma plus 1 this I rewrite it as P_3V_3 minus P_4V_4 by gamma minus 1 okay so that is what. So anyway this work is positive right so this is the positive quantity because it is work output term right during the expansion process and we can see that quantity is going to be positive.

So similarly if I consider the process from 1 to 2 and we evaluate this I would leave this as an exercise you will immediately see that if you follow a similar process we are going to get this expression okay. So that is the work interaction during the compression process obviously you see that this is going to have a negative sign because we are providing that is a sign convention it is used right work given to the system or work done on the system is negative so this is going to be negative.

So consequently the net-work output is going to be the sum of the 2 it is going to be P_3V_3 minus P_4V_4 divided gamma minus 1 plus P_1V_1 minus P_2V_2 by gamma minus 1 so this is the net-work output okay.

So now we simplify so we know that P_1V_1 power gamma equals P_2V_2 power gamma right so that is from the process relationship. So we immediately can rewrite P_2 by P_1 as V_1 by V_2 is the power gamma and V_1 by V_2 is the compression ratio so I get r power r. Similarly P_3V_3 power gamma is going to be equal to P_4V_4 power gamma so this will give me P_3 by P_4 to be equal to V_4 by V_3 power gamma and that is also equals to r power gamma correct.

So this essentially tells me P_2 by P_1 is equal to P_3 by P_4 why because both ratios are equal to r power gamma so obviously both ratios should be the same. So if I change the order I will get P_3 by P_2 to be equal to P_4 by P_1 and what is P_3 by P_2 equal to the pressure ratio r_p. So you see that P_4 by P_1 is also r subscript p right the pressure ratio okay. And another what to say equation which we are going to use here is a following if we take P_3 by P_1 we are going to need all these pressure ratios shortly I can rewrite this P_3 by P_2 times P_2 by P_1 okay what is P_3 by P_2 rp right.

What is P_2 by P_1? r power gamma right so we are going to utilize all these equations okay so now what we are going to do is that following. We are going to simplify this expression so let us take the expression for W_{net} let me take P_1V_1 by gamma minus 1 as common so what will I have? I will have P_3V_3 by P_1V_1 minus P_4V_4 by P_1V_1 plus 1 minus P_2V_2 by P_1V_1 immediately we can observe that V_4 and V_1 gets cancelled because they are the same right what is P_3 by P_1 let us take the first term P_3 by P_1 is going to be rp r power gamma we just derived it right.

So what we have is the following we have P_1V_1 whole to the power gamma minus 1 times rp r power gamma minus 1 minus rp plus1 minus r power gamma minus 1 okay so this is what we have. So this implies that the net-work output in one cycle is going to be equal to P_1V_1 to the power gamma minus1 sorry divided by gamma minus 1 multiplied by rp minus 1 times r power gamma minus 1 minus1 okay I can simplify that into product of these two factors right.

So this is the net work in one cycle so what is IMEP? It is this network in one cycle divided by the displacement volume. So now going up what can I say about the displacement volume V displacement is going to be equal to V_1 minus V_2 I can rewrite this as V_1 times 1 minus V_2 by V_1 and what is V_2 by V_1 by r. So I can get V displacement as I can write V displacement as V_1 by r times r minus 1 right so very simple just using the definition of compression ratio to just rewrite the displacement volume in terms of V_1 and obviously the reason will become very clear shortly right so that is my displacement volume so let us come down.

We can also here correct that is rp r power gamma and what is V_3 by V_1 it is 1 by r V_1 by V_3 is r V_3 by V_1 is going to be 1 by r so that is how the first expression will simplify P_4 by P_1 is rp we just got it here right P_4 by P_1 is rp right and going to substitute for that then I will have plus1. What is P_2 by P_1 it is r power gamma we get it from here and what is V_2 by V_1 once again 1 by r right it is a reciprocal of the compression ratio.

$$Now,\ P_1V_1^\gamma = P_2V_2^\gamma \Longrightarrow \boxed{\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = r^\gamma}$$

$$P_3V_3^\gamma = P_4V_4^\gamma \Longrightarrow \frac{P_3}{P_4} = \left(\frac{V_4}{V_3}\right)^\gamma = r^\gamma$$

$$\frac{P_2}{P_1} = \frac{P_3}{P_4} \Longrightarrow \frac{P_3}{P_2} = \frac{P_4}{P_1} \Longrightarrow \frac{P_4}{P_1} = r_p$$

$$\frac{P_3}{P_1} = \left(\frac{P_3}{P_2}\right) \times \left(\frac{P_2}{P_1}\right) = r_p r^\gamma$$

$$W_{net} = \frac{P_1V_1}{(\gamma-1)}\left[\frac{P_3V_3}{P_1V_1} - \frac{P_4V_4}{P_1V_1} + 1 - \frac{P_2V_2}{P_1V_1}\right] = \frac{P_1V_1}{(\gamma-1)}\left[\frac{r_p r^\gamma}{r} - r_p + 1 - \frac{r^\gamma}{r}\right]$$

So IMEP is going to be equal to the net work done in one cycle divided by displacement volume the net work done is this P_1V_1 power gamma minus 1 multiplied by r_p minus 1 times r power gamma minus 1 minus1 divided by the displacement volume that is going to be V_1 divided by r times r minus 1 right so we can immediately see that V_1 and V_1 cancel off.

$$= \frac{P_1V_1}{(\gamma-1)}\left[r_p r^{(\gamma-1)} - r_p + 1 - r^{(\gamma-1)}\right]$$

$$\Longrightarrow \boxed{W_{net} = \frac{P_1V_1}{(\gamma-1)}\left[(r_p-1)\left(r^{(\gamma-1)}-1\right)\right]} \rightarrow Net\ work\ in\ 1\ cycle$$

$$IMEP = \frac{W_{net}}{V_{disp}} = \frac{\frac{P_1V_1}{(\gamma-1)}\left[(r_p-1)\left(r^{(\gamma-1)}-1\right)\right]}{\frac{V_1}{r}(r-1)} \Longrightarrow \boxed{IMEP = \frac{P_1 r(r_p-1)\left(r^{(\gamma-1)}-1\right)}{(\gamma-1)(r-1)}}$$

So this implies that the indicated mean effective pressure of the Otto cycle becomes P_1 times r because the r comes from the denominator times r_p minus 1 times r power gamma minus 1 minus1 divided by gamma minus1 times r minus1.

So this is the expression for the IMEP for the Otto cycle okay as an exercise I want you to do the same derivation that is find an expression for IMEP for the air standard diesel cycle and the dual cycle. In terms of the initial pressure $P_1$and all other parameters are r_p ,r_c as appropriate okay and gamma okay. So you can immediate see that the indicated mean effective pressure is dependent on P_1 is the pressure at the start of the cycle right the initial state 1.

So going to back to our previous discussion we can immediate see that if I increase the value of P1 obviously the indicated mean effective pressure also would increase and the process of super charging would essentially increase the value of P_1. So now one can easily observe the direct benefit of increasing the inlet pressure right and this process of super charging is something which we will discuss in the next class okay so I will stop here for this class and we will continue with super charging in the upcoming class fine thank you.