Lecture-11
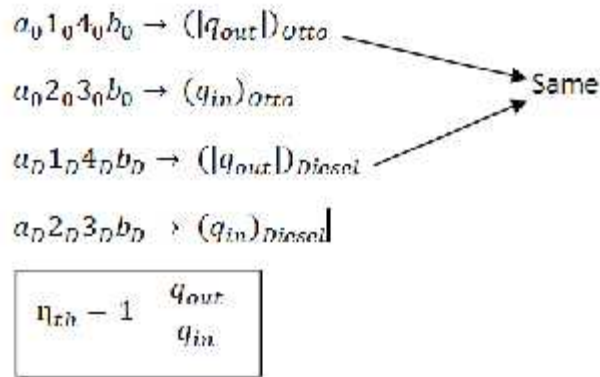**Dual Cycle and Engine performance**
Part 01

Ok, greetings, so welcome to today's class, so, just a quick recap of what we did yesterday. So in yesterday's class we considered the air standard otto cycle and air standard diesel cycle. We looked at the processes of those 2 cycles and we identified that the main difference between them was that of the heat addition process right. In the otto cycle heat addition was assumed to take place or almost constant volume.

In the diesel cycle, heat addition was assumed to take place at almost constant pressure, there is a main difference and we went and did some analysis using a conservation of energy and use the process relationships for isentropic process and ideal gas equation of state to get expressions for the thermal efficiency of the 2 cycles. So and we just started off at one case where we wanted to compare the otto cycle and the Diesel cycle.

So as we realized from the analysis, if you fix the same value of compression ratio r the air standard otto cycle has a higher thermal efficiency than that of the diesel cycle we just wanted to essentially investigated further. So we considered one case where we had the same compression ratio and heat addition, today I am just going to do one more example and then like we will move on to what is called as a dual cycle ok.

$a_o 1_o 4_o b_o \rightarrow (|q_{out}|)_{Otto}$

$a_o 2_o 3_o b_o \rightarrow (q_{in})_{Otto}$

$a_D 1_D 4_D b_D \rightarrow (|q_{out}|)_{Diesel}$

$a_D 2_D 3_D b_D \rightarrow (q_{in})_{Diesel}$

$$\boxed{\eta_{th} - 1 \quad \frac{q_{out}}{q_{in}}}$$
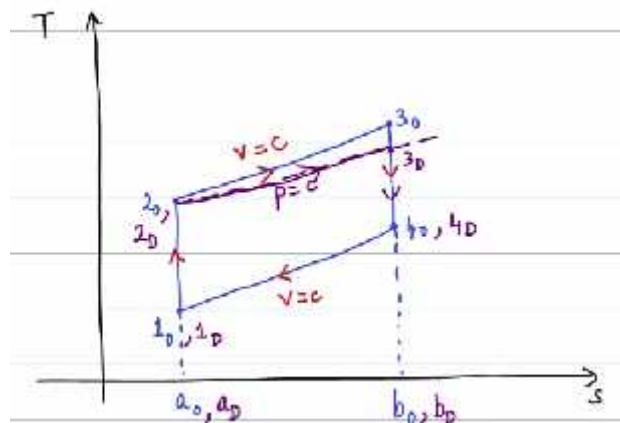
$Given \ (q_{out})_O - (q_{out})_D$

$(q_{in})_O \gg (q_{in})_D$

$$\boxed{(\eta_{th})_O \gg (\eta_{th})_D}$$

So let me do a second case where we still consider the same compression ratio for both the otto and the diesel cycle. But now let us consider the same amount of heat being rejected ok, so that is the scenario that we are going to consider now. So let me once again quickly draw the TS diagram for the otto cycle and then we will look at what happens in terms of the diesel cycles.

So let me call this 1o this is 2o and we have constant volume heat addition, go to 3o and then isentropic expansion come to 4o and then constant volume heat rejection ok. So we need to carefully mark all the processes when we draw these diagrams ok, so this is the TS diagram. Now as we have already discussed the area a o, 1o 4 o b o is equal to the heat rejected in the otto cycle and the area a o 2 o 3 o b o is a heat input given to the otto cycle right.

So that is something which we have already learned, now once again let us start from the same initial state ok. So 1 D is a same as 1o then what we do is that like, we have isentropic compression. So where should 2 D b same as 2 o why, because same compression ratio. So as we realized from the analysis T_2 as T_1 times r power gamma minus 1. So T_1 is the same for both cycles, r is the same that is the condition we are given.

So T_2 should be the same, so 2 D should coincide with 2o, so now let us draw the constant pressure line passing to 2 D and that is going to be something like this right. So that is the constant pressure line right passing through 2 D, so we are given that the heat rejected is now the same ok. So that means a D the area a D, of course a D is the same as a o now ok, a D 1 D 4 D b D should be equal to a o 1 o 4 o b o ok, so these 2 should be is the same.

So what does that tell us, because even in the diesel cycle, heat rejection is done at constant volume right ok. So a o 1 o and 1 D coincide a o and a D coincide, so what can we say about 4 D it should be the same as 4o right. Otherwise there is no way the area a D1- D 4 -D b- D will be the same as a o 1 o 4 o b o because in both the otto cycle and the diesel cycle heat rejection takes place at constant volume and there is only one constant volume line passing through 1o and 1 D.

So the point 4 D must coincide with 4o, so then the state b D will be the same as b o right. So we have identified 4 D now how do I identify 3 D, please remember 3 D, 4 D is isentropic expansion. So 3 D must lie on the vertical line passing through 4 D in the TS diagram. So and then 3 D should also lie on the constant pressure line passing through 2 D. So the intersection of the constant pressure line passing through 2 D and the constant entropy line passing through 4 D should be 3 D and what is that point it is here right.

So as a result this is my diesel cycle correct, so we have identifying this one, so please note the area a D 2 D 3 D b D is the heat input given in the diesel cycle. Now we know that the thermal efficiency is going to be equal to 1 minus Q_{out} by Q_{in} ok, we are given that Q_{out} in the Otto cycle and q out in the Diesel cycle or pretty much the same that is like I reject the same amount of heat energy ok in both the cycles, now what can you say about Q_{in} .

So please note q in the otto cycle is a o 2 o 3 o b o right, Q_{in} in the Diesel cycle as a D 2 D 3 D b D which is bigger otto cycle right. So then what can you say about the thermal efficiencies, please note that the expression for thermal efficiency is this. So the second term that appears a thermal efficiency Q_{out} which is the numerator is the same for both cycles, Q_{in} which as a denominator is larger for otto so the ratio is smaller for Otto cycle right.

So we are subtracting a smaller number from 1 in the Otto cycle, so which efficiency will be greater otto right. So the Otto cycle will have more thermal efficiency than the Diesel cycle as long as the compression ratio is the same right, so that is the main takeaway ok. But as we will discuss when we look at the actual combustion process, the phenomena of knocking typically restricts the maximum compression ratio in petrol engines or gasoline engines right.

So we are not going to have the same compression ratio in gasoline or spark ignition engines when we compare it, compare them in diesel engines right. So as a result, we will see that the compression ratios would not be the same, however if the compression ratios were the same, you see that the otto cycle is more efficient than the diesel cycle ok. We will discuss this point as we go now right as we go to actual engine performance and do the analysis, we will discuss it further ok.
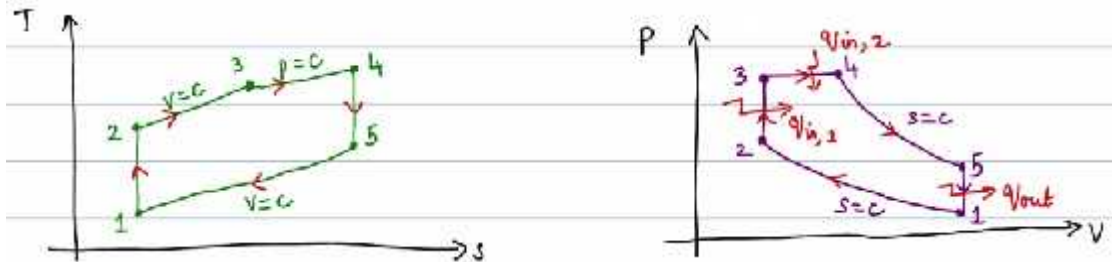
$$\eta_{th,Diesel} = 1- = 1 - \frac{|q_{out}|}{(q_{in,1}+q_{in,2})} = 1 - \frac{(u_5 - u_1)}{(u_3 - u_2) + (h_4 - h_3)}$$

$$= 1 - \frac{C_v(T_5-T_1)}{C_v(T_3-T_2)+C_p(T_4-T_3)}$$

$$Pressure\ Ratio, r_p := \frac{P_3}{P_2}$$

$$Cutoff\ Ratio, r_c := \frac{V_4}{V_3}$$

So the next air standard cycle that we will discuss is what is called as a dual cycle. So in the last class when we motivated the otto cycle and the diesel cycle you know the constant volume heat addition process and constant pressure heat addition process are typically motivated by looking at the actual engine curve right. In reality you know like if you look at the actual engine data,

you know we are not going to see the combustion process happening at almost constant volume or almost constant pressure ok, it is going to be a mix of both.

So the question is if the Otto cycle and the Diesel cycle do not approximate the actual cycle well enough, what does an alternative to do first analysis. So that answer is the dual cycle, so in the dual cycle, what we do is that, we consider that the heat addition process takes place in 2 phases, the first one is at constant volume, the second one is at constant pressure, of course we are making all these approximations.

So that we can derive analytical expressions which are simple enough to what to say do a pen and paper analysis. So that is the motivation, so in a dual cycle, we consider that the heat addition process takes place in 2 phases. So if I were to draw the TS diagram for the dual cycle, it will looks something like this. So I will start with state 1 what will happen is that like I will go to state 2 the compression processes going to be isentropic.

Then there will be 2 heat addition processes one at constant volume another at constant pressure. So first I will have a constant volume heat addition process then I have a constant pressure heat addition process then we will have isentropic expansion then the heat rejection process will is reasonable enough to be assumed at constant volume once again ok. So this is the dual cycle, so you can see that the heat addition process now consists of 2 sub processes ok constant volume process and a constant pressure process.

So if I were to draw the PV diagram for the same process, so and then I draw it here. So 1 to 2 will be isentropic compression, 2 to 3 will be constant volume heat addition, 3 to 4 will be constant pressure heat addition, 4 to 1 is isentropic expansion sorry 4 to 5, and 5 to 1 will be will be constant volume heat rejection. So the processes will take place like this ok, so consequently there are 2 heat addition process let us call Q_{in} 1 and Q_{in} 2 and this is the heat rejection process Q_{in} right, so that is the PV diagram of the dual cycle.

So I hope it is clear why the term dual came into be right, so the heat addition process has two stages one at constant volume then at constant pressure. So the idea is the whether the dual cycle

can better approximate the actual engine cycle ok. So in this analysis we are using the PV diagram just a quick assign, you know like in engine analysis you will also encounter the P - theta diagram ok P - theta diagram theta being the crank angle or the crank shaft angle ok.

So as we know in a 4 stroke engine, 1 operating cycle is completed in 2 revolutions of the crankshaft. So this is 1 cycle, so how many degrees will constitute 1 cycle and a P - theta diagram, suppose if we were to plot a P-theta diagram I start from 0 degrees right 1 cycle will go to 720 ok. So it will be spread out, so you would not see a loop like this ok because then you need to mark BDC, TDC repeatedly ok.

Here we mark at 1 volume and then we understand how what is happening, so there it will be spread out ok. So P minus theta diagram is also used by use for some analysis ok like in engine and you can obviously see that there is a 1 to 1 correlation between V and theta ok once you fix an origin, starting point, you can always figure out, you can always correlate V and theta right. So these diagrams are what are called as indicator diagrams ok, the term indicator diagrams is use to represent these diagrams ok fine.

So that is a quick aside right, so now let us come back to our analysis right of the dual cycle. So once again the thermal efficiency of the dual cycle is as we know from first of thermodynamics, what we derive is going to be 1 minus Q_{out} by Q_{in}. However now Q_{in} will consider will consists of 2 parts Q_{in} 1 plus Q_{in} 2 ok, so those will be the 2 parts. Now from yesterday's from the last classes analysis you know like we know how to write the expressions for Q_{out} right.

So let me put the magnitude to be consistent, so we are going to be look at the magnitude of Q_{out} ok. So we already know how to do this analysis right, so let me directly write the expressions. So what do you think will be q out now, it will be u_5 minus u_1 correct, so what will be Q_{in} 1, it will be u_3 minus u_2 because the constant volume heat addition process is from 2 to 3. Then what about 3 to 4, it will be now h_4 minus h_3 right because it is a constant pressure heat addition process ok.

So considering constant specific heats, we are going to get C_v times T_5 minus T_1 divided by C_v times T_3 minus T_2 plus C_p times T_4 minus T_3 ok, so that is the simplified expression ok.

$$1 \rightarrow 2 : T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)} \Longrightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = T_1 r^{(\gamma-1)}$$

$$2 \rightarrow 3 : \frac{T_3}{T_2} = \frac{P_3}{P_2} \Longrightarrow T_3 = T_2 \left(\frac{P_3}{P_2}\right) = T_2 r_p = T_1 r^{(\gamma-1)} r_p \qquad\qquad \frac{V_4}{V_5} = \frac{V_4}{V_3} \times \frac{V_3}{V_5} = \frac{r_c}{r}$$

$$3 \rightarrow 4 : \frac{T_4}{T_3} = \frac{V_4}{V_3} \Longrightarrow T_4 = T_3 \left(\frac{V_4}{V_3}\right) = T_3 r_c = T_1 r^{(\gamma-1)} r_p r_c$$

$$4 \rightarrow 5 : T_4 V_4^{(\gamma-1)} = T_5 V_5^{(\gamma-1)} \Longrightarrow T_5 = T_4 \left(\frac{V_4}{V_5}\right)^{(\gamma-1)} = T_4 \left(\frac{r_c}{r}\right)^{(\gamma-1)} = T_1 r_p r_c^\gamma$$

$$\Longrightarrow (\eta_{th})_{Dual} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)} = 1 - \frac{(T_1 r_p r_c^\gamma - T_1)}{[T_1 r^{(\gamma-1)} r_p - T_1 r^{(\gamma-1)}] + \gamma[T_1 r^{(\gamma-1)} r_p r_c - T_1 r^{(\gamma-1)} r_p]}$$

So here we are considering constant specific heats alright, so now the question that arises is the following right. So how do we then simplify right we have all temperatures, by now we are familiar the process right we have to use the process relationships, let us go from one process at a time and then like we eliminate the temperatures right. So but before doing that, you know like quick definitions that 2 parameters you know like the first one is what is called as a pressure ratio r P which is defined as P_3 by P_2.

So in the constant volume heat addition process the volume is constant but the pressure is increasing right. So the pressure ratio r subscript P tells me what is the ratio of P_3 and P_2 ok that we encountered in the constant volume heat addition process and we have already encountered the cut off ratio alright, cut off ratio r subscript C is going to be now V_4 by V_3. Because that is the 3 to 4 is the constant pressure heat addition process right.

So that is the cutoff ratio, now let us look at each process at a time. So now if you look at 1 to 2, what is 1 to 2, it is isentropic compression, so what is the process relationship will have T_1 V_1 power gamma minus 1 will be equal to T_2 V_2 power gamma minus 1. So that essentially results in T_2 being T_1 V_1 by V_2 to the power gamma minus 1 right and what is V_1 by V_2, r, so this is a standard relationship by now we are familiar with this right, so T_2 is going to be T_1 times r power gamma minus 1.

So then we consider the constant volume heat addition process 2 to 3, so we know that since the volume is constant the ratio of temperature is going to be equal to the ratio of pressures, how do we get that from the ideal gas equation of state right. So this is the process relationship for this constant volume heat addition process. So this will give me T_3 equals T_2 times P_3 by P_2 what is P_3 by P_2 r_P and then I substitute for T_2 I am going to get T_1 r power gamma minus 1 times r_P right.

So that is the second process then let us look at the constant pressure heat addition process, so what is going to be the process relationship for constant pressure heat addition process T_4 by T_3 is going to be equal to V_4 by V_3. So this implies T_4 equals T_3 V_4 by V_3 and V_4 by V_3 is r_C, then I substitute for T_3 we will get T_1 r power gamma minus 1 r_P r_C right, that is T_4. Then we consider process 4 to 5, which is isentropic expansion that is going to be the T_4 V_4 power gamma minus 1 that is going to be equal to T_5 V_5 power gamma minus 1.

So this will give me T_5 equals T_4 V_4 by V_5 gamma minus 1 and what is V_4 by V_5 , I can rewrite it as V_4 by V_3 times V_3 by V_5 , what is V_4 by V_3 is the r_C, what is V_3 by V_5 it is going to be 1 by r V_5 by V_3 is r, V_3 by V_5 is going to be the reciprocal of r right, so I will have r_C by r. So this will be r_C by r to the power gamma minus 1, if I substitute for T_4 what I am going to get r power gamma minus 1 will cancel off, I will get r_P r_C to the power gamma that is what I will get for T_5 right.

Now we are all set, so we go back and then look at the thermal efficiency of the dual cycle. So that is going to be 1 minus T_5 minus T_1 divided by T_3 minus T_2 plus gamma times T_4 minus T_3, so what have I done I have just divided by C_v right and C_p by C_v is going to be the ratio of specific heats correct. So now we go and substitute all the temperatures, so what will happen T_5 is going to be equal to T_1 r_P r_C power gamma minus T_1.

And in the denominator, T_3 is going to be T_1 1 r power gamma minus 1 r_P, T_2 is T_1 r power gamma minus 1 there is a first term then plus gamma times T_4 is T_1 1 r power gamma minus 1 r P r C minus T_3 right T_3 is what, T_1 r power gamma minus 1 r_P am I correct. So I am substituting the appropriate values of the temperatures that we have already derived.

$$(\eta_{th})_{Dual} = 1 - \frac{1}{r^{(\gamma-1)}} \left| \frac{(r_p r_c^\gamma - 1)}{(r_p - 1) + \gamma r_p (r_c - 1)} \right|$$

If $r_c = 1$, $(\eta_{th})_{Dual} = 1 - \frac{1}{r^{(\gamma-1)}} = (\eta_{th})_{Otto}$

If $r_p = 1$, $(\eta_{th})_{Dual} = 1 - \frac{1}{r^{(\gamma-1)}} \left[\frac{r_c^\gamma - 1}{\gamma(r_c - 1)} \right] = (\eta_{th})_{Diesel}$

For the same value of r, $(\eta_{th})_{Otto} > (\eta_{th})_{Dual} > (\eta_{th})_{Diesel}$

So if we simplify we can immediately see that this T_1 can be cancelled in the numerator and denominator right in the second term. So what we are going to have is the following, so you can pull out r power gamma minus 1 common from the denominator then we are going to have the numerator r P r C power gamma minus 1 divided by r P minus 1 plus gamma r P r C minus 1 ok. So this is what we have, so this is the expression for the thermal efficiency of the air standard dual cycle ok.

You can immediately observe that it depends on the compression ratio r the pressure ratio r P the cutoff ratio r C and gamma right. So those are the 4 parameters that affect the thermal efficiency of the dual cycle ok. Now we can immediately observe from this expression that if r C equals 1, what happens to the thermal efficiency expression. If r C is 1 what happens the above expression minus reduces to 1 minus 1 by r power gamma minus 1 which is nothing but the otto right.

If r C is 1, you see that the second term vanishes, this term vanishes and the denominator becomes r P minus 1 and r P minus1, r P minus1 cancel, we are going to get 1 minus 1 by r power gamma minus 1 am I correct. So that is the thermal efficiency of the air standard otto cycle, so if r C is 1 that means that you know like, what does it mean, there is no constant pressure heat addition right, the states 3 and 4 coincide alright, so that is a meaning.

So if r P is 1, what do you expect, of course now there is no constant volume heat addition process. So if you substitute r P equals 1, this is going to become 1 minus 1 divided by r power gamma minus 1 multiplied by r C power gamma minus 1 divided by gamma times r C minus 1. And what is this expression, if we go to the analysis of diesel cycle that we did, that is the thermal efficiency of the air standard diesel cycle correct.

So you can see that the Dual cycle boils down to the diesel cycle ok and in general for the same value of r ok compression ratio. We already learned that the thermal efficiency of the otto cycle is going to be greater than the thermal efficiency of the diesel cycle and the thermal efficiency of the dual cycle is going to be somewhere in between ok, so this can be shown right. So essentially the value of thermal efficiency of the otto cycle with the highest followed by the diesel cycle followed by the diesel cycle for the same value of compression ratio ok.

So these are 3 let us say we call 3 observations ok, so let me call them as a, b and c ok, so some points to note ok. So that completes a simple analysis of what are called air standard cycles, we learned the otto cycle, the diesel cycle and the dual cycle. And the main advantage of these cycles is that like, despite the assumptions that we make, it can be used to get a first cut idea about the particular engines performance right using some approximations you know like, so that is the main idea.