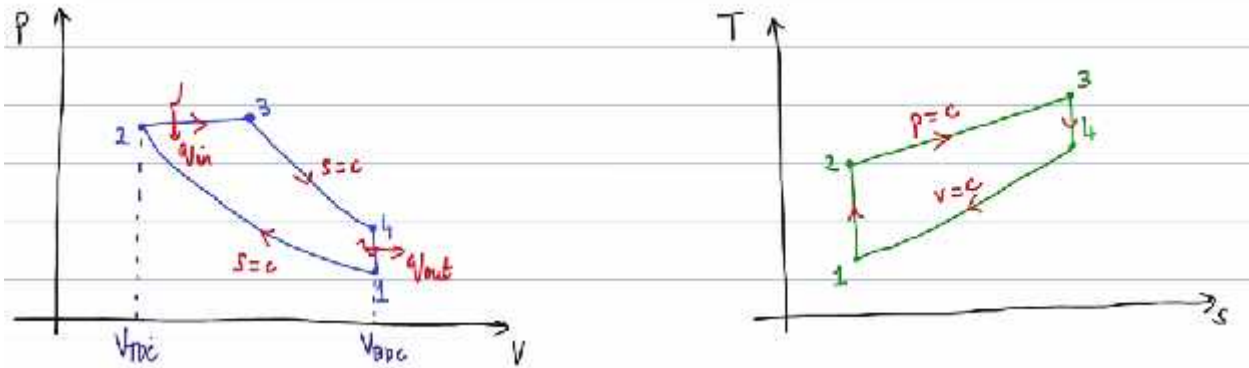**Fundamentals of Automotive Systems**
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Lecture-10
Otto Cycle and Diesel Cycle
Part 02

So now let us move on to another air standard cycle and as the name indicates, this typically utilize to characterize the behavior of compression ignition engines, right CI engines or diesel engines. And that is what is called as the Diesel cycle and as the name indicates, this was proposed by Rudolf diesel, right who was also a German automobile engineer, right. So what is the main difference between the air standard Otto cycle and the air standard Diesel cycle.

The air standard Diesel cycle once again it has 4 processes, there is only one difference ok. The heat addition process takes place at constant pressure ok, that is the important difference between the Otto cycle and the Diesel cycle. So, if I were to draw the PV diagram and the TS diagram for the diesel cycle the diagrams looks something like this. So, if I draw first draw the PV diagram 1 to 2 is going to be isentropic compression, the heat addition process it is assume to take place a constant pressure.

So instead of a vertical line in the PV diagram for the heat addition process as in the Otto cycle, we have a horizontal line right to characterize the heat addition process, ok. And 3 to 4 essentially is the expansion process isentropic and 4 to 1 is the constant volume heat rejection process. The heat rejection process is assume to take place at constant pressure, now what is the

motivation for this approximation you know we will see that typically when we look at data from a diesel engine due to the mechanism by which combustion takes place.

The PV diagram looks a little bit closer to the shape because in a diesel engine, only air is taken during the intake stroke, air is compressed and the fuel diesel is sprayed through fuel injectors over a period of time during which it combust and during that period of time the piston is already moving. So, the volume increases, but due to heat release, the pressure more or less remains the same alright.

So it is a reasonable approximation to make ok considering the way in which a diesel engine works and the manner in which the combustion process takes place in a diesel engine ok, so that is the main difference. So, if we label all the processes, 1 to 2 is isentropic compression 2 to 3 is constant pressure heat addition, 3 to 4 is once again isentropic expansion and 4 to 1 is constant volume heat rejection, ok, so these are the 4 processes.

If I were to draw the Ts diagram, 1 to 2 is once again isentropic compression 2 to 3 is constant pressure heat addition, 3 to 4 is isentropic expansion and 4 to 1 is constant volume heat rejection ok. This is pressure equals constant and is this volume equals constant ok, so it is so turns out then the constant pressure curves in the T_s plane have a lower slope than the constant volume lines ok in the T s plane ok.

So that is why you see that they are no longer parallel right in Otto cycle the 2 here processes the heat addition and the heat rejection process curves where almost parallel right to each other. But now they are not ok the constant pressure line or curve has lower slope than the constant volume curve ok in the T_s plane, so that is why we see a difference. Now, before we go to the analysis, there is an important parameter ok which we will use in addition to the compression ratio.

$$\text{Cut-off ratio,} \quad \boxed{r_c := \frac{V_3}{V_2}} \quad 2 \longrightarrow 3 : \int_2^3 \partial q - \int_2^3 \partial w = \int_2^3 de \quad \Rightarrow \quad q_{in} - (P_3 V_3 - P_2 V_2)$$

$$\Rightarrow \quad = u_3 - u_2$$

$$\Rightarrow \quad q_{in} = (u_3 + P_3 V_3) - (u_2 + P_2 V_2)$$

$$= h_3 - h_2$$

$$\eta_{th,Diesel} = \frac{w_{net}}{q_{in}} = \frac{q_{in} - |q_{out}|}{q_{in}} = 1 - \frac{|q_{out}|}{q_{in}} = 1 - \frac{(u_4 - u_1)}{(h_3 - h_2)}$$

$$= 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)}$$

Constant specific heat

$\rightarrow$ 2: $\quad T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)} \qquad \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{((\gamma-1))} \qquad \Rightarrow T_2 = T_1 r^{((\gamma-1))}$

$\rightarrow$ 3: $\quad \frac{T_3}{T_2} = \frac{V_3}{V_2} \qquad \Rightarrow T_3 = T_2 \left(\frac{V_3}{V_2}\right) = T_2 r_c = T_1 r^{(\gamma-1)} r_c$

So that is what is called as a cut off ratio, so the cutoff ratio r c in this case is defined as V_3 by V_2 , please note this is still volume and bottom dead centre. This point is still volume and top dead centre but now V_3 is not equal to V_2 right. So, the cutoff ratio r subscript C is defined as V_3 divided by V_2 ok obviously it is value is greater than 1 right. And why is it called cut off ratio you know like typically you know we approximate it as the period which the diesel fuel is sprayed into the combustion chamber right.

Around this point 3 that is where the fuel supply is cut off ok or stopped, so that is why you know like most probably it is called as the cut off ratio ok. So it is indicative of the phase or period over which diesel is sprayed into the combustion chamber ok, there is the parameter Rc. So, let us now do a similar derivation for the thermal efficiency of the air standard diesel cycle, having done the derivation for the otto cycle you know this becomes pretty straightforward.

Once again the thermal efficiency expression is going to be the network done by the net heat input right. The network done is once again Q_{in} minus the magnitude of the heat rejected divided by Q_{in} . So, this is going to be 1 minus magnitude of Q_{out} by Q_{in} and Q_{out} still remains the same, it

is going to the magnitude of q out is once again u_4 minus u_1. However, what happens to Q_{in} , for q in if you can consider the process 2 to 3 and apply the first law of thermodynamics.

And we integrate the differential form or the first law what happens this is Q_{in} minus you see that you know like delta w is no longer 0 right, because there is some expansion which is happening. So, this expansion work is going to be equal to P_3 sorry P_3V_3 minus P_2V_2, once I integrate PDV ok. But anyway the pressures constant, so I can have P_3V_3 minus P_2V_2 and on the right hand side I am going to get of course we neglect all the other the potential energy and the kinetic energy terms we are going to get u_3 and u_3 minus u_2 .

So as a result q in is going to be equal to u_3 plus P_3V_3 minus u_2 plus P_2V_2 and what is u plus P V called, the enthalpy right, so we are going to get h_3 minus h_2 as q in right. So going back in our expression for q in we are going to get h_3 minus h_2. So, now, once again we assume constant specific heats, so once we take constant specific heat what we get, we get 1 minus C_V times T_4 minus T_1 divided by C_p times T_1 minus T_2.

So, all of us know that C_p by C_V is going to be equal to gamma, so we are going to get T_4 minus T_1 gamma T_3 minus T_2 ok. So, once again we can observe that we have obtained the expression of thermal efficiency in terms of the 4 nodal temperatures. So, we follow a similar process to eliminate them, now we start looking at the individual process relationships. So, let us consider the process 1 to 2, so what is that, that is isentropic compression.

So, T_1V_1 power gamma minus1 is going to be equal to T_2V_2 power gamma minus 1. So as a result T_2 is going to be equal to the T_1 times V_1 by V_2 gamma minus 1. And that is going to be give me T_2 is T_1 times r power gamma minus 1 right same as before that does not change right, so that is for 1 to 2. If I consider the isentropic expansion process before going to isentropic expansion process, let me consider the constant pressure heat addition process.

So the process 2 to 3 is constant pressure heat addition, pressure remains constant. Then using the ideal gas equation of state, what can I say, T_3 by T_2 is going to be equal to V_3 by V_2 right. So then I will have T_3 equals T_2 times V_3 by V_2 and what is V_3 by V_2 rc, so we will get T_2 times rc.

And substituting for T_2 from here, what will I get, I will get T_1 r power gamma minus 1 times c right, that is what I will get for T_3.

$$1 \rightarrow 2: \quad T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)} \qquad \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{((\gamma-1))} \qquad \Rightarrow \quad T_2 = T_1 r^{((\gamma-1))}$$

$$2 \rightarrow 3: \quad \frac{T_3}{T_2} = \frac{V_3}{V_2} \qquad \Rightarrow T_3 = T_2 \left(\frac{V_3}{V_2}\right) = T_2 r_c = T_1 r^{(\gamma-1)} r_c$$

$$3 \rightarrow 4: \quad T_3 V_3^{(\gamma-1)} = T_4 V_4^{\gamma-1} \quad \Rightarrow \quad T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{(\gamma-1)} = T_3 \left(\frac{r_c}{r}\right)^{(\gamma-1)} = T_1 r^{(\gamma-1)} r_c \left(\frac{r_c}{r}\right)^{(\gamma-1)}$$

$$\Rightarrow T_4 = T_1 r_c^{\gamma}$$

$$\Rightarrow \eta_{th,Diesel} = 1 - \frac{(T_1 r_c^{\gamma} - T_1)}{\gamma(T_1 r^{(\gamma-1)} r_c - T_1 r^{(\gamma-1)})} \qquad \Rightarrow \qquad \boxed{\eta_{th,Diesel} = 1 - \frac{1}{r^{(\gamma-1)}} \left[\frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)}\right]}$$

Then we go a forward right, so we go and consider the process 3 to 4 which is an isentropic process, it is an expansion process. So once again, the process relationship will give me $T_3 V_3$ power gamma minus 1 will be equal to $T_4 V_4$ power gamma minus 1. So this implies T_4 is going to be equal to $T_3 V_3$ by V_4 gamma 1, what is V_3 by V_4. So, we immediately see that observe that V_3 by V_4 is not r right because V_3 is not at this is not the volume at the top dead centre.

So, what can I do, I can rewrite this as V_3 by V_2 multiplied by V_2 by V_4 correct, I am dividing and multiplying by V_2, what is V_3 by V_2, it is r subscript c the cutoff ratio, what is V_2 by V_4, it is a reciprocal of r right, it is 1 by r. So I am going to get rc by r, I hope everyone agrees right. So, V_2 by V_4 will be the reciprocal of the compression ratio, so T_4 I will get to this T_3 times rc by r to the power gamma minus 1.

And once I substitute for T_3, I substitute here for T_3, T_3 we have already derived that it is T_1 r power gamma minus 1 times rc. And then now I have rc by r to the power gamma minus 1, what will I get T_4 will be equal to T_1 rc power gamma correct, so this what I get. So, now we are done, right we substitute everything back into the expression for the Diesel cycle.

So, substituting we get 1 minus T_4 minus T_1 I substitute for T_4, I will get T_1 rc power gamma minus T_1 that is what I will get in the numerator divided by gamma T_3 is going to be equal to T_1

r power gamma - 1 rc minus T_4 sorry T_2 right, T_2 is going to be T_1 r power gamma minus 1 right, so that is what this one. So if I cancel out T_1 and then if I rearrange the terms I am going to get the thermal efficiency of the air standard Diesel cycle as follows 1 minus 1 by r power gamma minus 1 multiplied by rc power gamma minus 1 divided by gamma times rc minus 1 ok. This is how this is the expression for the thermal efficiency of the air standard Diesel cycle.

$$3 \rightarrow 4: \quad T_3 V_3^{(\gamma-1)} = T_4 V_4^{\gamma-1} \quad \Rightarrow \quad T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{(\gamma-1)} = T_3 \left(\frac{r_c}{r}\right)^{(\gamma-1)} = T_1 r^{(\gamma-1)} r_c \left(\frac{r_c}{r}\right)^{(\gamma-1)}$$

$$\Rightarrow T_4 = T_1 r_c^{\gamma}$$

$$\Rightarrow \eta_{th,Diesel} = 1 - \frac{(T_1 r_c^{\gamma} - T_1)}{\gamma(T_1 r^{(\gamma-1)} r_c - T_1 r^{(\gamma-1)})} \quad \Rightarrow \quad \boxed{\eta_{th,Diesel} = 1 - \frac{1}{r^{(\gamma-1)}} \left[\frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)}\right]}$$

Since $r_c > 1$, for any fixed value of r, $\eta_{th,Otto} > \eta_{th,Diesel}$ $\qquad \eta_{th,Otto} = 1 - \frac{1}{r^{(\gamma-1)}}$

Now we note that since there is a reason why I have written the expression for the thermal efficiency in this manner, if you compare it with the thermal efficiency of the air standard Otto cycle right, what was that, it was equal to 1 minus 1 by r power gamma minus 1.
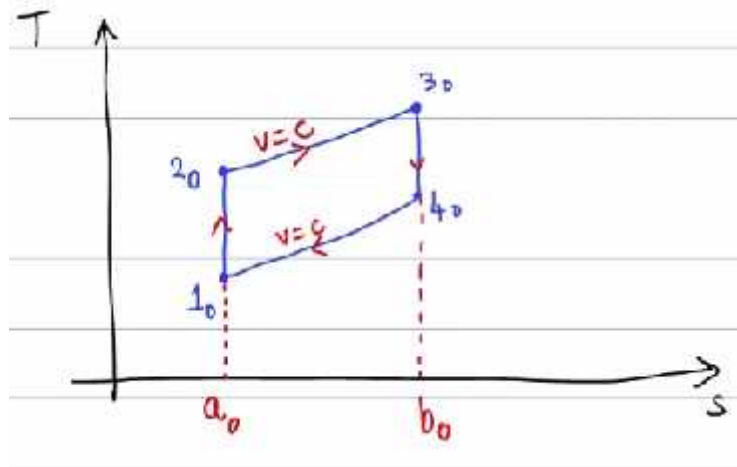
$a_0 1_0 4_0 b_0 \rightarrow (|q_{out}|)_{Otto}$

$a_0 2_0 3_0 b_0 \rightarrow (q_{in})_{Otto}$

$a_D 2_D 3_D b_D \rightarrow (q_{in})_{Diesel}$ } Equal

$$a_D 1_D 4_D b_D \rightarrow (|q_{out}|)_{Diesel}$$

$$\eta_{th} = 1 - \left(\frac{|q_{out}|}{q_{in}}\right) \quad Given \ (q_{in})_{Otto} = (q_{in})_{Diesel}$$

$$(|q_{out}|)_{Otto} < (|q_{out}|)_{Diesel} \quad \Rightarrow \quad \boxed{(\eta_{th})_{Otto} > (\eta_{th})_{Diesel}}$$

So I have purposefully written in this way, so that we can compare right as to what is happening, so let me write it, it is going to be 1 minus 1 divided by r power gamma minus 1 right. So you see that in the Diesel cycle the factor 1 by r power gamma minus 1 is multiplied by the term within the square bracket. So since rc is greater than 1 for any fixed value of r we can immediately show that thermal efficiency of the Otto cycle will be greater than the thermal efficiency of the Diesel cycle ok.

So in other words if we have the same compression ratio then the Otto cycle will have a higher thermal efficiency than the Diesel cycle ok. So that is a very important result let us test it out ok, so let us do a couple of simple examples and test out this result ok. So let us compare Otto and Diesel cycle, so let me start off with one case. So the first case we shall do is for the same compression ratio and heat addition.

So let us say, you know like we add the same amount of energy in both cycles and the compression ratio is the same. So let us figure out what happens to the efficiencies. So in this exercise we are going to use a TS diagram ok. So we will see that it is easier for us to understand and interpret the results by using a TS plot ok. So we will also draw the equivalent PV diagram. ok, so let us start with the TS diagram.

So let me first draw a base Otto cycle, so in the TS diagram, so as we know 1 to 2 is isentropic compression, I am putting the subscript o to indicate Otto cycle right. So then 2 to 3 is constant volume heat addition, then 3 to 4 isentropic expansion and 4 to 1 is constant volume heat rejection ok. So, these are the 4 processes in the air standard Otto cycle ok. Now let us say if I extend these lines to the absolute 0 temperature ok.

We know from thermodynamics that the area a o 1 o 4 o b o will be equal to the heat that is rejected in the Otto cycle right. See because the area under a curve process curve in the TS diagram is going to be the heat interaction right during that process ok. Similarly the area a o 2 o 3 o b o is going to be the heat input given to the Otto cycle ok, so that is the Otto cycle right. So, we are now going to reason out and draw the Diesel cycle on the same graph given the conditions that we are having.

So for fairness of comparison, we start with the same initial state, so 1 o 1d are the same ok, the initial state for the diesel cycle is the same as the initial state for the Otto cycle, right we started 1 D. Now, we have to locate 2 D, 3 D and 4 D since 1 D easier a D also will be here ok, it will coincide with a O right. Now where do you think 2 D will be, so please look at the conditions given, same compression ratio.

So, if you look at both the Otto cycle and the diesel cycle T_2 as T_1 r power gamma minus 1, T_1 is the same right for both otto cycle and the Diesel cycle in this analysis, r is the same. So, what can you say about T_2 should be the same right, so 2O and 2D coincide correct. Now, if I have to write down the same way a o, 2D-3D- bD is the heat added in the diesel cycle of course we need to find 3 D we are not done that.

But we know 2 to 3 in the diesel cycle is also constant pressure heat addition process and sorry a D1- D4 –Db- D is the heat rejected in the diesel cycle. In this example, in this problem we are given the fact that the heat added in both cycle is the same alright, so these areas should be the same Ok. So now using this fact how do we figure out the point 3 D, so please remember, in the diesel cycle the heat edition takes place at constant pressure.

So, let me draw the constant pressure line passing through 2D ok of course I can keep on extending it, but I am just going one way alright. So, why it is like this as we just discussed, you know the constant pressure curve has a lower slope then a constant volume. So, the constant pressure curve passing through the point 2D obviously has a lower slope than the constant volume line passing through 2D which is a same as 2O right.

Now, where will 3 D lie, will it, do you think will add lie to the right of 3 o 4 o or to the left of 3 o 4 o, it should lie somewhere to the right of the 3 o 4 o right. Only then, if you project this to b D, the area would be the same right under the heat addition curve. If the point 3 D were to lie to the left of the vertical line segment 3 o 4 o, we can immediately see that the area under the curve 2 D 3 D, which will be the heat added in the diesel cycle would be lower than the heat added in the Otto cycle which would violate the condition that has been given.

So, in order to meet the condition of the same amount of heat being added, the point 3D will lie somewhere to the right of the vertical line segment 3 o 4 o right, so I am sure all of us agree to this point. So, let me mark it here you know like just to show the relative location, now where will 4 D lie, 3 D 4 D is going to be isentropic expansion. So, 3 D 4 D should lie somewhere on the vertical line passing through sorry 4D should lie on somewhere on the vertical line passing through the point 3 D.

And 4 D 1 D is a constant volume heat addition process, so it should 4 D also should lie on the constant volume line passing through 1 D. So what we do, we just extend this constant volume line wherever it intersects the isentropic line passing through 3 D, that is 4 D right obviously. So this is ok, so that ok, so now we are almost through. So, we know that the thermal efficiency is given by 1 minus the magnitude of Q_{out}, Q_{in} .

Of course this will be the magnitude of Q_{out}, ok if I have to be very particular ok, so all right, so we are only taking the magnitudes right. So now we can immediately see that here in this problem Q_{in} the Otto cycle is the same as Q_{in} the Diesel cycle. And from the analysis, what can we say about the magnitude of the heat rejected in both cycles. So the heat rejected in the Otto cycle it is magnitude is given by a o 1 o 4 o b o, that is a o 1 o 4 o b o right.

Now, the heat rejected in the Diesel cycle is a D1- D4- Db- D, what can you say which is smaller in the Otto cycle right, so the heat rejected in the Otto cycle is smaller. So this imply is that what can you say about the thermal efficiencies. Thermal efficiency is 1 minus heat rejected by heat input, heat input is the same, the numerator. So the denominator in the second factor is the same, right for both Otto and Diesel cycle, the numerator is lower in Otto cycle.

So the ratio is going to be lower in the Otto cycle, so the thermal efficiency is going to be larger in the Otto cycle, right, so that is what we can conclude ok from this analysis ok. So as an exercise, what I want you to do is plot the PV diagram for the same example and observe and we would continue from here in the next class, thank you.