Control Systems
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Lecture – 72
Lag and Lag - Lead Compensation
Part - 2

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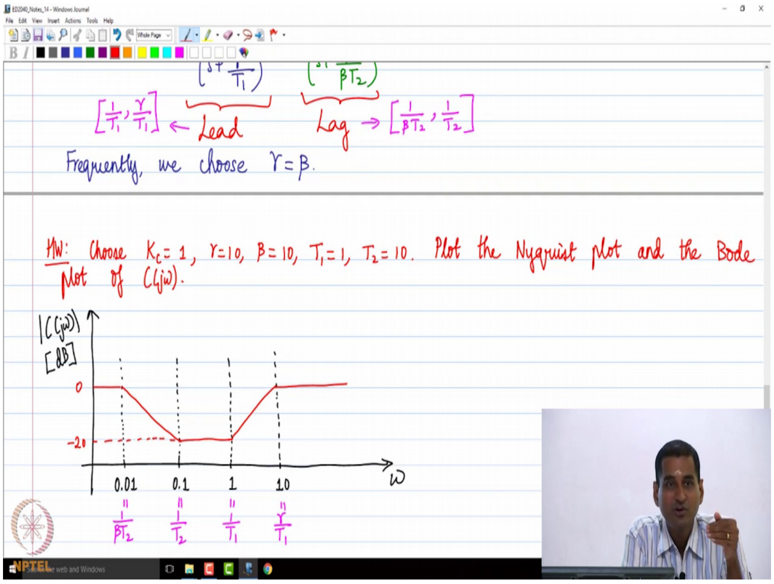

$$\left[\frac{1}{T_1}, \frac{\gamma}{T_1}\right] \leftarrow \underbrace{\left(s+\frac{1}{T_1}\right)}_{Lead} \qquad \underbrace{\left(s+\frac{1}{\beta T_2}\right)}_{Lag} \rightarrow \left[\frac{1}{\beta T_2}, \frac{1}{T_2}\right]$$

Frequently, we choose $\gamma = \beta$.

HW: Choose $K_c = 1$, $\gamma = 10$, $\beta = 10$, $T_1 = 1$, $T_2 = 10$. Plot the Nyquist plot and the Bode plot of $C(j\omega)$.

So, frequently you know like gamma is taken to be beta. So, what I am going to do is that like I am going to give you a homework problem ok. So, where you essentially draw the Nyquist plot and Bode plot of this particular transfer function, sinusoidal transfer function, and then like comment. So, let us say, we choose the following values, we chose K c to be 1, and gamma to be 10, beta also to be 10, and let us say T 1 to be 1, and T 2 to be 10 ok. So, let us say, we choose T 2 and T 1 in this fashion right ok.

So, what we are doing is that like I am I am choosing 1 by T 1 and 1 by T 2 to be 1 decade away right, so that is what I am doing right. And then like the lead compensator will essentially have a slope as far as the magnitude curve is concerned one decade from 1 by T 1. And similarly, the lag compensator will have a what to say frequency range of interest 1 decade below 1 by T 2 ok, so that is why that is how we just slip them as three equal components ok, I will give you the answer you just solve it right.

So, let us say you know like you essentially choose these values right, so what you need do is that plot the Nyquist plot, and the Bode plot of the of C of j omega ok, so that is your homework. So, I am just going to draw the Nyquist plot sorry not the Nyquist plot, the Bode plot, the log magnitude curve we more exact, so that like we will visualize how this going to look like right.

So, let me draw the magnitude of C of j omega, of course, in decibels ok. So, what is what are the critical frequencies, we are going to have 0.01 K will have. So, since 1 by beta T 2 will be 0.01 that is why I am taking from 0.01 right, 1 by T 2 will be 0.1 ok, then I have 1, then I have 10 right. So, these are the four frequencies right. So, what I am doing is that, I am just calculating the values of 1 by T 1, gamma by T 1, 1 by beta T 2, and 1 by T 2 right that is what I have marked for this particular example. So, what is 0.01 for this particular example, how do you get 0.01?

Student: (Refer Time: 03:27).

This is nothing but 1 by

Student: Gamma.

Beta T 2 right, 0.1 is essentially 1 by T 2, this is 1 by T 1; this is gamma by T 1. I hope now it is clear, how I got these four values ok, these are the two four frequencies. And the way, we are chosen these numbers, we have ensured that that the gaps in the logarithmic scale are the same that is just done by convention, you know that is it ok.

So, let us look at what happens to the I am just going to give you the answer, and then discuss. So, what will happen in this particular case is that the asymptotic plot, obviously, it will start from 0 ok, then it will reduce to minus 20, then stay at minus 20, and then it will go back to 0 ok. So, this is what happens with this of with the log magnitude curve of this particular transfer function the lag lead transfer function right.

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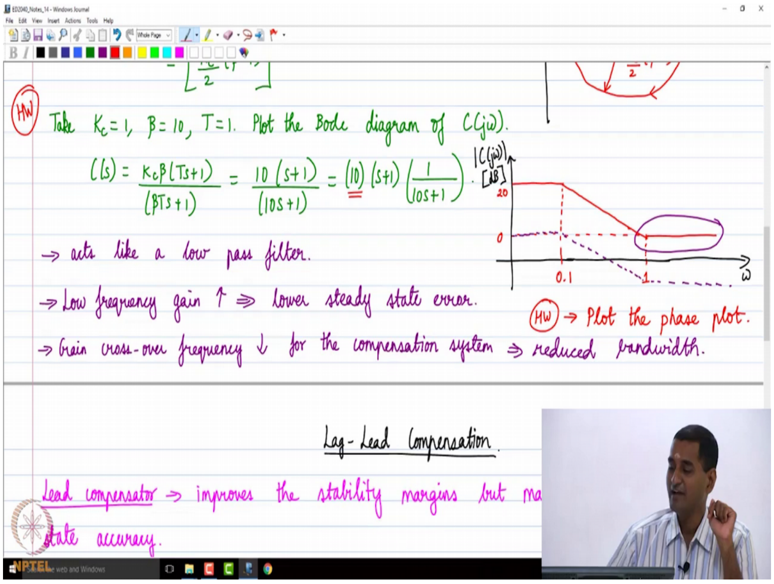

HW Take $K_c = 1$, $\beta = 10$, $T = 1$. Plot the Bode diagram of $C(j\omega)$.

$$C(s) = \frac{K_c\beta(Ts+1)}{(\beta Ts+1)} = \frac{10(s+1)}{(10s+1)} = \underline{\underline{(10)}}\,(s+1)\left(\frac{1}{10s+1}\right).$$

→ acts like a low pass filter.

→ Low frequency gain ↑ ⇒ lower steady state error.

HW → Plot the phase plot.

→ Gain cross-over frequency ↓ for the compensation system ⇒ reduced bandwidth.

Lag - Lead Compensation.

Lead compensator → improves the stability margins but ma

state accuracy.

So, you can immediately see that the lag part kicks in first right. In the lag lead compensator, the lag part kicks in first right, because you would you can immediately correlate it to whatever we learnt right. The lag compensators log magnitude plot is going to have a structure like this right. So, it is going to stay constant decrease, and then stay constant right, so that is what you are going to have in the first part, due to the lag part. And then the lead part kicks in, and then the magnitude value increases, and then becomes constant ok, so that is the structure for this lag lead compensator right. Yes please.

Student: Sir, lag is between difference we chosen T 1, T 2.

Yeah, it depends on the value chosen exactly right. So, if you look at it, you know like what is the, what can you say about the nature of this particular compensator what does it do.

Student: It will be like it does that (Refer Time: 05:52).

Yeah, it takes an intermediate frequency range right, and then attenuates that frequency range alone right, so that is structure, which is taken by this lag lead compensator right. So, you can immediately see that that is what is going to happen right.

So, please plot the what to say Nyquist plot, and also the phase plot ok, corresponding to this particular transfer function ok. And just make observations as before right, and that is something, which I am going to leave it you as an exercise or a home work, is it clear fine ok.

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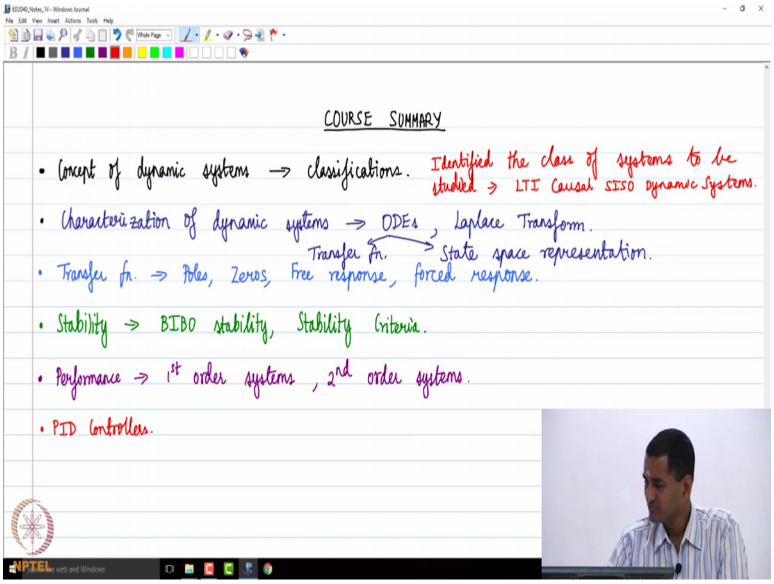

COURSE SUMMARY

- Concept of dynamic systems → classifications. Identified the class of systems to be studied ⇒ LTI Causal SISO Dynamic Systems.
- Characterization of dynamic systems ⇒ ODEs, Laplace Transform. Transfer fn. ⟷ State space representation.
- Transfer fn. ⇒ Poles, Zeros, Free response, forced response.
- Stability ⇒ BIBO stability, Stability Criteria.
- Performance ⇒ 1st order systems, 2nd order systems.
- PID Controllers.

So, this completes our what to say discussion as far as the theory part for this course on control system is concerned. So, what I am going to do is that like, I am just going to summarize whatever we learnt in this course, so that we get the big picture view point right.

So, and then you can also help in summarizing what we really learnt in this in these what to say lectures, and how we will use them right. So, let us let us do that. So, let us summarize all the topics that we learnt right. So, where did we start if you recall ?

Student: Stability.

Sorry.

Student: (Refer Time: 07:19).

Stability, you know like see think about from the first class right.

Student: (Refer time: 07:24).

Ok. So, I am just what you say essentially giving as bullet point’s right. So, we first started with what is the system drawing, what is the dynamic system correct. So, and then like what are the classes of dynamic systems based on whether it is linear, or non-linear, what is the definition of time in variance, causality, and so on right, single input, single output systems. And all those classifications, were first learnt right, so that is something, which we looked at ok. Then what did we do?

Student: Laplace transforms.

Laplace transforms ok, before that?

Student: Properties.

Properties of what?

Student: Systems.

Systems, of course, yeah. So, we looked at the classification, we also identified the class of systems that we are going to study right. So, now everything whatever we learnt, you know as far as this particular course is concerned, you know like is applied to what class of systems?

Student: (Refer Time: 08:57).Yeah.

Yeah. So, if you recall, we identify the class of systems to be studied as linear time invariant, causal, single input, single output.

Student: Dynamic.

Dynamic systems right, so that is the class of systems that we have analyzed throughout this particular course right.

So, then what did we did, you know like we looked at characterization of dynamic systems using mathematical models right, so that is what we learnt next right, how one can use essentially models to characterize.

And then we also learnt, what was that what is that typical class of models that we will use. If you remember, you know like lump parameter or spatially homogenous, dynamic continuous time right, deterministic models right that is the class of models, we essentially use. So, ultimately what we figure out as that like this essentially comes in the form of ordinary differential equations right.

So, consequently what we did, you know like we essentially revised a few important what to say mathematical tools particularly related to complex variables, and Laplace transform ok. We learnt how Laplace transform can be used to solve ODEs right. And since, for this particular class of systems, we will we will get linear ODEs with constant coefficient, there is only one solution, which is the exponential function right, so that is why exponential functions are pretty important. Then what did we learn?

Student: Laplace transform.

Sorry.

Student: Laplace transform.

Yeah of course, by Laplace transform, I refer to all it is properties examples that we did and so on right. So, then we looked at of course, we also did one more thing right. So, of course, in this is not in the same order, but then we learnt how to essentially represent the model, which is given in terms of ODEs in two representations right, the transform function representation, and the state space representation.

Of course, throughout this course, we looked mainly at the transfer function representation. Although we had a brief what to say discussion on what is the state space representation, how can it be obtained, and what is the equivalence between two right that is something, which we discussed.

Ah But we took the transfer function representation, then what did we do, we learnt about poles, zeros right, and then we also looked use the Laplace transform to look at free response right, forced response.

And we learnt that the transfer function is typically used to characterize the force response right, free response is essentially removed, because the initial conditions are taken to be 0. But, is that viable approach, you know like, because in reality we are always going to have a we may have a non-zero initial condition right.

: Non-zero.

But, then by now you would have realize that if we design stable systems, the coefficient that come in the solution, that depend on the initial conditions will be multiplied by exponential functions with the decaying, or a with a negative exponent.

So, what really happens is that if you consider the free response of a stable system that anyway decades to 0, and what persists the steady state value, and also the steady state response and even the transience to a large extent would be influenced by the force response by the input that we get right.

The free response would affect the input, if the initial conditions are non-zero, but the argument is just typically given is that like look any way the contribution of the free response is any way going to go to 0 right if you design a stable system, that is why we are looking at the force response predominantly right. Then what did we learn? We learnt two important topics right that of.

Student: Stability.

Stability, and performance right.

So, we learnt the definition of BIBO stability, and course by stability you know like, then we added an adjective there, we learn something called relative stability. So, people call this as absolute stability right. So, we learnt, what is called as a BIBO stability, what is the notion.

And then like we learnt stability criteria, we figured out that for BIBO stability, a necessary and sufficient condition was for all the poles to lie in the left of complex plane right. And then like we learnt the Routh's stability criteria to figure out you know, like how 1 could evaluate whether a given system is stable or not or conversely, how could one design controllers to ensure close loop stability right, so that is something we leant correct.

Then we also looked at performance, how did we quantify performance, if you recall, we looked at 1st order systems all right, 2nd order systems. And then like looked at you know like various parameters like time constant, raise time, peak time, over shoot and so on right, settling time. So, all those parameters, where used to essentially quantify performance, correct.

So, then we looked at predominantly PID controllers right. So, we learned the structures, what is the structure of the PID controller, what was the a contribution of each part proportional, integral, and derivative, what are the pros and cons of each component in the PID control what to say control structure and essentially we did a few examples right and we learnt, you know like when we got an intuitive feel of when to use what right, because when we have poles on poles of the origin or poles on the imaginary axis for the plan, what to do and when we have poles in the right of plane, which controller would surface and so on right. So, those things we figure out by doing a few examples.

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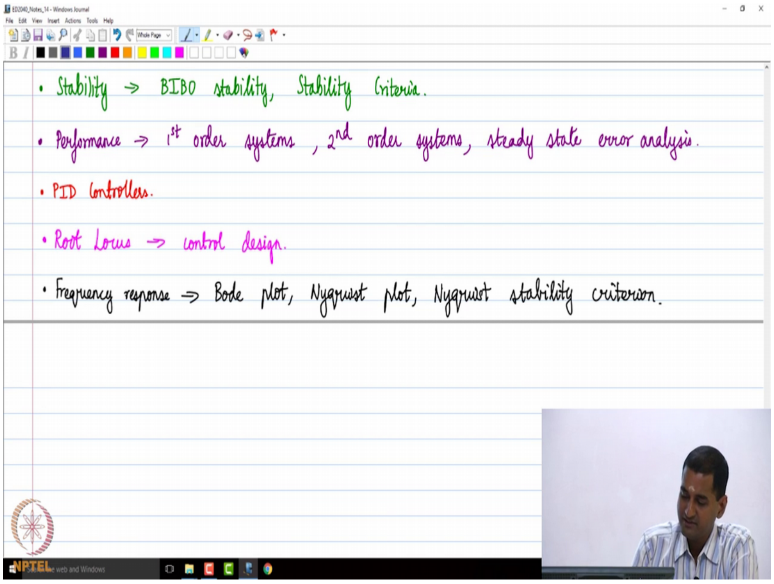

- Stability ⇒ BIBO stability, Stability Criteria.
- Performance ⇒ 1^{st} order systems, 2^{nd} order systems, steady state error analysis.
- PID Controllers.
- Root Locus ⇒ control design.
- Frequency response ⇒ Bode plot, Nyquist plot, Nyquist stability criterion.

Then if you recall, we studied about the root locus right. So, root locus essentially was use to plot the locus of the closed loop poles, and then this was used for control design correct. So, if I if I am given the open loop transfer function, if I vary one parameter, which may be a controller parameter, how do the locus of the closed loop pole change, you know like and that was used to essentially design the closed loop system for stability and performance right. So, after this we moved to frequency response.

Student: I think you missed the steady state steady state error.

Yeah fine that is right you know like, so where would I put it, let me put it under performance ok, because any way thank you. So, I think we yeah I missed out writing steady state error analysis right. We learnt various definitions of definitions of various parameters kpe, kve, kea and so on right, and when to use what. And we looked at different types of system, type 0, type 1, type 2 and so on right and what is the implication of those systems.

We looked at frequency response, then we learnt what it was, and then how it can be visualized using Bode plot, Nyquist plot right. So, and then we learned the Nyquist stability criteria, transfer and so on ok, so that that was another thing.

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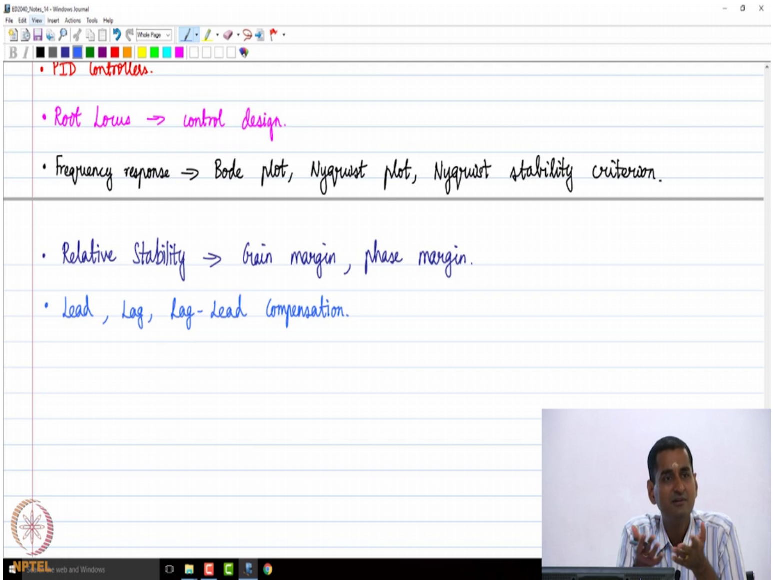

- PID controllers.
- Root Locus ⇒ control design.
- Frequency response ⇒ Bode plot, Nyquist plot, Nyquist stability criterion.

- Relative Stability ⇒ Gain margin, phase margin.
- Lead, Lag, Lag-Lead Compensation.

And finally, what we did was that like we essentially looked at relative stability right. We looked at various parameters that can be used for quantifying performance in the frequency domain like gain margin, phase margin, and so on right. We then, we looked at cut off frequency bandwidth and so on right, so that is something, which we did.

And finally, of course, we looked at lead, lag, and lag-lead compensation right, so that is another just choice of controller transfer functions that you have right, depending on the type of system that you have you choose either. And I hope that this course essentially also serves as a starting point right. If you want to essentially learn, advance control theory, you know like this whatever we learnt in this particular course would be the fundamentals based on, which you would built right, so that is the expectation of this particular course.

So, if you go back, and look at what were the learning outcome outcomes of the course, you know that we discussed in the very first class, you know like essentially that is been listed under the summary right. So, we have learned what is called as classical control theory right, for a particular group of systems fine. So that is about it as far as the theory is concerned. So, I thank you for your attention.