Control Systems
Prof. C. S. Shankar Ram
Department of Engineering Design
Indian Institute of Technology, Madras

Lecture – 71
Lag and Lag-Lead Compensation
Part - 1

So, in the previous class, we looked at Lead Compensation, and we did an example as far as the design of the lead compensator is concerned. In today's class, I am going to introduce you to lag compensation ok, when it is used, you know like what is the structure of a lag compensator and what are its features right. And then we will also look at what effects does it have on the entire system, and we will also learn what is called as lag lead compensator ok. So, I am just going to explain the concepts to you with respect to Lag Compensation and Lead Lag Compensation.

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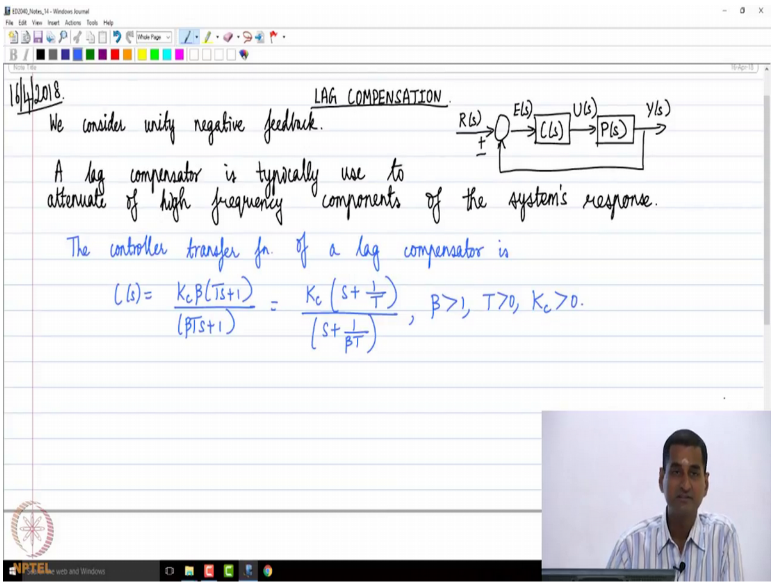

So, let us look at lag compensation. So, once again we consider unity negative feedback ok. So, the same structure that we adopted when we discussed lead compensation is taken here right.

So, the question becomes when do we use a lag compensator right. So, a lag compensator is typically used to attenuate or essentially reduce right. So, the magnitude of high frequency components of the systems response ok, so that is what a lag compensator typically components ok, high frequency components of the systems response that is the role of a lag compensator.

So, it what it essentially does is that is it essentially reduces or attenuates the high frequency components of the systems response. As the name indicates it introduces a phase lag, but by and large when you use a lag compensator, we are not unduly worried about the phase margins. So, we the base system itself, will have enough phase margin that the addition of a phase lag. See phase lag means, it is negative phase angle right, so obviously, you know like our phase margins are going to decrease a little bit, but we are not unduly worried about that right, so that is when you can apply a lag compensator. But, then like we essentially want to attenuate the high frequency components, you know then we use a lag compensator and design the entire close loop feedback system.

So, let us look at the structure of a lag compensator, the controller transfer function, or the transfer function corresponding to a lag compensator is the following. So, let us look at the structure. So, this is given by C of s is equal to K c beta times T s plus 1 divided by beta T s plus 1 ok, so that is the transfer function corresponding to a lag compensator. So, this can be rewritten as K c times s plus 1 over T divided by s plus 1 over beta T, and here of course, beta is greater than 1 T is greater than 0, of course, we take K c also to be greater than 0 ok, so that is the structure of a lag compensator.

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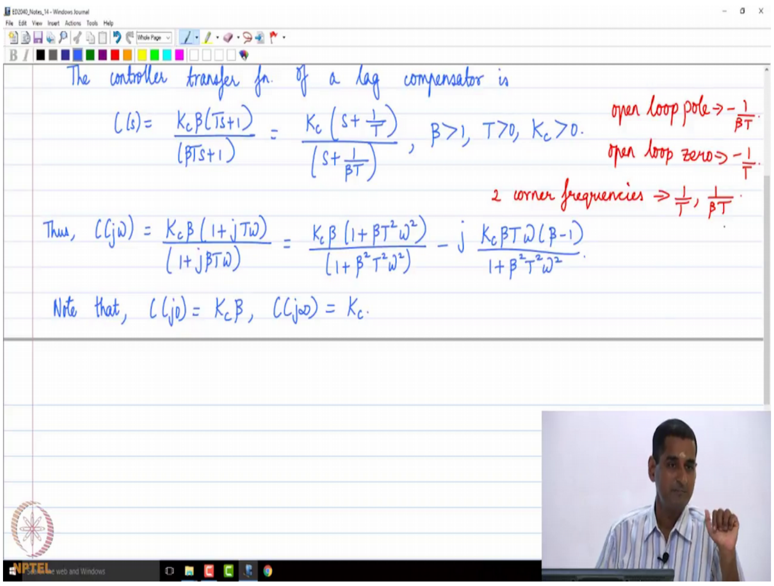

So, immediately we can see that this introduces an open loop pole at minus 1 over beta T and an open loop 0 at minus 1 over capital T right. So, we can immediately observe that there is an open loop pole, and an open pole 0 that is being introduced by this lag compensator.

So and what about the 2 corner frequencies? So, what are the 2 corner frequencies corresponding to this lag compensator? So, once again there are going to be 1 by T, and 1 by beta T ok. So, those are the two corner frequencies corresponding to the numerator term, and the denominator term respectively ok, so that is what happens with the lag compensator right.

So, now let us look at the sinusoidal transform function for this lag compensators. So, C of j omega will be K c beta times 1 plus j T omega divided by 1 plus j beta T omega ok, so that is what will what is the sinusoidal transfer function corresponding to the lag compensator ok. So, if we multiply and divided by the conjugate term of the denominator, so what we will get is the following, we will get K c beta times 1 plus beta times T square omega square divided by 1 plus beta square T square omega square minus j times K c beta T omega times beta minus 1 divided by 1 plus beta square T square omega square ok, so that is what we will have pretty straight forward.

So, these are all like I am sure all of us are familiar with this process by know right. I am just multiplying and dividing by the conjugate of the denominator right, so that is what we do right. So, we can immediately note that as omega tends to 0 that transfer function essentially tends to K c beta, and as omega tends to infinity the transfer function tends to K c.

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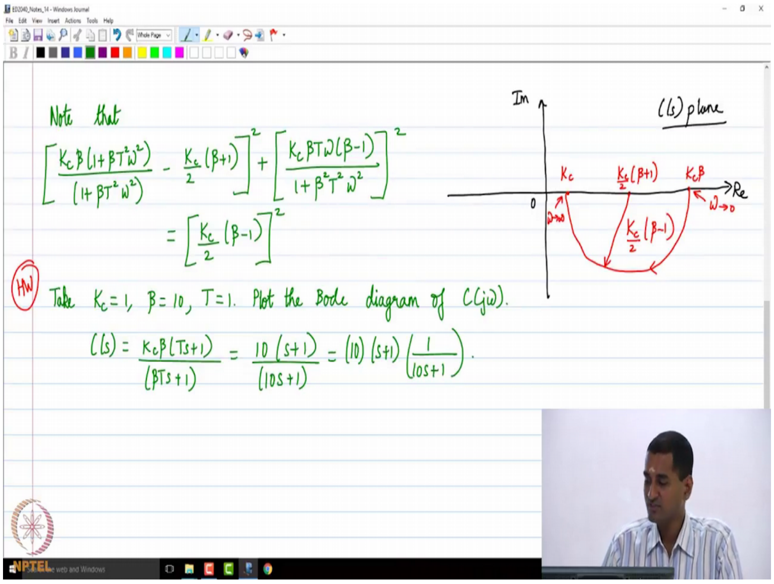

So, you see that at either extremes once again the imaginary component is 0 and so, if you want to plot the what to say Nyquist plot for this sinusoidal transfer function, so what can you immediately observe, I as far as the location of the Nyquist plot is concerned? The complex plane, let us call it as a C s plane. So, what can you observe regarding the location of this Nyquist plot?

So, if you look at the real component and the imaginary component right, so for all omega you can immediately see that the real component is going to be positive right. What about the imaginary component? Of course, leaving aside the two limiting values of 0, and infinity it is going to be negative, or in general it is going to be non-positive right. So, which quadrant we are going to have this Nyquist plot in? The fourth quadrant right, so in this complex plane that is where the Nyquist plot for this factor would lie in right.

So, you can immediately of course, once again this is the locus is going to be in the form of a semi-circle. So, once again we can note that if I take the real part ok, and subtract the center of this particular what to say term, which is going to be K c by 2 times beta plus 1 whole square, because how do I get the center, because the two limiting values are K c beta and K c right, you take the mean of the loop right. So, you will get K c b by 2 times beta plus 1 plus I square the imaginary component K c beta T omega beta minus

1 divided by 1 plus beta square T square omega square whole square that is going to be equal to what do you think should be the radius?

Student: (Refer Time: 08:55).

I am sure by now we are all familiar with the what is happening right. So, what should be the radius?

Student: 25.

Yeah. You just take the difference and divide by 2 right. So, essentially we are going to get K c by 2 times.

Student: Beta minus 1.

Beta minus 1.

Student: Whole square.

Whole square right, so that is what we are going to have here. So, if we take the if you want to draw the Nyquist plot, the center is going to be at K c by 2 times beta plus 1. So, this is going to be K c beta please note that beta is greater than 1 right, and this is going to be K c right.

So, what is going to happen is it, the Nyquist plot is going to go something like this ok, so this is going to be a semicircular path ok and this is where it will start as omega tends to 0, this is where at it tends as omega tends to infinity right and there radius of this particular semi-circle is going to be K c by 2 times beta minus 1 ok, so that is what is going to happen at the Nyquist plot of this particular sinusoidal transfer function.

So, now if we continue, so I am going to leave you with a few exercise problems ok. As this pretty similar to what we did in the lead compensator ok. So, you can see the analogy, you know like as you, or you can see the similarities and differences, once you do the problems right.

So, let us take you know like the value of K c to be 1, beta to be 10, and T to be 1 ok. Plot the Bode diagram of C of s of C of j omega right, so that is what is the first exercise.

Of course, I am going to give you the answer, so that like we can discuss some concepts right.

So, but then like I am sure, by now all of us know how to plot these diagrams. So, if I plug in these parameters, what is going to happen to C of s? I am going to get please note that the transfer function for the lag compensator is K c beta times T s plus 1 divided by beta T s plus 1. So, if I substitute, I am going to get 10 times s plus 1 divided by 10 s plus 1 right, so that is what I will have. So, this I can rewrite as three factors right, 10, s plus 1, and 1 by 10 s plus 1 right, that is what we are going to do.

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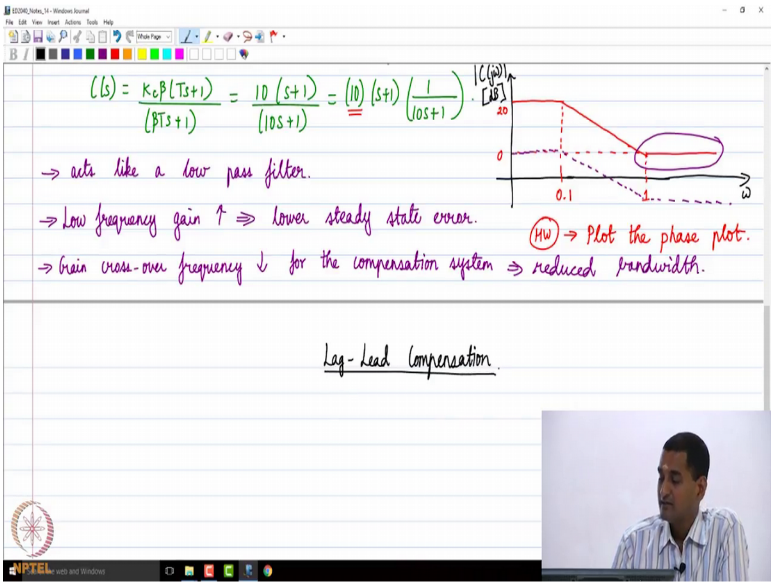

We are essentially going to plot the what you say, what you need to do is essentially plot the magnitude asymptotes, for the log magnitude curves for each of these three factors and add right, so that will give me their net Bode plot, the log magnitude curve.

So, I am just going to give you the answers straight away right, so that we discuss it. So, what is going to happen is the following. So, this will start at 20 decibels right. So, what is what are the 2 corner frequencies now?

Student: 1 by 10.

It is going to be 0.1 and 1 right, 1 by T and 1 by beta T. So, you will get the 2 corner frequencies as 0.1, and 1; 1 by beta T, obviously, is less than 1 capital T right, because beta is greater than 1 right. So, given the values, you know like we get the corner frequencies as 0.1 and 1.

So, what is going to happen is that, due to this contribution of 10, initially we will have 20 decibels, still I reach 0.1 right, correct. Then what is going to happen, I am going to essentially have a down slope at minus 20 decibels per decade, till I reach the other corner frequency, after which I will go for a horizontal line, in the log magnitude curve right, so that that is what is going to be the magnitude plot. I am just only drawing the asymptotes right, so that is what we are doing.

So, I hope it is clear, how we got it right, because after the second corner frequency, we add a what to say asymptote with a positive slope of 20 decibels per decade that cancels out the minus 20 decibels per decade that is why we get a straight line, beyond the omega equals 1 right, so that is what we have.

Now, you can immediately see that from this particular bode plot, you can immediately observe that this by enlarge acts like a low pass filter right. So, it is essentially attenuating the high frequency components relative to the low frequency components correct, so that is the characteristic of a lag compensator right.

So, essentially of course, once again you know like plotting the what to say, phase diagram is something I leave it to you as an exercise ok, plot the phase plot for this particular transfer function. But, immediately you from this particular what to say magnitude plot, we can immediately see that the what to say the lead compensator acts like a low pass filter that is first observation. Then what else can we observe?

Of course, there are two things right, if I leave it the way it is, what is going to happen, the low frequency gains are going to be increased right. Please note that, if I keep this as C of s, I multiply with the plan transfer function. What am I doing? At low frequencies, I am adding 20 decibels right. 20 decibels means, you know like I am multiplying the gain by 100, please remember sorry not by 100, 10 right, by a factor of 10. See 20 decibels in the log magnitude curve means the factor is 10 right, so that is what is the low frequency gain of this particular controller transfer function, correct. So, if I do this, what can you tell about the steady state errors?

Student: Decrease.

They will decrease, because the steady state what to say, the low frequency gains will increase, then you remember Kpe, Kve, and all those parameters will increase, which will reduce the steady state errors. So, you see that the low frequency gain increases this implies that lower steady state error that is another feature.

But on the other hand, if I do not want this, and then like if I want essentially, see increasing the low frequency gain means, you know like I am also I the cost of the system also increases right. But, then if I want to really attenuate the high frequency components, what is what should I do? I should push down this curve right by 20 decibels, because look at this here, the high frequency component are only are left as they are, it is not really an attenuation.

Of course, comparatively it is an attenuation, but in a on a absolute scale, the high frequency gains pass through as they are right, the way, this bode diagram is drawn. But, if I really want to attenuate their high frequency components, what will I do, I will just push down this bode diagram by let us say, 20 decibels for example, right.

Then what is going to happen, the modified bode diagram may look something like this ok, it may shift down, and go like this right. If I want to really attenuate the high frequency components. So, if I do this, what is going to happen to the gain cross over frequency, and the cut off frequency? So, you immediately see that the gain cross over frequency would it decrease or increase?

Student: Decrease.

It will decrease right, because what is a gain cross over frequency, it is a frequency at which the magnitude of open loop transfer function is 1 right, now I am pushing it down right. So, I multiplying the plant transfer function by a transfer function, whose magnitude characteristics essentially are becoming lower right at this in this frequency range right, lower than 1 in this frequency range.

So, essentially what will happen is that, obviously, I will choose the corner frequencies in a frequency range of interest that is what you will do when you do your case studies ok. These are not arbitrarily chosen, the values of beta T and all depend on the plan that you are given. So, the corner frequencies are chosen in a sense way that it will be in a range or region around the original gain cross over frequency of the system that you are given.

So, now what will happen? The gain cross over frequency was a combined system was a compensative system will decrease. So, what will that mean? This will mean reduced bandwidth right. So, of course, reduced phase margin of course, for this mainly what we are concerned about is reduced bandwidth also.

So, of course, that is that is expected right, because a low pass filter is supposed to attenuate the high frequency components. But, the flip side is that when you try to do that, the bandwidth decreases, if the bandwidth decreases, the systems response becomes a little bit slower right, so that is some that is a that is a price we need to pay right, if we want this feature right. Please remember what happened in the lead compensator, we essentially increased the gain cross over frequency, the bandwidth increased right. But, what was the price that we paid in the lead compensator? It acted as a.

Student: High pass.

High pass filter right.

So, consequently, if I what to say if I wanted to adjust the magnitude curve without amplifying the high frequency components too much, the steady state characteristics where effected. So, but here, you know like it is a tradeoff right, you can see that the bode plot is just the other way around right. So, here we want to attenuate the high frequency components, the flip side is that the what to say, the bandwidth decreases ok, so that that is a limitation with the what to say, lack compensator ok.

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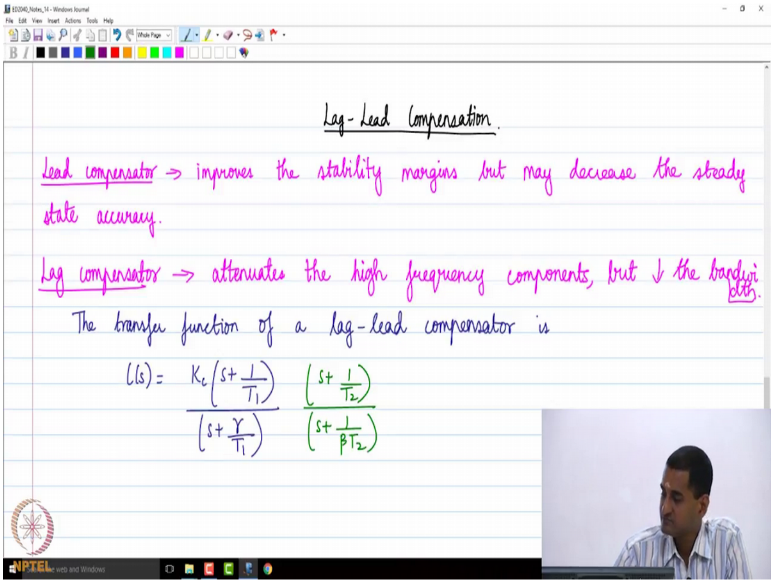

Ah So, the if you want to combine, the advantages of both a lack compensator, and a lead compensator right, while addressing their limitation, you know like people use sometimes people use what is called as a lag lead compensator. So, what is this lag lead compensator right?

Please recall that, if we use a lead compensator, a lead compensator improves this stability margins that is it improves the phase margins, and so on right. So, those are called as this stability margins ok. So, typically a lead compensator is designed to improve this stability margins, but may decrease a steady state accuracy that is what we just discussed right, so that is a what to say an outcome, which is not desired right from a lead compensator. So, this may reduce the steady state accuracy.

On the other hand, a lag compensator right so, attenuates the high frequency components, but decreases the bandwidth right, so that is that is something, we just observed. So, it can lead to system with lower bandwidth or reduced bandwidth right.

So, hence sometimes you know like people use a lag lead compensator right. So, just essentially do the combine the benefits of both these compensators depends on the scenario right. So, I am just going to give you the structure, and then like discuss it, and I will leave the Nyquist plot, and bode plot of the lag lead compensator as a home work problem right.

So, let us look at what is the controller transfer functions. So, the transfer function of a lag lead compensator is ok. So, what is how is the controller transfer function written, C of s is typically written as K c times s plus 1 by T 1 divided by s plus gamma by T 1, let me complete writing it, then we will discuss it ok. So, then multiply it by s plus 1 by T 2 divided by s plus 1 divided by beta T 2 ok, so that is the transfer function.

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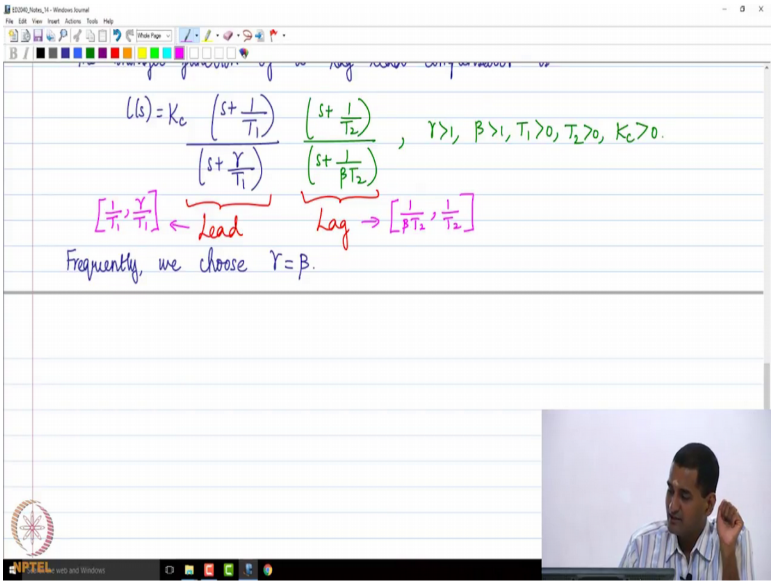

So, here gamma is greater than 1, beta is greater than 1, T 1 is greater than 0, T 2 is greater than 0, of course, typically K c is also greater than 0 right.

So, of course see, when we discuss a lead compensator, we use alpha, alpha was less than 1. Say by conventionally, you know like when we talk about lag lead compensation, you know like that alpha is replaced by 1 by gamma. So, I am sure, you can observe by now right. So, alpha was less than 1, consequently gamma is greater than 1 that is the only change.

So, you can immediately see that this part, so what I will do is that, let me write this K c factor just outside, so you can immediately observe that this part is the lead compensator, it provides the phase lead, this is this corresponds to the lag compensator ok, so that is that is what those that is the split between the two terms.

Of course, you can immediately see that the complexity of the controller increases right, is not it right, because what you are having is now what to say, two open loop 0's, and.

Student: Two open loop.

Two open loop poles being introduced by the controller transfer function right. So, the design becomes more and more complex right.

So, but then, it combines the advantage. So, some sometimes you know like people may use it, and frequently we choose gamma to be equal to beta ok, so that is a typical design choice, which is made right. So, there is the value of gamma and beta are choose to be the same.

So, immediately you will see the that the spread of the frequency range would be the same, because here you would see that the corner frequencies are going to be. So, in this lead compensator, you will immediately see that what are the corner frequencies, it will be they will be 1 by T 1, and gamma by T 1, so obviously, gamma by T 1 is greater than 1 by T 1.

See previously, we learn them as 1 by T and 1 by alpha T. Since, 1 by alpha is now gamma, we just have it as gamma by T 1. And for the lag compensator, the range of frequency is going to be 1 by beta T 2 and 1 by T 2 right and those are the two corner frequencies, I am just writing the two corner frequencies.

So, if you choose gamma to be equal to beta, what is going to happen is that the range is going to be pretty much just equal in the logarithm scale right, so that is essentially the ratios are going to be the same right. As long as, you know like you have you design what to say T 1 and T 2 carefully, of course, that that is something, which we need to do carefully right, so that that is another aspect we need to essentially look at.