Control Systems
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Lecture – 70
Lead Compensator Design
Part – 2

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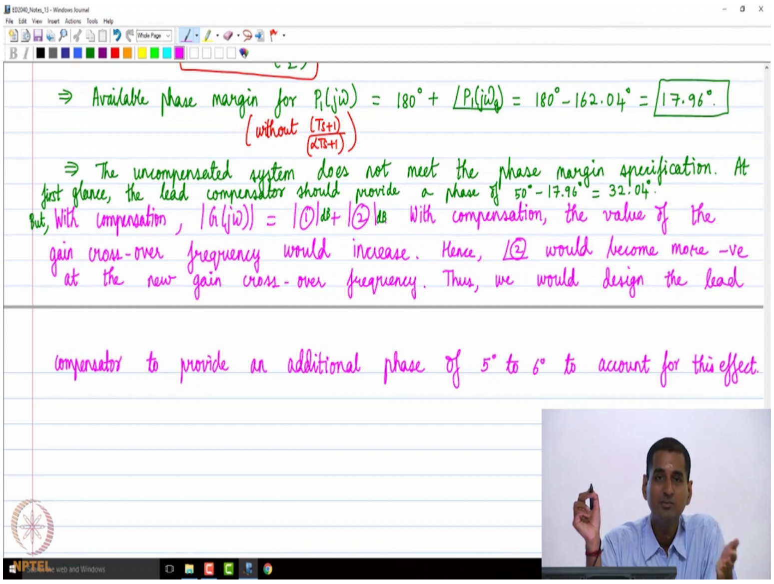

With compensation, the magnitude of G of j omega is going to be magnitude of 1 plus magnitude of 2 in decibels right. So, with compensation consequently the value of the gain cross over frequency would increase ok. Hence, the phase of 2 would become more negative at the new gain cross over frequency right.

Thus, what we do is that in order to avoid any conflict, you know like, because the phase margin may decrease below 50 degrees due to this phenomena. Thus, we would design the lead compensator to provide an additional phase of 5 degrees to 6 degrees to, account for this effect ok. So, that is what we are going to do ok, because the phase of the second factor is going to become more negative.

So, if I still keep the phase contribution of 1 to be 32 degrees what will happen is that the sum will fall below 50 degrees right; so, that is what is going to happen. So, what we, we do not know the new cross over gain cross over frequency yet, so, but as a rule of thumb what we do is that we typically add 5 to 6 degree degrees to the number, which we have calculated just know.

So, instead of 32, we design the, gain sorry, lead compensator to provide an phase of around. Let us say 38 degrees ok; so, that is what we typically do ok. I, I hope it is clear, what essentially we are doing.

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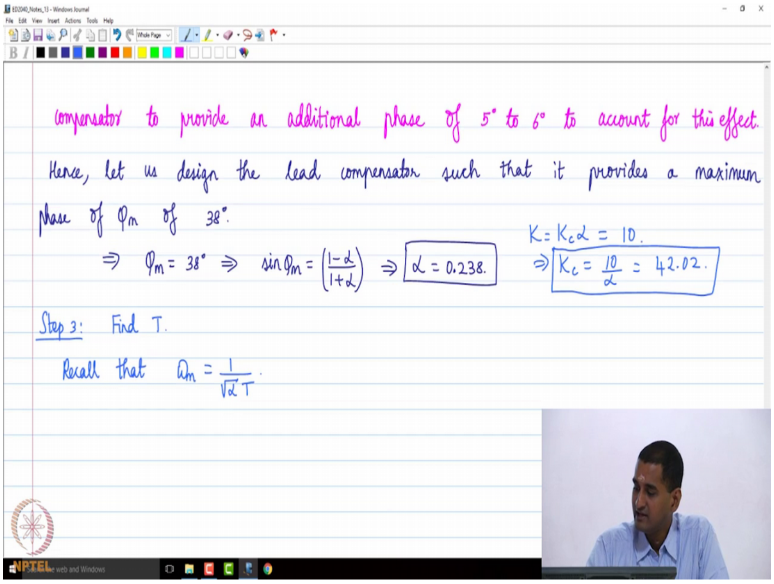

Hence, let us design the lead compensator, such that it provides a maximum phase of phi m, you member what phi m was right, phi m was the maximum phase contribution right, for the for the lead compensator factor.

So, of 38 degrees, let us say we add 6 degrees to 32 degrees. Let us say we know, we say that the gain, sorry the lead compensator. Let it provide a maximum phase of 38 degrees that is what we are going to, design for. So, this implies that phi m is 38 this implies that what was sig; sig of phi m, if you recall from the previous class.

Student: 1 minus alpha divided by.

It is going to be 1 minus alpha divided by.

Student: 1 plus alpha.

1 plus alpha right; so, if you use this, what will happen is that you will get the value of alpha to be 0.238 ok. So, that is what we will have ok. Is it clear? How we got alpha?

So, we essentially calculate what was the existing phase margin without compensation then we find out what is the additional phase that should be provided to the lead compensator, by looking at the required phase margin and then what we do is, that we add a correction term, because the gain cross over frequency is going to be shifted to the right with compensation alright.

So, that is why we just added a correction term to the required, phase of the lead compensator that is what we have done. Is it clear? Now, let us go to step 3, step 3 is to find, the value of t. So, in a sense we have, we have found alpha, if you have found alpha have we found K c.

Student: Yes

Yes. So, if alpha is 0.238 immediately you note that K equals K c alpha. What was K?

Student: 10.

10.

Student: 10 10.

Right. So, this implies that K c is going to be 10 divided by alpha. So, immediately you will see that, that value comes to our own 42 0.02. So, what were the three parameters, that are involved in the lead compensator K c alpha and T. So, we have found K c and alpha. What is left behind?

Student: T

Capital T. So, let us find capital T. That is what we are going to do. So, let us say find capital T ok. For this recall that omega m is going to be equal to 1 divided by square root of alpha times T.

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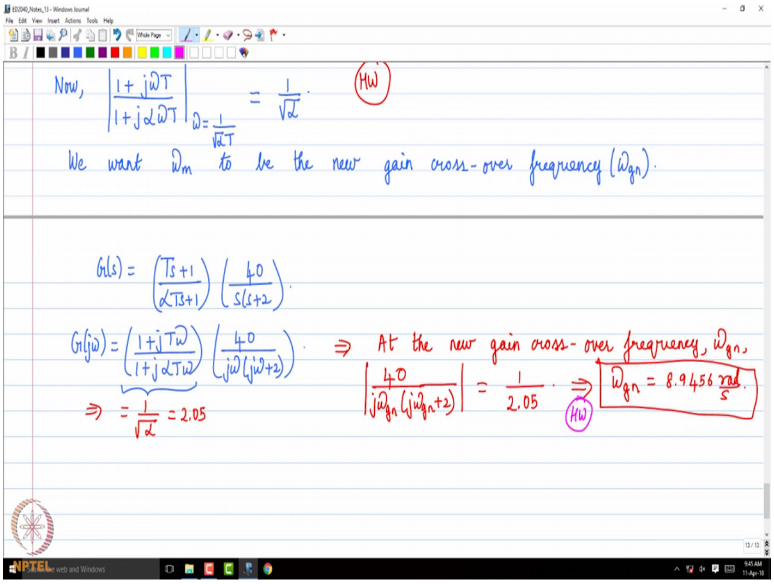

So, this expression I leave you to you as homework. The magnitude of 1 1 plus j omega T divided by 1 plus j alpha omega t at omega equals 1 divided by square root of alpha times T that is going to equal to 1 by square root of alpha ok.

So, that is something I leave it to you as homework ok, pretty straight forward right.

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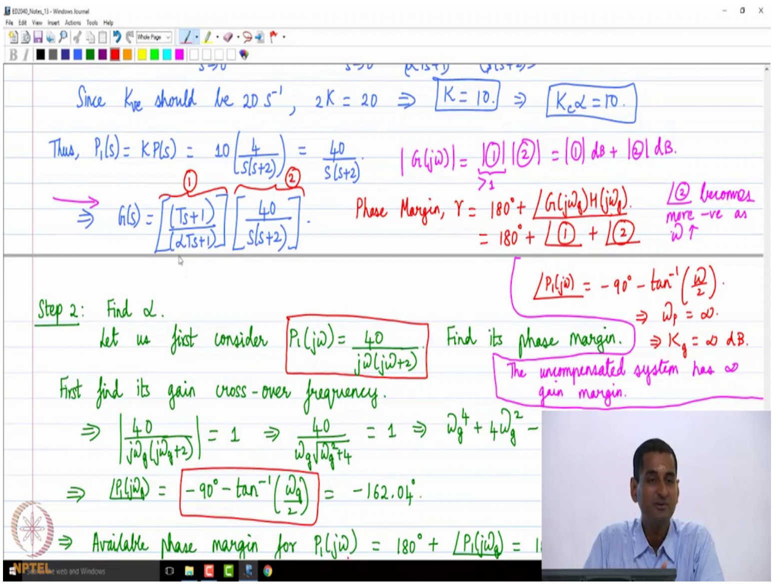

So, what is this magnitude I am calculating? I am calculating the magnitude of the sinusoidal transfer function corresponding to T s plus 1 divided by alpha T s plus 1 that is what I am doing right, that is this ok.

So, this I leave to you as homework pretty straight forward you just substitute the, what you say, the frequency at which you get the maximum phase right and please note that this should be our new gain cross over frequency, why? Because our, what to say lead compensator is going to provide the maximum phase at omega equals omega m and that is what I want as the new gain cross over frequency would I not why? Because that is when my phase margin would be most positive right.

So, we want, we want omega m to be the new gain cross over frequency. So, what is G of s, it is alpha sorry T s plus 1.

Student: Yes.

Divided by alpha T s plus 1 times 40 divided by s times s plus 2 right. So, this is what we already have. So, consequently G of j omega is going to be equal to 1 plus j T omega divided by 1 plus j alpha T omega multiplied by 40 divided by j omega times j omega plus 2.

So, we want omega m to be the new gain cross frequency ok. So, let us say we call, this to be omega G n. Now, at the new gain cross over frequency omega m the value of 1 over, what is it? This particular factor is going to be equal to this value is going to be equal to 1 divided by square root of alpha, which is essentially calculated as 2.05 right.

So, thus this implies that at the new gain cross over frequency omega G n the magnitude of 40 divided by j omega G n times j omega G n plus 2 should be what see at the, what to say frequency omega m, which should be the new gain cross over frequency. The magnitude of the first term is 2.05 right. So, I want to find, what is what should be the magnitude of the second term. So, that the product becomes 1. So, what should be the magnitude of second term.

Student: 1 1.

Should be the reciprocal of this term right. So, this should be 1 divided by.

Student: 2.

2.

Student: 0 0.

05, is it clear pretty straight forward. So, you use this to solve omega G n to be 8.9456 radians per second ok. Please, check it out ok. So, once again this I leave it to as one intermediate homework ok, pretty straight forward, we already done for two cases. So, please do that. So, that is it yeah.

Student: Sir, that sir at least 50 degrees sir.

Student: And working out we should use the equality or based on.

So, we are just, we are just adding a factor of safety and we are essentially dealing with equalities yes ok. It is a you are right, it says at least 50 degrees, if you want you can work with inequalities. So, but what we are doing is that like ah.

So, in order to get specific values, you know like for design purpose. You know like we just, what to say put some additional buffer and then or we are working with equalities ok. Finally, we will come back and check that I am going to leave it you as homework.

So, we you have to go back and check whether it is satisfying the performance requirements right, So, we are almost there right.

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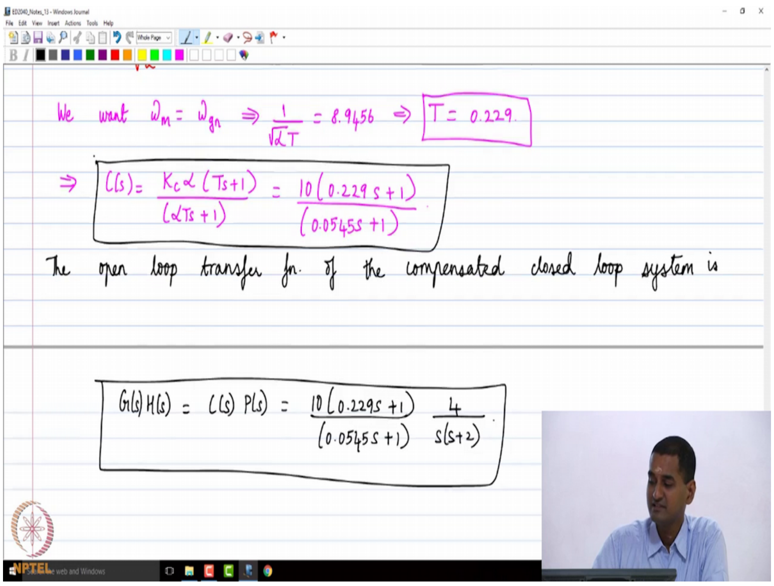

So, now, note that we want omega G n oops, we want omega m. What is omega m? Omega m is the frequency, where the lead compensator provides the maximum phase ok, to be omega G n and what is omega m? 1 divided by square root of alpha T.

So, this implies that 1 divided by square root of alpha T should be equal to 8.9456. So, from here you can get T, you will get T to be 0.229, because alpha you already know right, alpha value has already been calculated right, it was 0.238. So, you plug it in and then like you substitute and then like, you will get the value of capital T to be 0.229.

So, that is it we are done right. So, this implies that the controller transfer function is which is K c alpha times T s plus 1 divided by alpha T s plus 1 that is going to be equal to 10 times 0.229 s plus 1 divided by 0. not 545 s plus 1 ok.

So, this is the controller transfer function. So, then the open loop transfer function of the compensated closed loop system, closed loop system. What is that going to be is going to be G of s times H of s, which is going to be nothing, but C of s times.

Student: P of s.

P of s right, because H of s is 1 C of s is this 10 times 0.229 s plus 1 divided by 0. not 545 s plus 1 and the plan transfer function is 49 sorry, 4 divided by s times, s plus 2 ok. This is the open loop transfer function and this is the controller transfer function ok. This is the, lead compensator that has been designed ok.

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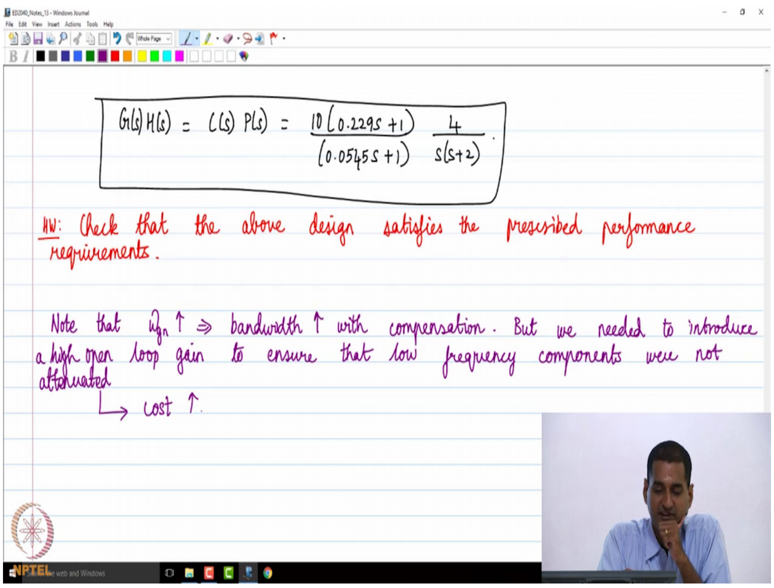

So, as homework check that the above design satisfies the prescribed performance requirements ok, so that is the check which I want you to do. So, you go back and check whether K v is, what did we want K v to be 20 right correct. So, you can immediately see that K v is 20. You know you multiply by a sub stake limit extending 0 K v is 20 right.

So, check whether gain margin is at least degrees and phase margin is at least 10 decibels please, do that as, homework ok. You will see that, the design will be a specified right. So, but you can make one more observation right. Please note that the value of omega g n has increased, which would mean that the band width of the system as increased right. So, but then what was your observation if the band width increased.

Student: Sir (Refer Time: 15:40).

So, note that omega g n increased implies the bandwidth as increased with compensation ok, but what is the price we need to pay.

Student: (Refer Time: 16:10).

So, the price was that, if you go back and think about it, but we needed to increase the open loop gain ok, open loop. What do I mean by open loop gain is, if you substitute s equals, what you say you take the steady state value ok., sorry, you take K v e right.

So,, you initially, we had it as two for the plan right. We had to increase it to 20 right to essentially, ensure that the low frequency components and where not attenuated. So, that that is something which, which we discussed yesterday right, because with a, if you recall, if you go back to yesterdays, class. We saw that one, limitation with the lead compensator was that the low frequency components where attenuated right.

So, you see that the low frequency components where attenuated to. In fact, what to say, essentially address. This we need to push this curve up. So, that is why we had an extra factor of 10. You remember, we had a gain K, you know like; so, which was 10 right.

So, we needed to introduce I would say, we needed to yeah introduce a, a high open loop gain ok, to ensure that low frequency components were not attenuated, but what is the flip side.

So, that is why we calculated K to be 10, if you recall right, the first step right. So, this has two effects. One is cost increases when you increase the gain of the system. Second thing is also the high frequency noise gets slightly amplified right. So, that is that is something which we need to remember right.

So, I just want to reiterate those two points. Once again right, through this example fine. So, this is an example which essentially shows you the process for designing a, lead compensator. We will stop here for, this class and when we come back, what we are going to do is that like, the, final theory class. What we are going to learn, is about, lag compensator and lag lead compensator ok. I am just going to tell you what is the, purpose and structure of those compensators and then we will also see you know like, when to use them right, then we will recap the entire course once in the next class fine.

Thank you.