Control Systems
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Lecture – 69
Lead Compensator Design
Part – 1

So, let us get started with today's class. So, today we are essentially going to look at an example of how to design a Lead Compensator ok.

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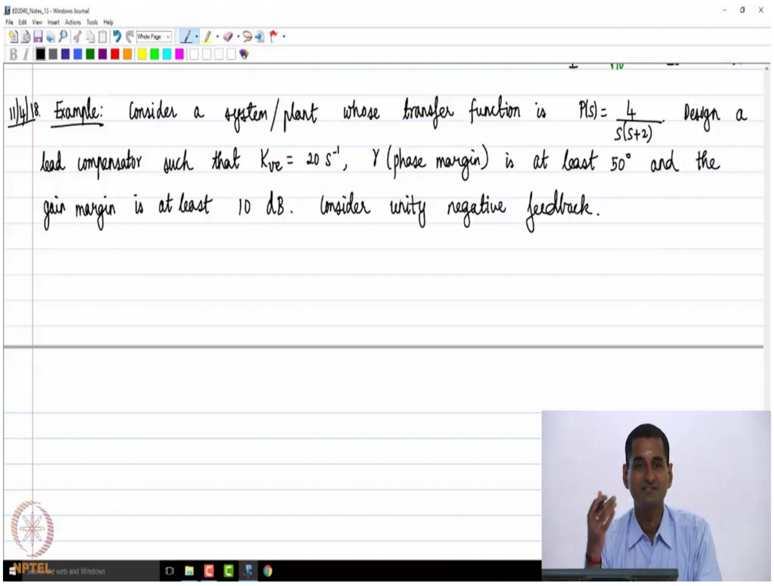

That is going to be our exercise for today. So, let us do that design from the start right. So, that is what we are going to do ok. So, let me write down the problem statement, then we will get started with the design process ok.

So, the example that we are going to consider is that like we consider a system or a plant whose transfer function is the following ok. So, we consider a system or a plant whose transfer function is P of s that is given as 4 divided by S times S plus 2 ok.

So, that is the plant transfer function ok. So, what we need to do is that we need to design a lead compensator such that the static velocity error constant K v e; if you remember what was K v e that is the static velocity error constant right.

So, that is 20 the phase margin gamma that should be at least 50 degrees and the gain margin is at least 10 decibels ok. So, and of course, consider unity negative feedback ok. So, that is what we want to do.

So, we can have unity negative feedback ok. So, that is the problem statement right. So, as far as the design of this particular lead compensator is concerned, so, we have a plant to begin with you know which is whose transfer function is 4 divided by S times S plus 2 right. So, do you think the plant is stable or unstable?

Student: Marginally stable.

Or marginally stable?

Student: (Refer Time: 02:54).

Of course,. So, it depends on how you look at it right. So, we you immediately observe that there are pole the plant poles are it minus 2 and 0 right. It has the pole at the origin. So, you give a step input anyway the output is going to become unbounded right. So, you call it either margin is stable or unstable also right because the definition of BIBOs stability means that the output should be bounded for all possible bounded inputs right.

So, essentially we want to stabilize the close loop system right. So, that is also main objective. At the same time, you know like we also want to ensure that some performance specifications are met right; when we design controllers or compensators using frequency domain techniques, you know like as we discussed yesterday, the performance parameters are going to be specified in terms of gain margin, phase margin; I know like steady state error characteristics and so on right.

Here, K v e essentially is the static velocity error constant if you remember right, we will define that once again and so that essentially tells us you know like what will be the magnitude of the steady state errors, you know like when we give a step input or a ramp input and so on right.

So, that is something which we would use for our design process ok. So, now in order to get started, you know like please remember that these our basic feedback diagram ok, let me draw it here.

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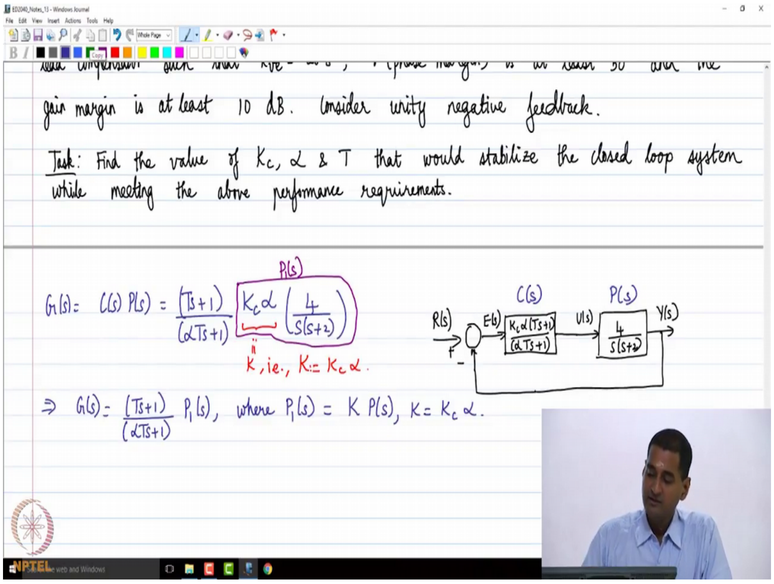

So, the plant transfer function is going to be 4 divided by S times S plus 2 right. So, this is U of s and we get a y of s out of it right.

So, what do? We want to do we want to design unity negative feedback system. So, this is R of s the reference inputs. So, we have negative feedback. The feedback path transfer function H of s is 1 and then, now we need to design a lead compensator here right.

So, what is the transfer function of a lead compensator? It is going to be K c alpha times T s plus 1 divided by alpha T s plus 1 right. So, that is the transfer function of a lead compensator.

So, the theta sk for us is to find the values of K c, alpha, and T that would stabilize the close loop system while meeting the above performance requirements right. So, that is the problem statement that is task before this right.

So, that is what we need to do when we solve this problem right. So, let us lets go step by step ok. So, what we are going to do is the following. So, let us lets do it step by step.

So, please note that G of s which is going to be equal to C of s times P of s. So, please remember this is our controller transfer function. This is our plant transfer function right. So, I am using the same what to say notation as we have been doing through the course right.

So, the controller transfer function is going to be K c alpha times T s plus 1 divided by alpha T s plus 1 and I am just rewriting in this form; I am just putting K c alpha here and the plant transfer function is going to be 4 divided by S times S plus 2 right. So, that is what we have.

So, this is C of s and this is P of s right. Let us call this K c alpha is defined as some K that is the parameter K is defined as K c times alpha ok.

So, this would essentially tell me that the forward path transfer function that is G of s which is the product of the controller transfer function and the path plant transfer function is going to be T s plus 1 divided by alpha T s plus 1 times P 1 of s ok; where, the transfer function P 1 of s is going to be equal to K times P of s ok. So, that is what we have.

So, what are we taking as P 1 of s, this particular block ok. So, this is my it is what we call as P 1 of s right. So, I am just rearranging the terms you know that is that is pretty straight forward ok. So, of course, here please note that you know like here K is essentially K c alpha right. So, that is something which we already know.

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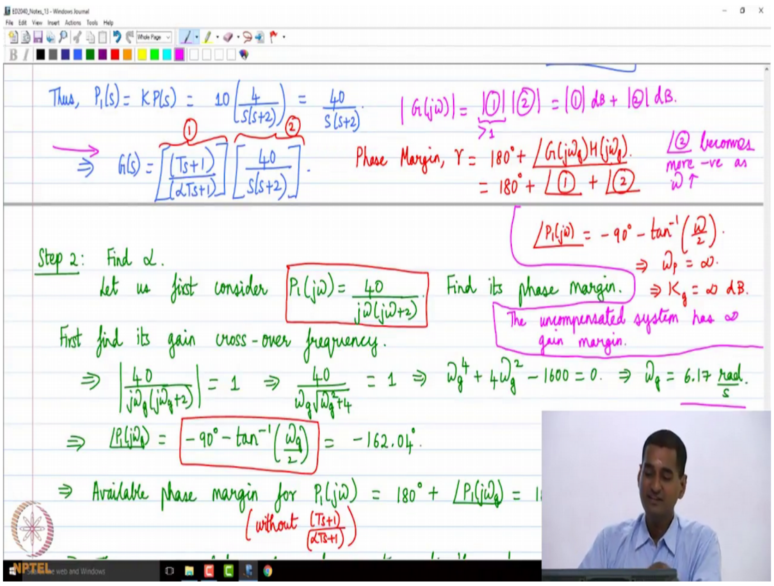

So, having rewritten in this form let us proceed and see what we do ok. So, the step first step is to adjust K right such that the desired value of K v e is obtained ok; so that is the first step. So, in the first step what we do? We adjust this K this parameter K so, that we what to say get that desired value of K v e which was essentially specified as 20 right.

So, recall that what was K v e? If you recall our discussion on steady state error characteristics, K v e we defined when we were dealing with the you know like errors to a units ramp reference input do you recall ok. So, that was essentially defined as.

Student: (Refer Time: 09:24).

Limit extending to 0 s times G of s times H of s right. So, here H of s is 1 in this particular problem ok; please note that we are considering unity feedback right. So, H of s is 1 in this problem right. So, this is the definition of K v e. So, what will we have? We will have this to be limit extending to 0 s times; G of s is going to be this.

So, we are going to get T s plus 1 divided by alpha T s plus 1 times K times 4 divided by s times s plus 2 ok. So, that is what we have. So, now what is going to happen? What are we get if we substitute the limit extending to 0, take the limit extending to 0, we are going to get?

Student: 2 K.

2 K right? That is what we are going to be left with. Please note that this s and s will cancel and then, like we are going to be left with 2 K. So, since K v e should be 10 second. So, we immediately get that 2 K should be equal to sorry not 10 right 20 right.

So, 2 K should be equal to 20 that implies that K should be equal to 10 right; that is what we have. So, the value of K should be 10. So, this implies that K c alpha is nothing but 10 ok. So, this is it is what we get ok.

So, you see that we are using that required value or the performance specification on K v e to get the value of K right. So, once we do this let us let us rewrite the open the forward path transfer function and P 1 of s.

So thus, we can immediately see that P 1 of s which is K times P of s that is going to be equal to 10 times 4 divided by s times s plus 2 that is going to be equal to 40 divided by s times s plus 2 ok. It will become clear, why I am doing this ok.

So, let us let us look at this way. So, this implies that G of s is going to be equal to T of s plus 1 divided by alpha T s plus 1 ok. This is going to be one block ok. So, times 40 divided by s times s plus 2. This is the second block ok. So, I hope everyone agrees ok. So, I hope things are clear till this point right.

So, what have we done? We have essentially consider the structure of a lead compensator we are just integrated that with the plant transfer function which was been given to us and calculating the open loop transfer function and equating it to the desired value of K v e, we have found the value of the parameter K which is K c alpha and ok. So, we have rewritten the what to say essentially open loop transfer function in this way.

Please note that this is the open loop transfer function right, why because H of s is 1. Now what is the next specification we have been given? So, what we have done is that we have now essentially use this specification right K v e equals 20 per second right. Now, we are given that the phase margin should be at least 50 degrees.

How did we calculate the phase margin for this class of negative feedback systems? We have to essentially calculate 180 degrees plus the phase of the open loop transfer function and the gain cross over frequency right. So, that is the equation for calculating the phase margin right if you recall ok.

So, the phase margin should be at least 50 degrees. So, you immediately see that the phase margin, gamma is going to be 180 degrees plus the phase of the open loop transfer function at the gain cross over frequency right. This is the general formula right. In this case H of s is 1. So, I am just considering only G of H. Now, you see that the phase of the open loop transfer function comes from 2 components. What is the first component? It is this. T s plus 1 divided by alpha T s plus 1 ok.

The second component is this ok. So, that we have already seen this in the previous class that the lead compensator provides the positive phase right. Is it not? So, the factor T s plus 1 divided by alpha T s plus 1 would provide a positive phase to the open loop transfer function ok. So thus, it will essentially increase it is phase margin right; but the question is that by how much should I increase the phase margin right.

For that I should figure out what is the contribution of 40 divided by s times s plus 2 the phase margin. See what is the, suppose if I do not have a what to say any compensation, what is the phase margin of the uncompensated system right? How do I find that? I essentially find what is the phase margin which I will get if I just consider 40 divided by s times s plus 2 ok.

So, that is what we were going to do now. So, if I calculate what will be the phase margin which I will get with just 40 divided by s times s plus 2. I can know how much additional positive phase, I need to provide by using the first factor T s by plus 1 divided by alpha T s plus 1 because we know that when we have product of 2 transfer functions right, the net phase is going to be the sum of the 2 individual blocks ok. So, we already know that ok. So, that is what we are going to do.

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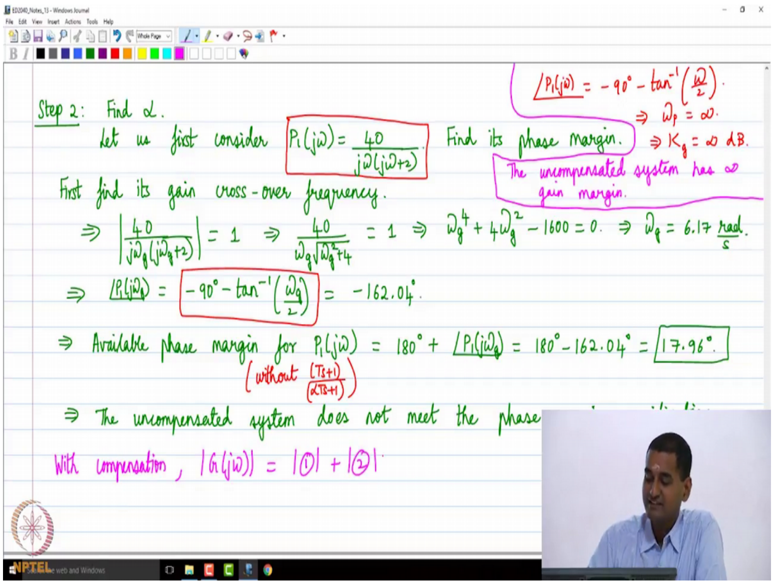

So, the second step is to essentially do this and find alpha ok. So, the second step is to find alpha ok. So, for that let us first consider P 1 of j omega which is 40 divided by j omega times j omega plus 2 right. Find its phase margin ok?

So, that is what we are going to get? So, why are we doing this? Because you see that immediately, this phase margin is going to be if I split this into 2 factors, this is going to be the phase margin of this factor 1 plus phase margin sorry phase of factor 2 right. Let us first figure out what is 180 plus 2; then we will know what should be 1.

So, that I achieve at least gamma of 50 degrees that is that is the or idea here right. So, let us find its phase margin. So, how do I find the phase margin of this factor? What should I first do? I have to find the gain cross over frequency for this factor right. So, I will find the gain cross over frequency of this particular factor? First, find it is gain cross over frequency omega G. So, what is the definition of gain cross over frequency?

It is the frequency at which the magnitude of this particular sinusoidal transfer function becomes 1 right. So, that is the definition. So, this implies that by definition 40 times the magnitude of j omega g j omega g plus 2 should be 1 or in other words, what we should get is 40 divided by omega g times square root of omega g square plus 4 should be equal to 1 right.

We have already done this exercise. So, if you do this you will get omega g power four plus four omega g square minus 1600 is going to be equal to 0 right. So, this something which is, which we already know. So, if you simplify this, you will get omega g to be equal to if you do the calculations and simplify this you will get omega g to be around 6.17 radians per second ok. So, that is what you will get ok.

So, this will imply that the phase of P 1 j omega g is going to be equal to minus 90 degrees minus tan inverse of omega g by 2 right. So, that is something which we already know right.

Because 40 is not will provide a phase of 0 1 by j omega will provide phase of minus 90; 1 by j omega plus 2 will provide a phase of minus tan inverse of omega g by 2 right. So, this is something which we already know right. If you calculate this you will get the phase of this what to say factor P 1 j omega at the gain cross over frequency to be minus 162.04.

So, this implies that the available phase margin for P 1 j omega which is the on compensated system. Suppose, if I do not multiply the second factor T s plus 1 divided by alpha T s plus 1. So, what this is what this means is, it without right T s plus 1 divided by alpha T s plus 1 ok.

So, that is going to be equal to 180 degrees right plus P 1 the phase of P 1 and j omega g that is going to be 180 degrees minus 162.04 ok; that is going to be how much is that? That is 17.96 degrees.

So, that is close to around 18 degrees right. How much phase margin do we want? What is the performance specification? At least 50. So, are we meeting the phase margin specification? No. So, immediately see that the uncompensated system does not meet the phase margin specification ok. So, that is what is happening.

So, this implies that the uncompensated system does not meet the phase margin. Sorry, the phase margin specification ok. Does uncompensated system satisfy the gain margin specification?

Of course, will continue with this, but before we carry on see what was the third requirement the gain margin should be at least what was that? Should be at least 10 decibels right. So, that was the thirds performance specification. Does the uncompensated system meet the gain margin specification?

. So, what do we mean by uncompensated system once again? P 1 of j omega. You will immediately see that the phase crossover frequency for this is infinity. So, the gain margin is once again going to be infinity decibels right.

So, it does meet the gain margin specification; but it does not meet the phase margin specification ok. I am not repeating it because we have already done that done this as the example right in the previous class. Because you see that here this is the second order system right which does not have any pole or 0 in the right of plane.

So, immediately see that the phase cross over frequency is going to be infinity right. Because what is phase cross over frequency? The frequency at which the phase becomes minus 180, when will the phase become minus 180? This is the phase right. You forget omega g in general the phase of P 1 j omega is going to be minus 90 minus tan inverse of omega by 2.

So, this implies that the phase crossover frequency is infinity right because that is going to become minus 180 only at infinity. So, this implies that the gain margin of the uncompensated system is going to be infinity decibels right.

So, that is what it is right. Because when you go to infinite frequencies that is higher frequencies the value of the magnitude plot log magnitude curve of P 1 j omega is going to go towards negative and negative value right. It is going to 10 to minus infinity; why because the slope of the curve will go away as minus 40 decibels per decade. You can immediately see that n is 2; m is 0 right, for this particular plant transfer function right.

So, the slope of the log magnitude curve will go like minus 20 times n minus 1 decibels per decade that is something which we already seen. So, the slope of the log magnitude curve will go as minus 40 decibels per decade. So that means, that essentially the value of the magnitude of P 1 j omega goes to minus infinity decibels as omega tends to infinity and gain margin in decibels is that negative of that. So, gain margin tends to plus infinity right.

So, you see that the uncompensated system does satisfy the what to say if a gain margin requirement right. So, this, essentially this is an assign. So, the uncompensated system has infinite gain margin ok. So, but then the phase margin is not sufficient right so that is why we need to do some design here alright. So, let me put it this way ok. So, within a box ok, essentially we do not meet the phase margin specification.

So, now what should be the minimum phase that the factor T s plus 1 divided by alpha T s plus 1 should add as a lead compensator to ensure that the phase margin goes to at least 50 degrees? See the factor 2, P 1 of s has a gives a margin total that is 180 plus 2 is what 18 degrees around 18 degrees. So, what should be 1? So, that phase margin is at least 50 fifty minus 18 right; what is that?

32 right; around 32 degrees is what should be the phase contribution of the factor T s plus 1 divided by alpha T s plus 1. But there is a catch here right; what is the catch here right? Let us let us go back and look at the bode diagram that we constructed yesterday.

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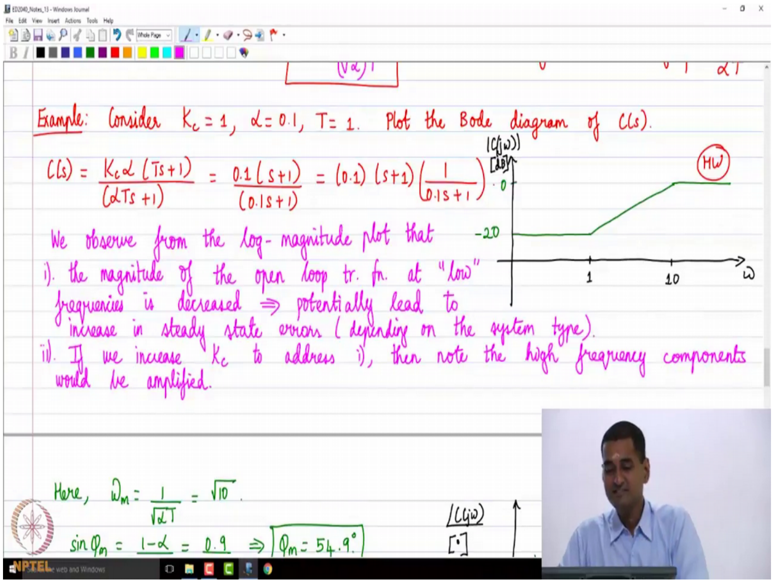

See yesterday, you know like if you forget this factor point 1, right and just consider T s plus 1 divided by alpha T s plus 1; what do you observe? You see that the magnitude curve is going to start at 0 decibels and then like it is going to increase and then it is going to settle down 20 decibels alright at steady state right. So, as omega tends to infinity and we are going to get the maximum phase at the geometric mean of that 2 corner frequencies this is something which we have already seen.

Now, what is going to happen at this geometric mean? You immediately see that this factor T s plus 1 and alpha divided by alpha T s plus 1 would have already added some magnitude, some positive decibels right because the magnitude of a T s plus 1 divided by alpha T s plus 1 will not be 0 right. It will be some small positive; finite positive number.

So, what is going to happen is that the open loop transfer function which is now the product of factors 1 and 2 will essentially be influenced by this magnitude of 1 which will come and consequently the gain crossover frequency would be slighted through the right. The gain crossover frequency would increase.

What is the definition of gain crossover frequency? It is the frequency at which the magnitude becomes 1 or 0 decibels right. For the uncompensated system, the gain crossover frequency was 6.17 radiance per second ok.

Now, with compensation, you immediately see that the magnitude of G of j omega is going to be the magnitude of factor 1 plus magnitude of factor 2. What I call as 1 and 2 here right.

Now, you see that the magnitude of factor 2 has a gain crossover frequency of 6.17 radiance per second; but at that frequency the magnitude of 1 is not 0 right because this going to some slight positive decibels right because that is that is the frequency at which I want to add some phase right.

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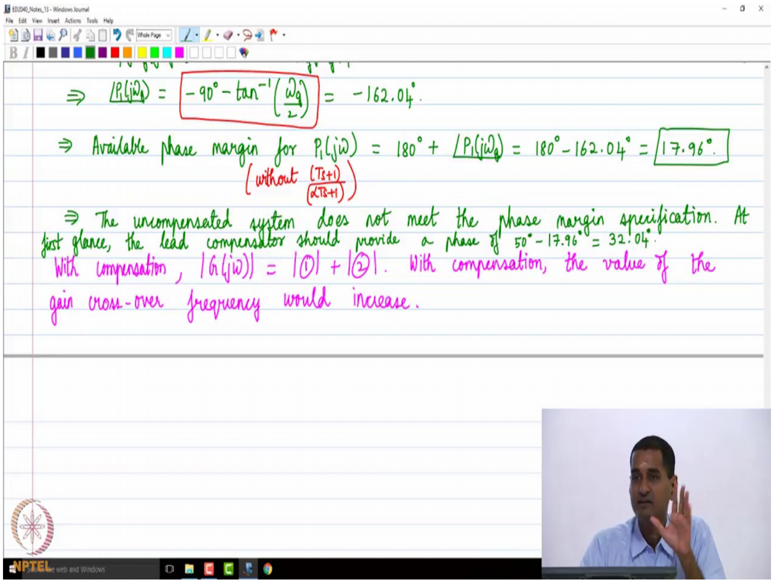

So, you immediately see that with compensation, the value of the gain crossover frequency would increase; would increase and if this value of gain cross over frequency increases, you immediately see that the phase of this factor 2 will become more negative at the increased gain cross over frequency. So, all though we may want to provide a 50 degree phase margin the effective phase margin would be lower because of the shift in gain crossover frequency ok.

So, let me repeat these points once again ok. Let us go through these points once again. So, what have we done? So, I the open loop transfer function is this right. So, everyone is convinced about that right. So, now, there are two factors 1 and 2 right. Two is what we call as the Transfer function of the compensated system because it is not going to effect the phase margin as such right. So, you see that for the compensated system we calculated a gain crossover frequency and a corresponding gain margin right ok.

So, now we have considering the we are considering the compensated system. In the compensated system this factor 1 comes into play right. T s plus 1 divided by alpha T s plus 1. Now previously, we consider only 40 divided by s times s plus 2; for that factor the gain crossover frequency was 6.17 radians per second.

With this additional factor which will contribute as positive magnitude in decibels; that means that the gain cross over frequency of the compensated system would increase. I am sure all of us agree right because the gain crossover frequency is going to be the frequency at which the magnitude of the open loop transfer function is 1 right.

So, now you see that the magnitude of G of j omega is going to be magnitude of 1 times magnitude of 2 or magnitude of 1 in decibels plus magnitude of 2 in decibels right. Previously, we equated the magnitude of 2 to 1 and then, got 6.17; 6.17 radians per second as the gain cross over frequency. But, now I am I am essentially introducing a new factor 1 whose magnitude is going to be greater than 1 at this frequency.

So, now what is going to happen? Consequently, this would imply that the gain crossover frequency of the new system would increase right. Is not it? Because I am multiplying the magnitude of 2, with the number, which is greater than 1 right.

So, at 6.17 radiance per second, the magnitude of 2 is 1; but the magnitude of 1 will be greater than 1. So, consequently it will no longer, it will no longer be the gain crossover frequency of the compensated system right for which the open loop transfer function is the product of 1 and 2 right.

So, but if the gain crossover frequency increases what will happen? Note that the phase of 2 becomes more negative as omega increases right. So, phase contribution of factor 2 right. Now with the new gain crossover frequencies is more, the factor two will be more negative.

So, if I just keep the phase contribution of the lead compensated to be 32 degrees, I am going to be still not be in a position to meet the phase margin because it will be less than 50 because what has happened the gain crossover frequency has shifted to the right. So, consequently the phase contribution of factor 2 has become more negative. So, the gain margin would reduce.

So, what we do is that as a rule of thumb, we essentially say that the uncompensated system does not meet the phase margin specification and at first glance, the lead compensator should provide a phase of 50 degrees minus 17.96 degrees right which is going to be what 31, 32.04 degrees that is what we can observe at first glance right.