Control Systems
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Lecture – 68
Lead Compensation
Part – 2

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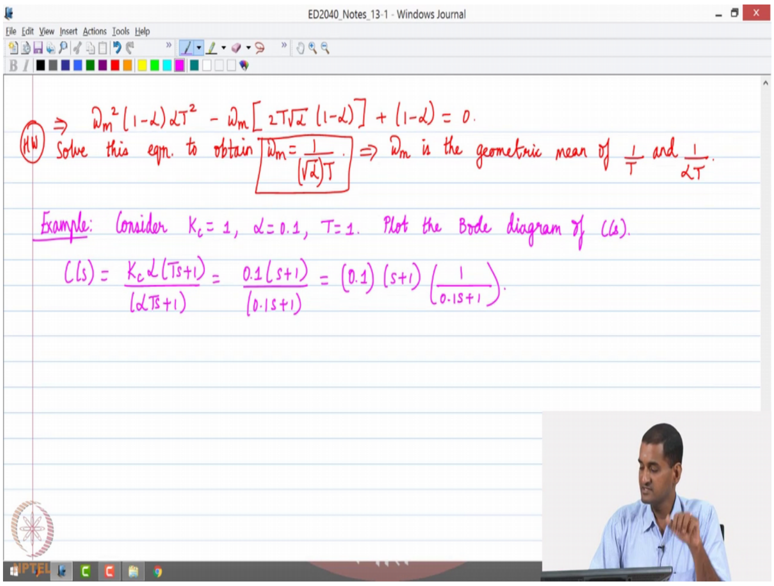

This is the expression for; this is the expression for the frequency at which we get the maximum phase. So, let us do an example ok let us uses entially take some values for the parameters involved in the lead compensator. So, you see that in the lead compensator, there are three parameters which have to be figured out, its K c, alpha and capital T right.

So, those are the three parameters involved in the controlled transfer function. So, let us, let us say we consider k c to be 1 ok. We consider alpha to be let us say 0.1 and T to be equal to 1.

So, let us plot the Bode diagram of this controller transfer function C of s ok. So, the exact Bode diagram I will leave it to you as homework, but the construction process, because by now we are familiar with how to construct bode diagrams so, but I am just going to give you the answer and also discuss some interesting observations from that right.

So, with these values, please recall that the controller transfer functions of a lead compensator is K c alpha times T s plus 1 divided by alpha T s plus 1. If we substitute these values we are going to get 0.1 times, T is also 1. So, I will get s plus 1 divided by 0.1 s plus 1 right.

So, this can be rewritten as 0.13 factors and then s plus 1, then 1 divided by 0.1 times s plus 1 right. So, those are the three factors that are involved in this particular transfer functions. So, now, if I want to plot the log magnitude plot right for the bode diagram.

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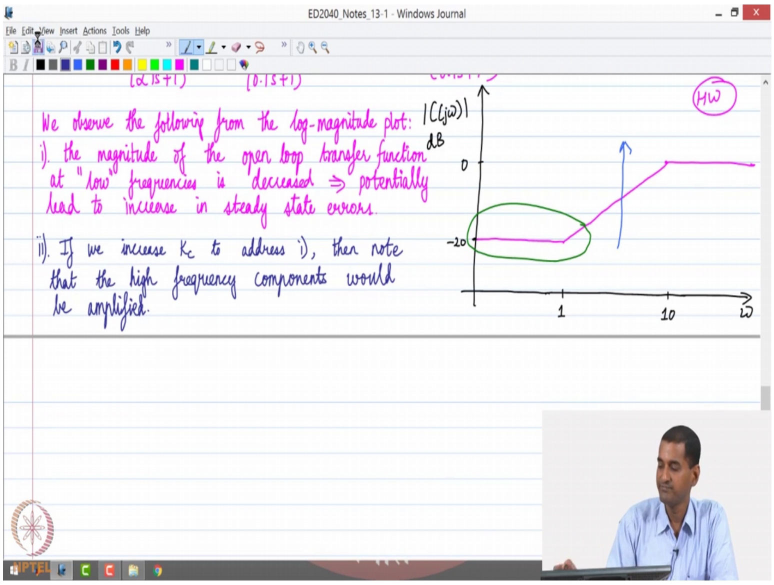

So, let us do that right. So, let us plot the log magnitude plot. So, we can immediately observe that the magnitude of c of j omega if I plot it in decibels. So, the two corner frequencies are 1 and 10 alright.

So, that is something which we can immediately observe right those are the two corner frequencies and we can immediately observe that the Bode diagram is going to look something like this ok. I am drawing the final bode diagram; of course, only the asymptotes.

So, we are, we are going to have a contribution of minus 20 due to the fact that 0.1, tell the first corner frequency of 1, then I we are going to have a straight line with the slope of plus 20 decibels per decade between the two corner frequencies. So, as a result I will go from minus 20 decibels to 0 decibels as long, as for as the log magnitude is concerned.

Then after that I will settle down at 0 decibels, because the factor 1 by 0.1 s plus 1, we will come in to play right. So, that is going to be their log magnitude curve for this particular transfer function right. So, once again you know like plotting the three individual, what to say log magnitude curves and adding them, is left to as an homework assignment, but it is pretty straight forward to see that this will be the final asymptotic log magnitude plot, that is what I am drawing. I am only drawing the asymptotes, not the actual magnitude plot right.

So, one could immediately observe the following. So, we observe the following from the log magnitude plot. So, what do we observe? we immediately observe that this particular lead compensator is going to essentially attenuate or decrease the magnitude of the low frequency components right, because we can immediately observe that the low frequency components of the signal are going to be attenuated by the lead compensator right. So, that is something we can immediately observe right. The low frequencies are attenuated here.

So, consequently the magnitude of the open loop transfer functions. Please note that the open loop transfer function is C of s times P of s right, because we have unity feedback. In general the open loop transfer function is going to be G of s time H of s. Here H of s is one and G of s is C of s times P of s right. So, the open loop transfer function at low frequencies is decreased ok. So, that is, that something which happens.

So, this would immediately imply that this can potentially lead to increase in steady state error right, because please remember, you know like if the low frequency values are reduced the value of k p e k v e and all essentially are effected right, so they go down.

So, as a result you know like the study state errors may increase right, due to what to say this particular aspect, this particular characteristics of the lead compensator. Of course, this depends on the type of the system, whether we have type 0, type 1, type 2 and so on right, which we have already looked at right, depending on the type of the system. The study state errors corresponding to certain inputs may increase with this particular aspect.

Now, if I want to ensure that you know like that is not the case, then what should I do? What I should is that, I should push up this magnitude plot. Let us say up by 20 decibels right for example, all right, considering this particular example. Suppose if I push, shift this entire log magnitude curve by 20 decibels by multiplying by a factor of 10 as far as the controller is concerned, what will happen is that, the low frequency components will now have a magnitude of 0 decibels as far as the controller transfer function is concerned.

So, consequently the values of k p k v e k a e are going to be not affected right. So, the steady state error characteristics will still be retained as far as the original system is concerned, but what will happen, we immediately see that the high frequency components will now be magnified right. There will be amplified, because I have to shift the entire bode plot up by 20 decibels.

So, consequently the high frequency components, gets shifted out by also 20 decibels right in this example. So, consequently the magnitude of the high frequency components will increase right. So, that is something which can be observed from here.

So, we can immediately know that the second observation is that, if we increase K c right to address the first point, then know that the high frequency components would be amplified ok, that is the flip side of having, what to say shifting this factor right.

So, if you, if we essentially increase a value of K c to address this problem we say that the high frequency components are also what to say essentially amplified right. So, that is something which is an observation. Now, if I plot that before plotting the phase is plot right.

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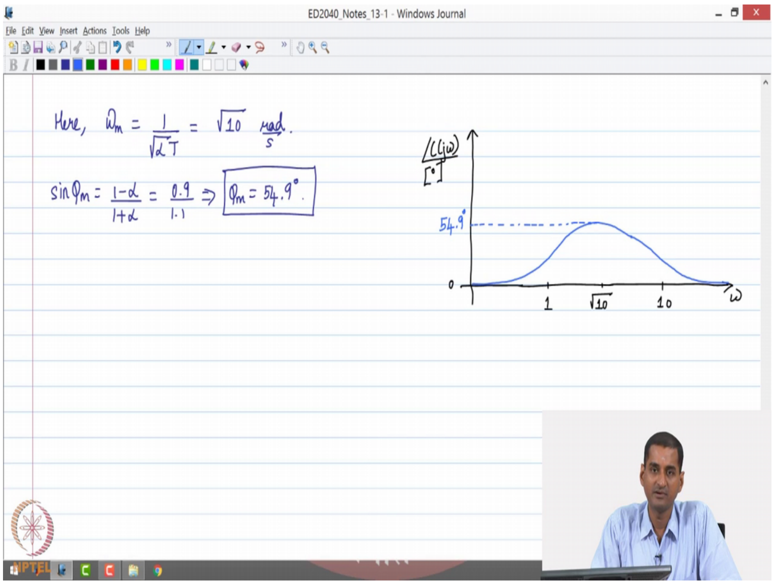

. So, in this example note that omega m is going to be equal to square root of 1 divided by square root of alpha T, which is essentially square root of 10 radians per second right and sin of I phi m is essentially 1 minus alpha divided by 1 plus alpha, which is essentially 0.9 and divided by 1.1. This implies that 5 m is essentially around 54.9 degrees. So, that is the maximum phase which is added by this particular lead compensator ok, the example that we are considered.

So, if I plot phase plot for this particular example right. So, this, this is also something which I leave it to you as some work exercise so, but I am just going to plot the final plot. So, if you plot the phase plot of C of j omega in degrees what we are going to get is, the following. So, 1, this is 10 so on.

A logarithmic scale square root of 10 which is a geometry mean is going to be in the, in between the middle of 1 and 10 right, and let us say we start from 0 degrees and then let us say you know like we reach a peak. So, if we plot the phase plot, what is going to happen is? It is going to look something like this, it is going to start from very low values right and then as frequency increases ok, it will reach a maximum at square root of 10 and then it will start to decrease and ultimately it will go to 0.

So, at square root of 10, whatever is the maximum value that we have calculated to be 54.9 degrees ok. So, that is a maximum phase for this particular lead compensator.

So, to summarize a lead compensator, is typically use to an the given system or plan does not have a sufficient phase margin to begin with. So, what we intent to do is, essentially to add a positive phase to the system, to ensure that we increase the stability margins right, but what was the trade off here. You know like we immediately saw that the magnitude characteristics is like a high pass filter.

So, essentially what happen happens is that, like it amplifies high frequency noise, if we a, if we are not very careful right, but there is a tradeoff once again right in order to not amplify the high frequency noise. We keep the low frequency components at a high lower magnitude then the study state error characteristics are going to be effected right, the study state errors may increase.

So, in order to avoid that, if I push the magnitude plot up, the high frequency components are going to get amplified right. So, there is a tradeoff here, but the, but the good thing is, it will essentially give us a positive phase lead which will increase the face margin of the system.

So, this is the theory behind lead compensator. In the next class we will essentially do a problem, where we will start with the given system, you know like and we will see how to design a lead compensator for a given set of performance, specification, requirements right.

We will go through all the steps and then like calculate the values of K c alpha and T to meet the given performance requirements ok. So, that is that is all with today's class, you know like we will earn and example of lead compensation in the next class.

Thank you.