Control Systems
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Lecture – 67
Lead Compensation
Part – 1

Greetings welcome to today's class. So, we have been looking at frequency response methods in the previous class we looked at stability margins, what was called as relative stability right, and how to essentially characterize system performance using gain margins, phase margins and so on right.

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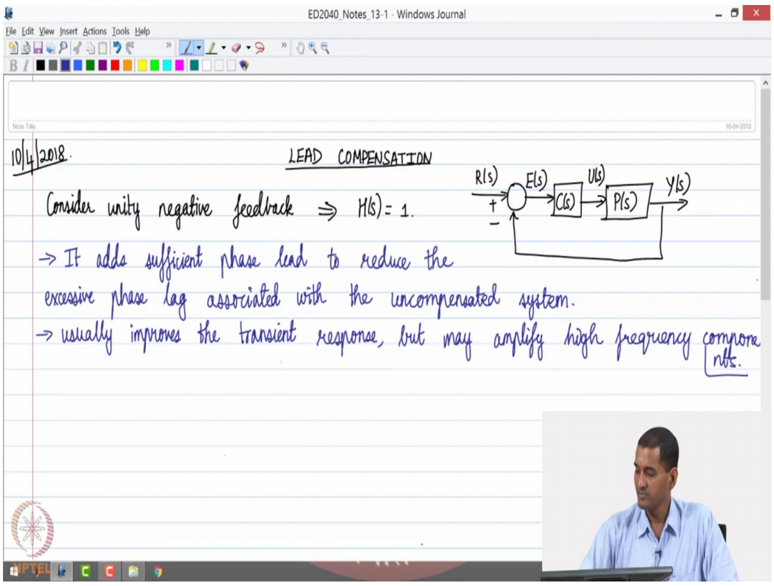

So, in that connection, you know like today we will essentially learn how to design a controller, which is called as a lead compensator using frequency response methods ok so, that is going to be the object of today's class.

So, we will considered this standard feedback loop that we have been dealing with in this particular course. So, we consider unity negative feedback. This implies that the feedback path transfer function H of s is going to be 1, so that is the implication. C of s is the controller transfer function and P of s is the plant transfer function.

So, now what does a lead compensator do right. So, typically what a lead compensator does is that, it adds a sufficient amount of phase lead to reduce the excessive phase lag if any; you know that is associated with the uncompensated system right.

So, excessive phase lag associated with the uncompensated system. See typically you know like we can have a plant, you know like which does not have a significantly high phase margin for example. Then what happens is it the lead compensator adds phase to the system or the plan, and ensures that the phase margins are essentially driven to a level which is acceptable right.

So, that is what a lead compensator does? And by and large a usually improves the transient response, we are going to see how, but it may also amplify high frequency signals ok, so since some cases.

So, it can amplify high frequency components in the output. So, that may be a an issue particularly if you have noise for example, high frequency noise for example. We will see how these points can be addressed right as we go long.

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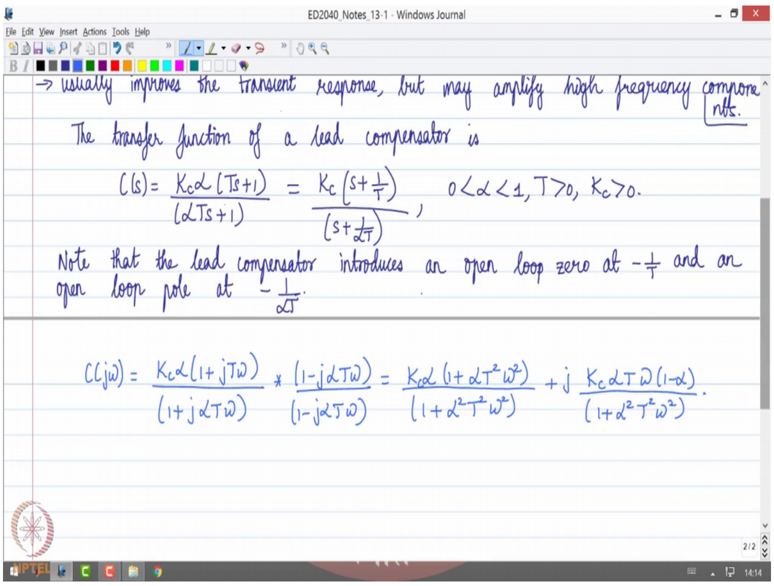

So, let us look at the structure of the lead compensator. So, typically the transfer function of a lead compensator takes a form C of s is equal to K c alpha times T s plus 1 divided by alpha times T s plus 1.

So, this can be rewritten as K c times S plus 1 by T divided by S plus 1 by alpha t ok. So, that is the structure of the lead compensator. So, here alpha is a, what to say positives real number between 0 and 1 and of course, T is greater than 0 and K c is also greater than 0.

So, immediately we can observe that, the lead compensator introduces an open loop pole and open loop 0. So, one can immediately observe that the lead compensator introduces an open loop 0 at minus 1 over t and an open loop pole at minus 1 over alpha T ok.

So, that is what happens in a lead compensator. So, it introduces an open loop zero at minus 1 by T and an open loop pole at minus 1 by alpha T ok. So, now, let us analyze the sinusoidal transfer function associated with this lead compensators. So, essentially we substitute S equals j omega and see what happens right. So, then C of j omega becomes K c alpha times 1 plus j times T omega divided by 1 plus j alpha T omega.

So, this can be rewritten as K c alpha divided by 1 plus alpha square. Of course, we need to do it carefully; this will become K c alpha. Of course, what I am doing is an, let me explain what I am doing here. So, I am just multiplying and dividing by the conjugates. So, if I multiply and divide by 1 minus j alpha T omega, what we are going to get is, the following.

K c alpha times 1 plus alpha T square omega square divided by 1 plus alpha square t square omega square plus j times K c alpha T omega times 1 minus alpha divided by 1 plus alpha square T square omega square ok, so that is what we will get ok.

So, that is the real component and the imaginary component of the sinusoidal transfer function associated with the controller transfer function of a lead compensator.

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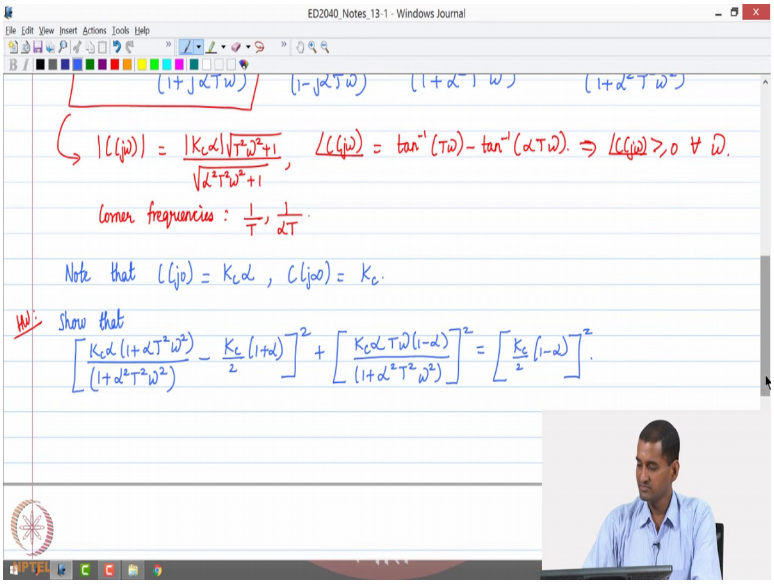

So, before we look in to this, you know like from the. I would say from the function C of j omega we can immediately observe that the magnitude of a C of j omega is nothing, but K c alpha times square root of T squared omega square plus 1 divided by square root of alpha square T square omega square plus 1.

And the phase of C of j omega is going to be tan inverse of T omega minus tan inverse of alpha T omega ok. So, that is what will happen to the a phase of this transfer function C of j omega ok.

So, we can immediately observe that the phase of C of j omega is always going to be greater than or equal to 0 for all omega right, so that is something which we can observe right. So, for all frequencies you know the phase is going to be non negative right. So, that is something which we can immediately observe.

And also if you look at the structure of the transfer function, we can immediately see that the corner frequencies of the transfer function corresponding to a lead compensator are at 1 by T and 1 by alpha T ok.

So, those are the two corner frequencies associated with the lead compensator. So, anyway we will use these corner frequencies in plotting the bode diagram right, corresponding to this transfer function. So, now, if we look at the, what to say real and imaginary component of C of j omega, we can immediately notice that the value of C of j omega at omega equals 0 is just K c alpha right, because the imaginary component vanishes. And as omega tends to infinity once again, the imaginary component vanishes and C of j infinity just K c.

So, we can immediately observe that these the transfer function C of j omega is, sinusoidal transfer function C of j omega starts on the positive real axis for omega equals 0 and ends on the positive real axis at as omega tends to infinity right. So, that is an observation we can readily make from this particular equation right.

Now, this is something which I am going to leave as homework ok, one can easily show that. So, if you take the real component, which is K c alpha times 1 plus alpha T square omega square divided by 1 plus alpha square T square omega square minus K c by 2 times 1 plus alpha whole square, plus the imaginary components square K c alpha T omega times 1 minus alpha divided by 1 plus alpha square T square omega square whole square, that is going to be equal to K c by 2 t times 1 minus alpha the whole square.

So, that is something which we can show, you know like, so I am going to leave this as a home work exercise, pretty straight forward ok. So, one can easily show that this is true right; that is we subtract a K c by 2 times 1 1 plus alpha from the real components square it and then take the imaginary components square it and then add that to resulting quantities, we will get K c by 2 times 1 minus alpha whole square right.

So, this immediately tells us that the locus of the sinusoidal transfer function C of j omega is going to be in the form of semi circle which is going to essentially have its center at on the positive real axis at K c by 2 times 1 plus alpha and having a radius of K c by 2 times 1 minus alpha right so, that is the essentially an equation of a circle right.

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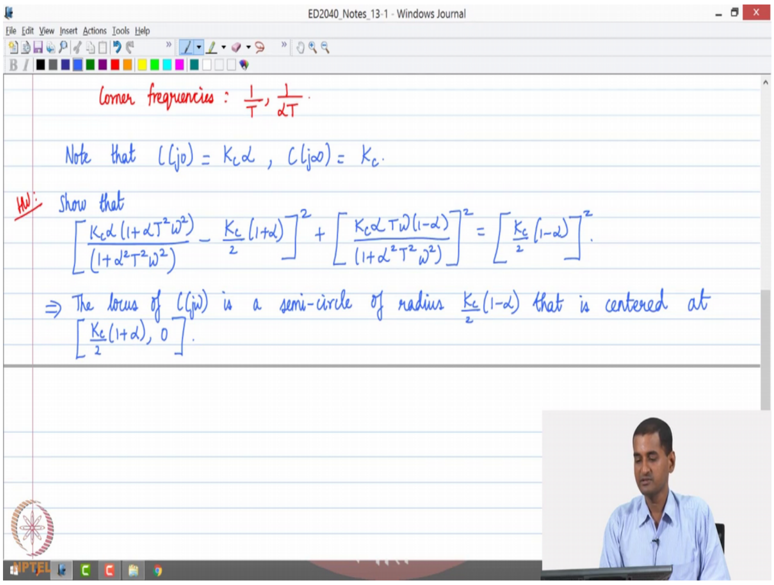

So, we can immediately observe that the locus of c of j omega is a semi circle of radius K c by 2 times 1 minus alpha ; that is centered at K c by 2 1 plus alpha comma 0. So, that is what happens in the case of the locus of C of j omega.

So, that’s, that is the, what to say structure or a shape of the particular transfer function that we are considering ok. So, let us let us plot the Nyquist plot for this particular transfer function. So, if we plot the Nyquist plot ok.

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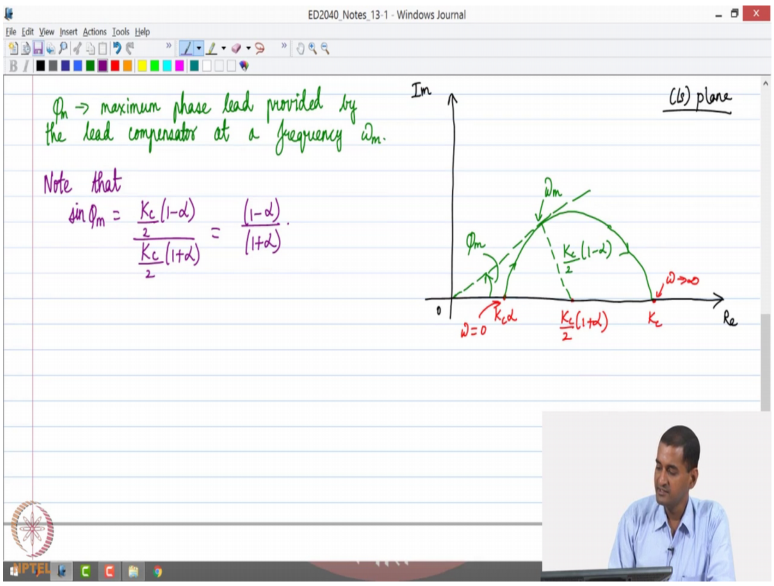

Let me consider the real and the imaginary axis ok. So, we can immediately observe the one thing right, from the real and the imaginary components of C of j omega, we can readily observe that for all omega between 0 and infinity right, the real part is always going to be positive and the imaginary part is also going to be positive, you know like except other limiting values of 0 and infinity right.

So, as a result the Nyquist plot of this particular, function is going to be in the, first quadrant right, in the complex plane. So, let us call this complex plane as the C of s plane.

So, in the C of s plane , we can immediately see that, the value of the sinusoidal transfer function is that K c alpha right, and it goes to K c as omega tends to infinity and, the center of the circle is going to be at the middle. It is a K c by 2 times 1 plus alpha and, what is going to happen is that the, the, locus of this curve, the Nyquist plot is going to be a semicircle ok so, that is what is happening.

So, this is, the Nyquist plot for this particular, transfer function. So, this is where it will start as omega equal 0 and this is where it goes to as omega tends to infinity right. So, let me write it here ok. So, this is the place, where it starts at omega equals 0.

Now,, we can immediately see you observe one thing right. So, if I want to figure out, of course, we can immediately see that, the radius of this, semi circle is going to be K c by 2 times 1 minus alpha right.

So, that is already some, which is known and we can immediately observe, from this Nyquist plot that, the phase of this particular, sinusoidal transfer function is always going to be a non negative right. Of course, this 0 at omega equals 0 and omega tending to infinity for all other frequencies, it is positive right and, the, maximum phase angle, which can be obtained from this particular, transfer function can be figured out by drawing a tangent, from the origin right to this semi circle

And if we look at,, this particular, line segment that is going to be the radius of the, the length of the line segment is going to be the radius of this, particular semi circle right.

So, now, let us, call this angle as phi m ok. So, let me, let me write it here. So, this is phi m. So, what is phi m? Phi m is going to be the maximum phase lead provided by the lead compensator at a frequency, which we denote by omega m.

So, ok, what is omega m? Omega m is this particular frequency, you know like where ever it provides the maximum phase you know like, that is essentially omega m ok. So, phi m is the maximum, phase angle right. So, we can immediately observe that , we can immediately note that, it is pretty straight forward to calculate phi m. We see that sign of phi m is going to be equal to K c by 2 times 1 minus alpha divided by K c by 2 times 1 plus alpha.

So, this will essentially give me 1 minus alpha divided by 1 plus alpha right. So, that is what I am going to get for, phi m right now.

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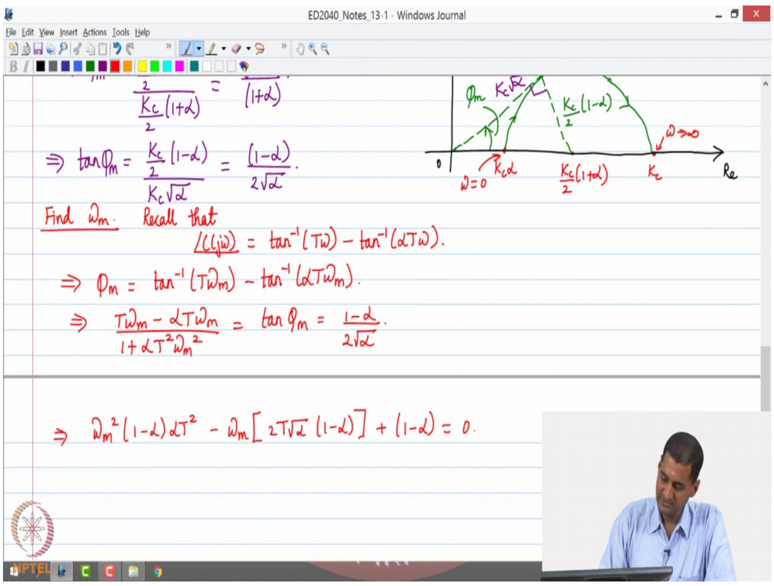

We can also observe that tan phi m, which is going to be the opposite divided by the adjacent side and the length of this adjacent side is going to be K c times square root of alpha right.

So, because this is a right angled triangle, you can use the, Pythagoras theorem to figure out the, length of this,, what is the line segment from the origin to the point, where the tangent touches the semi circle, that is going to be K c times square root of alpha.

So, as a result tan phi m is going to be equal to K c by 2 times 1 minus alpha divided by K c times square root of alpha.

So, this will give us 1 minus alpha divided by 2 times square root of alpha ok, so that is what we will get for tan phi m. Now, as a next step, let us find omega m. So, what is omega m? Omega m is the frequency at which we get the maximum, phase angle right.

So, for that you know, we just, need to essentially go to the expression for the phase of the sinusoidal transfer function C of j omega, please recall that, the phase of C of j omega is going to be tan inverse of T omega minus tan inverse of alpha T omega ok.

So, this implies that phi m is going to be equal to tan inverse of T omega m minus tan inverse of alpha T omega m, because we get the maximum phase angle of phi m, when the frequency is, omega m right. So, that is what we are using.

So, this will immediately tell us that T omega m minus alpha T omega m divided by 1 plus alpha T square omega m square is going to be equal to tan phi m, which is nothing, but 1 minus alpha divided by 2 times square root of alpha right.

So, we just are reusing the trigonometric, identity for expression, you know like, which essentially relates tan inverse of, a minus tan inverse of b right. So, that is what we are, essentially, used right that trigonometric formula has been used.

So, with this expression, we can immediately observe that we can get a quadratic equation and omega m square as follows. So, omega m square times 1 minus alpha times, alpha t square right minus omega m times 2 t square root of alpha times 1 minus alpha plus 1 minus alpha that is going to be equal to 0 ok.

So, that is the equation we are going to get for, omega m ok. So, one could solve this equation.

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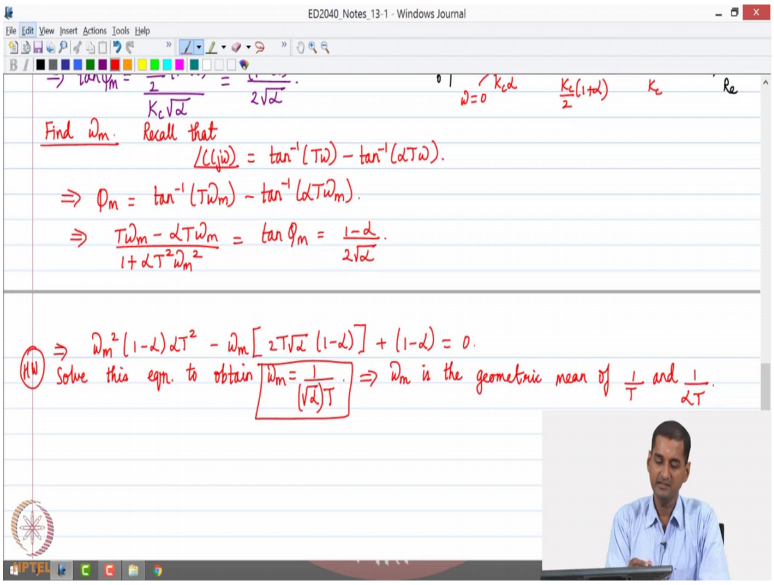

So, solve this equation, to obtain omega m as square root 1 divided by square root of alpha times T ok. So, this is something which I am going to, once again, leave, leave you as a, homework exercise right.

So, we see that the maximum, phase is obtained at a frequency of 1 divided by square root of alpha times T right. Now, if you recall, what were the two corner frequencies right. The two corner frequencies, where 1 by T and 1 by alpha T. So, you we can immediately observe that omega m is the geometric mean of the two corner frequencies right.

So, that is something, which we can, observe, from this expression. So, we can observe that omega m is the geometric mean of 1 by T and 1 divided by alpha T ok. So, that is something which we can, immediately, find out right from this expression.