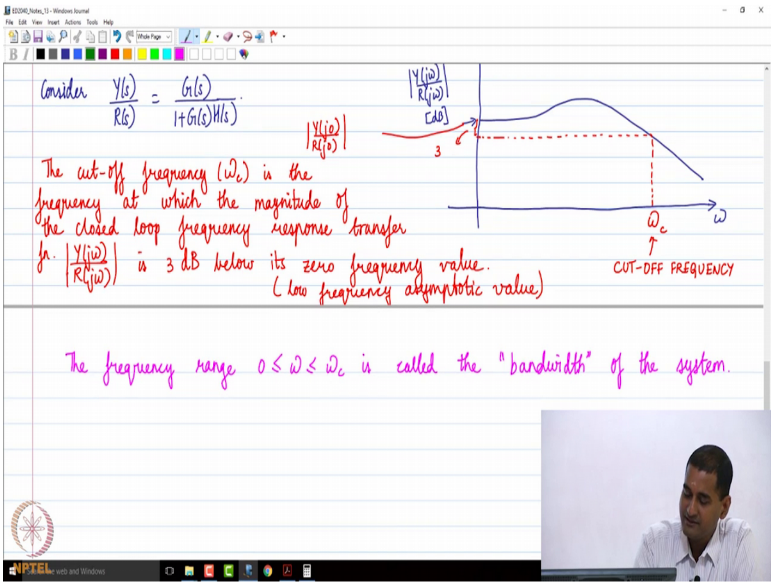
Control Systems
Prof. C. S. Shankar Ram
Department of Engineering Design
Indian Institute of Technology, Madras

Lecture - 66
Relative Stability 2
Part - 2

So, now we are going to define a few more terms and all these terms put together you know like define performance specifications in the frequency domain right. So, let us once again consider the closed loop transfer function right. Consider Y of s divided by R of s to be equal to G of s divided by 1 plus G of s H of s. Of course, we assume that the closed loop system is stable right.

(Refer Slide Time: 00:34)



So, let us say you know like I plot the magnitude plot of the closed loop sinusoidal transfer function ok. So, please note that you know like see one important point which I want to highlight here is a gain margin and phase margin were defined for the closed loop system using the open loop systems frequency response that is very important, right. Kindly recall that we only looked at the transfer function G of j omega H of j omega which was the open loop systems sinusoidal transfer function right to comment on some parameters related to the closed loop system that is something which is critical for us to remember ok.

So, we are done with gain margin and phase margin I am just moving to what to say two more definitions of parameters that are important. So, let us say I plot the magnitude plot of the closed loop frequency response transfer function in decibels. So, let us say you know like the low frequency asymptote are some z some constant value and then it goes like this some arbitrary curve right.

Now, what is this value? It is this value at the low frequency asymptote. So, this value I can call it as the magnitude of Y of j 0 divided by R of j 0. Essentially I am saying that it is the asymptotic value as omega tends to 0. Now what we do is that we take a number which is 3 decibels lower than this number whatever that number is the low frequency gain some people will call it as the d c gain or the low frequency gain and so, on right.

So, essentially you take a number value of the magnitude which is lower than this number by 3 decibels and find out as to at what frequency ok. The magnitude plot the magnitude of this closed loop sinusoidal transfer function becomes lower than the low frequency value by 3 decibels. This frequency is what is called as a cutoff frequency ok. So, cutoff frequency is this frequency. So, let me write down the definition for cutoff frequency.

So, the cutoff frequency omega c is the frequency at which the magnitude of the closed loop frequency response transfer function ok. The magnitude of Y of j omega R of j omega is 3 decibels below its 0 frequency value ok, 0 frequency value is the low frequency asymptotic value that is that is what it is ok. Please note that the low frequency asymptotic value need not be 0 decibels all the time it is just that for the building blocks we considered it was 0 ok, but it need not be zero you would have realized it when we did an example in for the bode diagram right. The low frequency asymptote was non zero decibels it was some nonzero number it can happen no issues at all ok.

So, we are only looking at what is the what to say frequency which is at which the magnitude is 3 decibels lower than the low frequency asymptote value right. So, with this definition the frequency range 0 sorry I would say you know like a frequency range 0 sorry till omega c is called the bandwidth of the system this is an important definition.

So, because you can notice from the magnitude curve right, the system allows frequency components till omega c to pass through it right. Below omega after omega c it sorts of attenuates all the other frequencies right. So, that is what happens right. So, the frequency range 0 to omega c is called as the bandwidth of the system. Now one may ask the question hey what is so unique about this number of minus 3 decibels or 3 decibels this is the difference of 3 decibels. What do you think is unique about 3 decibels you know please find out and tell me ok.

So, why should I take consider 3 decibels as my threshold right; say I like 5 decibels why can I not define omega c as a number frequency at which the magnitude is 5 decibels lower than the low frequency value I am not doing it right. So, I am defending it as 3 decibels lower than the low frequency value. Why 3 decibels?

Student: (Refer Time: 07:22) 1 by root 2 times.

1 by root 2 times.

Student: Sir, regarding that regarding the graph (Refer Time: 07:28) take 1 by 2 1 by 2 (Refer Time: 07:30).

Why you know?

Student: (Refer Time: 07:34).

Say yes, definition I agree I am just asking you what is the physics behind it?

Student: Summing to a root mean square.

Root means square something to do with root mean square.

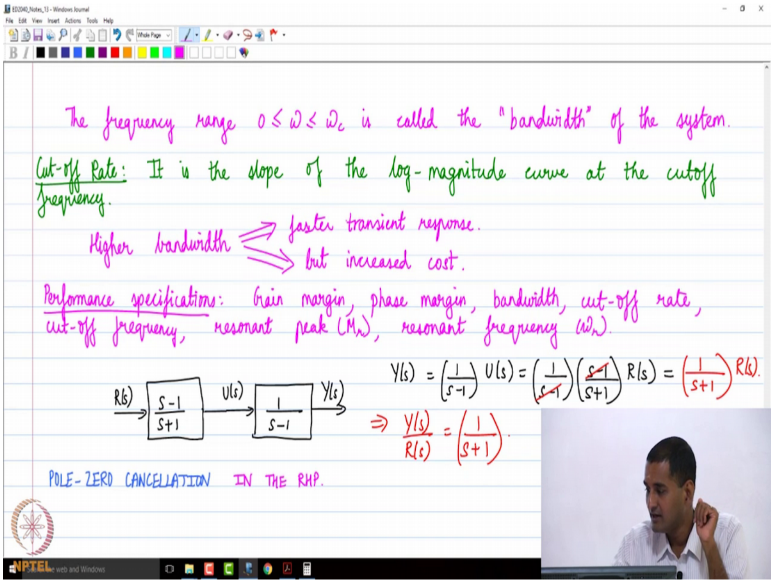
Student: (Refer Time: 07:50).

Sorry.

Student: (Refer Time: 07:53).

Corner frequency; please find out that is the second question for the day right So, please find out and tell me right why that three why a number of 3 decibels right or 3 decibels in this case right; so please ok. So, another parameter which we want to essentially define is what is called as a cutoff rate.

(Refer Slide Time: 08:23)



So cutoff rate it is the slope of the log magnitude curve at the cutoff frequency ok. It essentially tells me how fast the magnitude is decreasing right. So, that is the cutoff rate and by the way you know like ideally we want as high a bandwidth as possible ok. So, it does not matter whether I am designing a practical system or I am building a sensor with some dynamic characteristics or an actuator with some dynamic characteristics, you know like higher bandwidth means you know like you will have a faster transient response right.

So, that is essentially something which is what to say desirable right, so see because suppose if I have a high bandwidth actuator right. So that means, that I ask you to let us say move an object all right then it can do that action much faster right so, than a lower bandwidth actuator, but at the same time leads to increased cost. So, there is always a tradeoff between a cost and performance higher bandwidth typically means that better faster transient response better response characteristics, but then the cost is more ok.

So, that is the price we pay see for example, if you go for actuation systems right if you use an electromechanical actuator you are going to get a very fast response right so, but then the cost of an electromechanical actuator is going to be high complexity is going to be high right. On the other hand if you use a pneumatic actuator the bandwidth of the pneumatic actuator is going to be relatively lower, but the cost is also lower right. So, there is a tradeoff between a cost and performance.

Student: (Refer Time: 10:24).

We will essentially look at it right. So, how is bandwidth related to response? So, we will ok. So, that is essentially what to say a thing and so, once again you know like if you use frequency response methods. So, the performance specifications are typically going to be a given in terms of these parameters ok, that is gain margin phase margin, phase margin and then like bandwidth cutoff rate and then like of course, cutoff frequency ok.

These are all parameters and then resonant peak you remember that M r right for that second order factors that we looked at right the resonant frequency all those things are these parameters these are parameters which can be used.

See in using when you what to say did design using transient response right this using design using frequency response ok, we are going to learn controllers from tomorrow right. So, when we did design using transient response what are the parameters that were used to quantify performance? Rise time, settling time, peak overshoot peak time and so, on right.

Those are all the parameters that we use right. So, when we will use frequency response we are going to look at what to say these parameters the question that arises is whether how are the two related right. So, see how is the bandwidth related to other parameters right? How can I say bandwidth means faster response? See for example, if I show that higher bandwidth means a lower rise time would you agree that the system response faster ok.

So, the there are some relationships you know like we would address them right. So, these are performance specifications in the frequency domain ok. So, I would stop here you know like and we would continue with a controller design using frequency response tomorrow, but I am going to leave you with a question right. So, I hope I we have not discussed this question till now. Let us say right I have this as my system ok, let us say this is my system right 1 by a plant transfer function is 1 by S minus 1. Is it stable or unstable?

Student: Unstable.

Unstable right, the pole is at plus 1 right. So, one aspect of this course is to essentially stabilize unstable systems by doing feedback right, but rather than taking all this effect effort right can I not do this, that is it that is say I pre multiply by a block which essentially does this then what will happen? You know Y of S is going to be equal to 1 by S minus 1 U of S and essentially this will become 1 by S minus 1; U of S is going to be S minus 1 divided by S plus 1 R of S and then look what happens right I cancel this S minus 1 and S minus 1. So, I am going to be left with 1 by S plus 1 R of S.

So, if I look at it from a different perspective now Y of S by R of S is going to be 1 divided by S plus 1 all right, stable. So, have you not stabilized an unstable system? So, what I have done is what is called as a pole zero cancellation right. We briefly might have discussed this before, but more importantly I have done pole zero cancellation in the right half plane right because I am cancelling a pole in the right half plane with a zero in the right half plane right. So, this solves all our problems right then I do not need to have expensive closed loop feedback systems and so, on right.

Student: These two system will independently unstable one because (Refer Time: 15:11) system in reality and that one will be other systems so.

Ok.

Student: So they both are independently unstable right.

But when I cascade them would they not become stable.

Student: But that is (Refer Time: 15:25)

See because anyway this block R of S and U of S, it is stable right; the zero is in the right of plane its non minimum phase right, but then like S plus 1 is there in the denominator right. So, can I not compensate for whatever instability which is happening by designing a suitable control input.

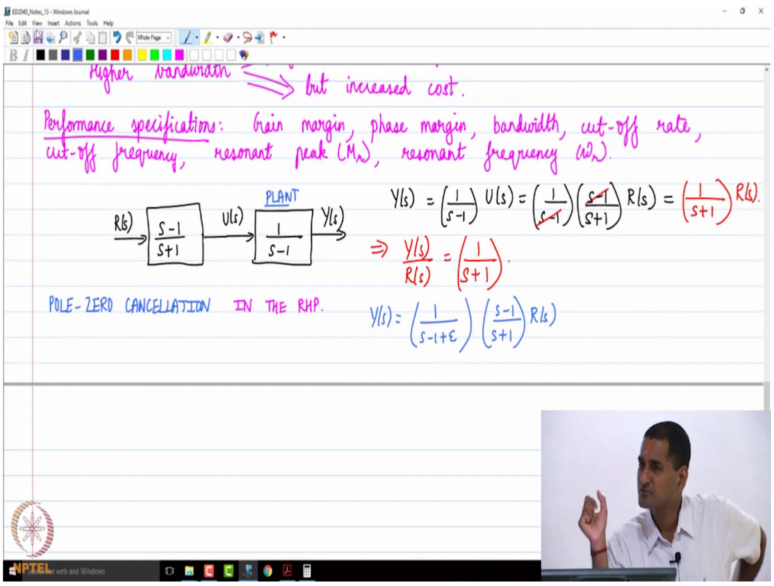
Student: (Refer Time: 15:47) that system very unstable right, the cascaded system. Like error is there, (Refer Time: 15:54)

Ok.

Student: Theoretically, if everything is perfect that system will be stable but in practical life some errors will be there or something might not be (Refer Time: 16:04).

So, I hope everyone got what he was trying to say right. In real systems that were always going to be some parametric uncertainties like these are all like idealized approaches right. So, you never know like, see for example what essentially it means is that is a following right.

(Refer Slide Time: 16:31)



So, this term can be Y of S can be 1 by S minus 1 plus epsilon right and then yes even if you build a perfect compensating block ok, this is my compensating block this is my plant right my plant may have some uncertainties. So, my pole may be around 1 plus or minus epsilon can happen then you will not have a perfect pole zero cancellation that is very difficult to achieve, right.

And even if you have, if you say epsilon is very small if you take the Inverse Laplace Transform, you will see that you will have a term like e power epsilon t epsilon may be very small, but as t progresses it will blow off to infinity. So, initially the system may appear to be stable, but then as time progresses it will become unstable; so not a great idea ok. So, one should never do pole zero cancellation is in the right half plane for this reason ok. So, that is something which I wanted to convey as an assay right.

So, anyway we will continue with what to say design of controllers tomorrow based on the frequency responses.