Control Systems
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Lecture – 64
Relative Stability 1
Part – 2

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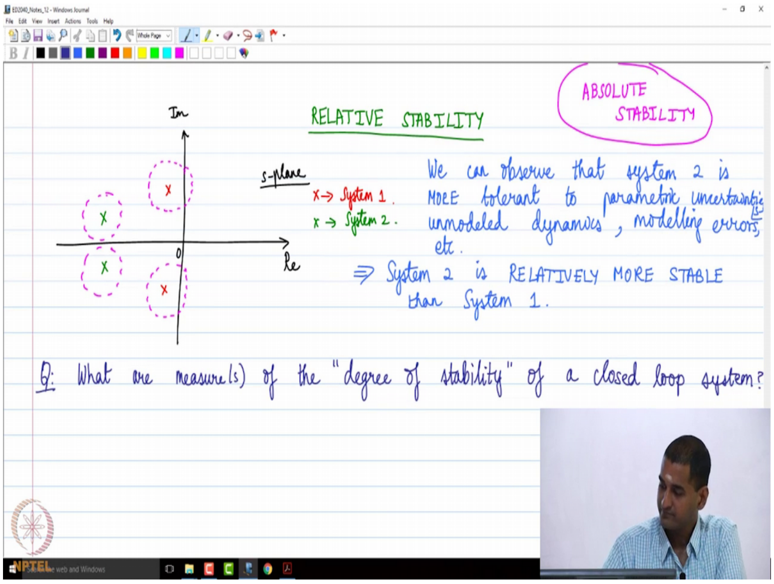

Essentially the question that we are asking ourselves as far as relative stability is concerned ok. So, now that we understand what is a relative stability right, what do we mean by that right. So, the question that we need to ask ourselves see this is a very qualitative discussion is it not?

See, you plot the open loop poles that is a poles of the transfer function of the system transfer function, and say hey look you know this system is relatively less stable than the other one right, but then we need to have quantitative measures which essentially help us in taking a decision right when we do control design right. So, then the questions that we need to ask ourselves is that are the following.

So, the one question we can ask is the following you know what are measures of the degree of stability of a closed loop system that is a that is a question that we need to ask ourselves right. To answer this let us revisit our standard ah what do you say feedback system that we have been considering where the forward path transfer function is G of s ok.

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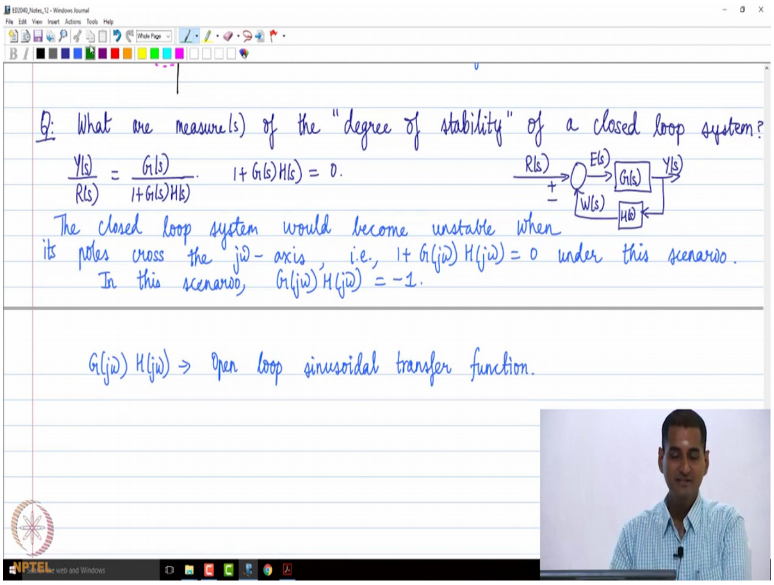

The feedback path transfer function is H of s ok. And there is an error E of s right. So, then please note that Y of s divided by R of s is going to be equal to G of s divided by 1 plus G of s H of s so that is what we are going to have right ok. Now, what is what is going to have a we are going to look at is essentially we are going to look at the closed loop characteristic equation ok, because the roots of this equation on the closed loop poles.

Suppose, let us say the closed loop system is stable to begin with right then we know that all closed loop poles are in the left of complex plane ok. Now, due to some parametric uncertainties the closed loop system what to say is on the verge of instability, but then you see that if my closed loop poles are in the left half plane to begin with, they can essentially transition to the right of plane only when they go through the. What is the boundary between the left half plane on the right of plane?

Student: J omega

J omega axis right, so that is essentially that is why it is called as a stability boundary right. So, essentially the j omega axis or the imaginary axis is the boundary between the left half plane and the right half plane. So, if my closed loop system is stable to begin with you know the closed loop system poles if they are migrating closer and closer to the right of plane, they have to necessarily pass through the j omega axis right which is a stability boundary before they go to the right half plane. Do you agree?

So, we can use that to our advantage ok. So, the point is the system or the closed loop system would become unstable when its poles cross the j omega axis right. So, of course, the presumption is that to begin with the closed loop system is stable all right that is when all this discussion makes sense right, so that is on the j omega axis what will happen the closed loop characteristic equation will become this right, because s s cause is ss of the form j omega. Do you agree right? So, this what will happen to the closed loop characteristic equation.

So, so in this scenario then j omega g j omega h j omega will be equal to minus 1 so that is when what is the scenario that is the closed loop poles are in the left half plane and they are migrating to the right half plane due to whatever reason right some parametric concern at disturbances and so on. So, then they need to cross the j omega axis let us say you stop when they when they are crossing the j omega axis some closed loop pole right. So, the closed loop characteristic equation is going to satisfy g is going to essentially become 1 plus g j omega h j omega equals 0. I can rewrite it as g j omega h j omega is equal to minus 1. So, what is G of j omega H of j omega?

Student: (Refer Time: 05:35)

That is what we have been reading under frequency response right, it is the open loop sinusoidal transfer function right. So, that is the [funk/function] function G of j omega H of j omega. So, let me write it one more time. So, G of j omega H of j omega is the open loop sinusoidal transfer function so that is what it is ok. Now, what are we interested in then. So, the question is the system would become unstable when the Nyquist plot of G of s H of s right passes through the minus 1 point because if there is some value of S, which makes the value of G of s H of s to be minus 1 right, then the system would become unstable right.

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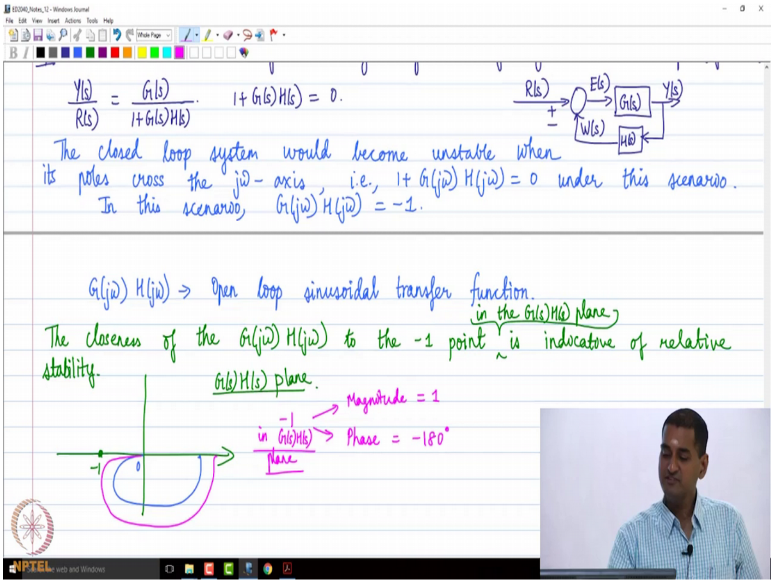

So, or in other words, I can reinterpret this as the following the closeness of the G of j omega H of j omega to the minus 1 point is indicative of course, minus 1 point in the G of s, H of s plane is indicative of relative stability so that is what we one could conclude right from this. So, see what does it mean? Let us say I have the G of s H of s plane right. So, let us say this is my minus 1 point right. So, ok.

So, let us say you know like I have let us say some I want to say a Nyquist plot right. So, let us say what some systems Nyquist plot which essentially let us say starts from the origin and then goes like this. And or let us let us do it in this way right. So, let us say comes like this and goes like this ok. So, another system which essentially comes and goes like this ok. So, you see that the second system is going to be closer to the minus 1 point, you perturb it, you stretch the Nyquist plot a little bit it can pass through the minus 1 point maybe right.

So, the closer the block us to the minus 1 point the less relatively stable that system is right, so that is what we can conclude about conclude from this particular discussion on a relative stability. Now, note that the minus 1 point in the G of s H of s plane means the magnitude is 1, the phase is minus 180. I am writing minus 180, because right you see that typically we deal with proper transfer functions you will understand it shortly. So, so of course, minus 1 point in the in G of s H of s plane indicates that the magnitude of the open loop transfer function is one at that point and the phase of the open loop transfer function is minus 180 degree. Do you agree? So, I am just writing in terms of magnitude and phase. We are going to use this idea to define a few parameters that can quantify relative stability ok, so that is what we are going to do now ok. So, are there any questions now? Yes.

Student: (Refer Time: 09:46).

Sorry which one.

Student: Statement of (Refer Time: 09:50).

So, that follows from this because you see that see what is the scenario we are considering, let us say we go back to this right. You plot the Nyquist plot of system 1 and system 2, it may look like what I have plotted right. So, because the closer it is to the minus 1 point you know like if you perturb it, it can pass through minus 1. What does passing through minus 1 mean as far as the Nyquist plot of the open loop transfer function as that means, that S equals g omega satisfies the closed loop characteristic equation that means that a closed loop pole is going to be of the form j omega.

In other words you are going to have closed loop poles on the imaginary axis and that is what we are discussing right, so that is why we are breaking down the problem or not breaking down or converting the problem of analyzing closed loop stability once again into a problem of analyzing the open loop transfer function, see that is a big advantage with frequency response methods.

So, one advantage of frequency response methods if you go back and think through is that even a Nyquist stability criteria you know like we are looking only at the open loop transfer function. Even here I am looking only at the open loop transfer function not at the closed loop transfer function right. So, you are need to we are need to calculate the closed loop transfer function. You give me the open loop transfer function I can I can analyze its Nyquist plot, and then tell you some information of the closed loop system ok, so that is the ah impact here ok.

So, you see that I can rewrite this closed loop characteristic equation in this form all right. So, the as G of j omega H of j omega equals minus 1 if there were a closed loop pole on the j omega axis that is a scenario we are considering ok, yeah ok. So, a minus 1 point in the G of s H of s plane means at the magnitude of G of s H of s is 1 and the phase of G of s H of s is minus 180 degrees ok, is it clear ok, those are two important ideas ok. So, based on this you know like we are going to essentially have a few definitions ok.

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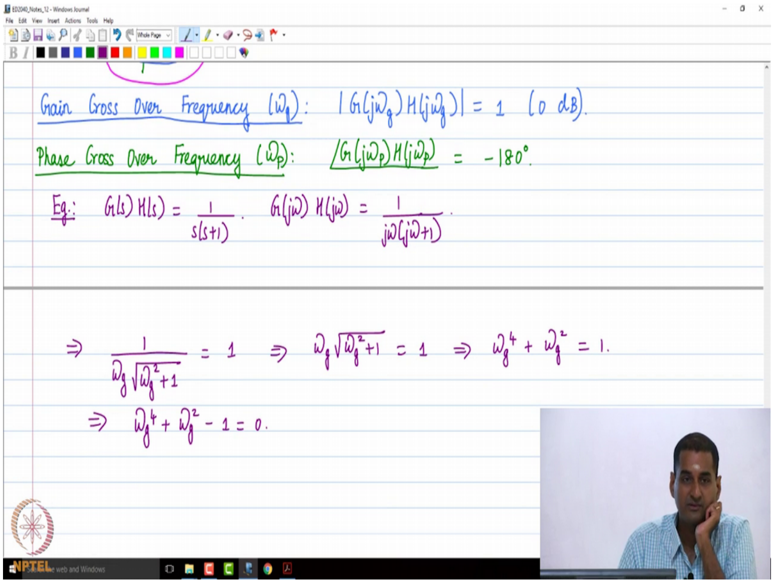

So, let me essentially write down those definitions ok. The first definition is the following ok. So, we are going to define what is called as a gain cross over frequency ok. I will define these two frequencies then we will come back in the next class and then define two important parameters ok. I just want to spend some time discussing those right.

So, what is this gain crossover frequency ok? It is the frequency its denoted by omega g ok. The gain crossover frequency omega g it is the frequency at which the open loop sinusoidal transfer function magnitude becomes equal to 1. So, the magnitude of the open loop transfer function should become unity or 0 decibels ok. If you convert to 0 decibel measure obviously you if you take logarithm of one you will get 0 right, so that is the definition of the gain crossover frequency. It is the frequency at which the magnitude of the open loop transfer function becomes 1 right.

Then we have another definition all these are definitions please (Refer Time: 13:29) that ok. We have another definition which is what is called as a phase crossover frequency omega p. What is the definition of phase crossover frequency? It is the frequency at which the phase of the open loop transfer function becomes minus 180 degrees ok, so that is the definition.

So, of course, you can immediately see how these definitions are motivated right, because why the one point has a magnitude of one and a phase of minus 180 degrees right that is motivating these two definitions ok. So, of course, if at if omega g equals omega p for some particular open loop transfer function, what does it immediately imply that the open loop transfer function is passing through the minus 1 point at that frequency, is it not think about it right. If omega g equals omega p for some open loop transfer function, then at that frequency the magnitude of open or transfer function is 1, phase is minus 180. What point does it correspond to the minus 1 point in the G of s H of s plane right.

So, so let us quickly do this example at the example that we considered today. So, let us say we consider G of s H of s to be 1 divided by S times S plus 1. G of j omega H of j omega to be equal to 1 divided by j omega times j omega plus 1 right. So, what is the gain crossover frequency, can you calculate it please? So, how do you get the gain crossover frequency? Gain crossover frequency, you equate the magnitude to 1 right.

So, how do I get gain crossover frequency? What is the magnitude of this factor? It is going to be 1 over omega g times square root of omega g squared plus 1 equals 1 am I correct? Then how can I calculate the omega g from here? Of course, I cross multiply this will imply that omega g times omega g square plus 1 is equal to 1. Then what can I do, I can square it, if I square it I will get omega g power 4 plus omega g square right hand side will be 1 ok. So, this will imply that omega g power 4 plus omega g square minus 1 is equal to 0 right.

So, if you solve this is a this is a fourth order polynomial in omega g, but then you see that it is quadratic in omega g square all right it has only omega g power 4 omega g squared and constant up. So, if you say omega g square is some omega g hat you will see that you will get a quadratic equation. You use that notion to essentially solve for this if you solve this, please do it ok.

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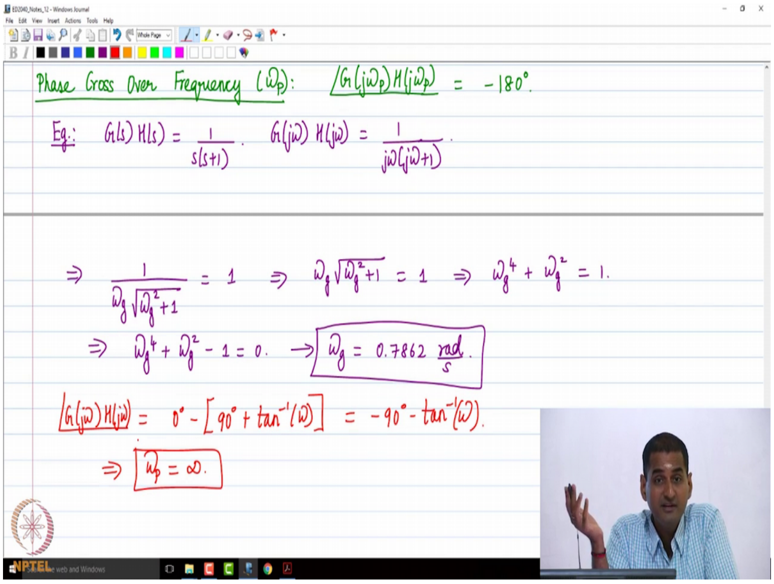

And check this answer, you will get omega g to be 0.7862 radians per second please check this answer so that is what you will have, so that is the gain crossover frequency for this particular open loop transfer function. So, this is the this is the only frequency at which the magnitude becomes one for this particular open loop transfer function. Now, how do I calculate the phase crossover frequency? What is the phase of G of j omega H of j omega? It is going to be 0 minus 90 plus tan inverse omega. Do you agree? I think by now I think all of us should be familiar with these concepts right.

So, the numerator phase is 0, denominator due to 1 by S you will have [mi/minus] minus 90 and then minus tan is a omega right. So, this is going to give me minus ninety degrees minus tan inverse omega. So, when will this phase become minus 180 degrees right? So, the question becomes when will this phase become minus 180 degrees that means, tan inverse of omega p should be 90 degree right.

When will that happen only when omega tends to infinity. So, my phase crossover frequency is infinity or in other words this essentially means at you know like the particular open loop transfer function phase becomes minus 180 degrees only when omega tends asymptotically to infinity which you can easily figure visualize even from the bode plot. See think about how the phase plot of this particular factor will look like; the phase will go only to minus 180 only when omega tends to infinity right so that is why the phase sorry the phase crossover frequency is infinity ok.

So, we will stop here. So, we will build on these two frequencies in the next class; and then like I will define parameters which can be used to quantify relative stability and we would use that ok. We will discuss that in the next class fine.

Thank you.