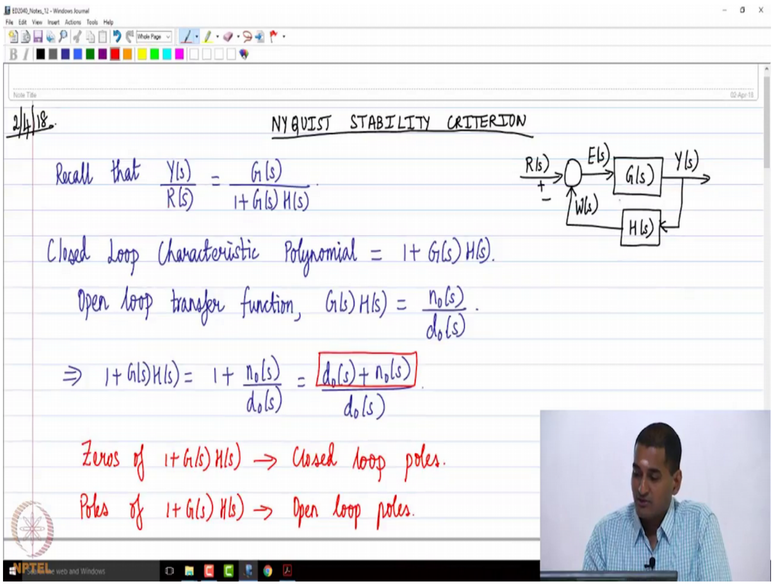
Control Systems
Prof. C. S. Shankar Ram
Department of Engineering Design
Indian Institute of Technology, Madras

Lecture – 63
Relative Stability 1
Part – 1

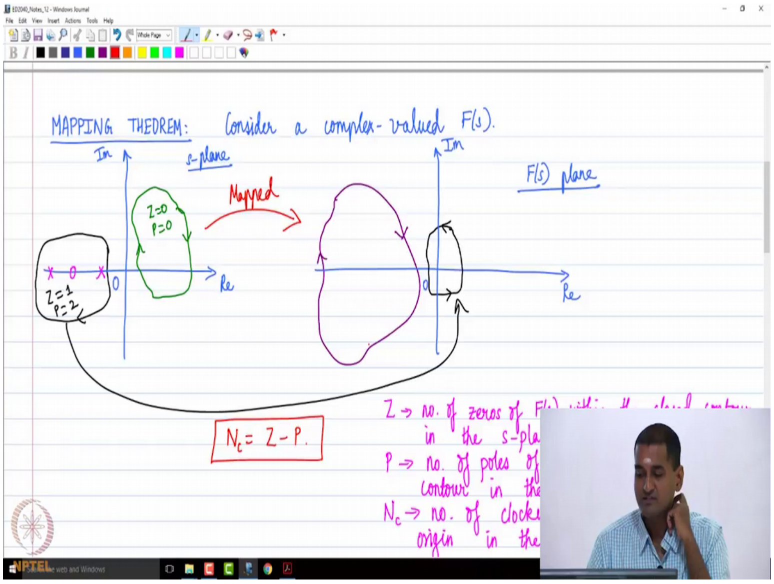
Let us get started with today's class you know like we were looking at the Nyquist stability criteria.

(Refer Slide Time: 00:21)



So, what was our starting point our starting point was this standard feedback loop that we have been considering, whose closed loop transfer function is G of s divided by 1 plus G of s H of s the closed loop characteristic polynomial is 1 plus G of s H of s and, we saw that we are interested in the zeros of this closed loop characteristic polynomial right because, the zeroes of the closed loop characteristic polynomial, or the closed loop poles.

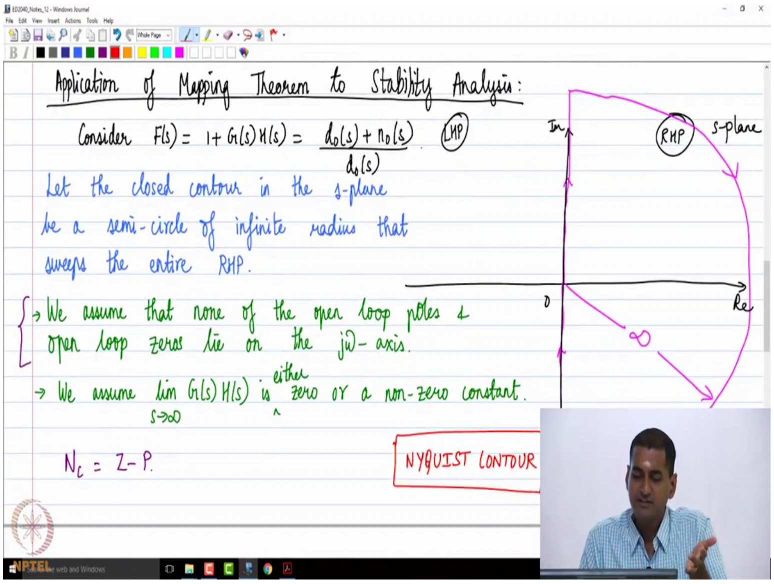
(Refer Slide Time: 00:49)



So, in this connection you know like we studied what is called as a mapping theorem. So, what is a map what is the mapping theorem you know like it essentially in general deals with any complex valued function F of s, if you take a contour in the s plane, then map a closed contour in the s plane and then map it to the F of s plane.

So, the mapping theorem talks about how many encirclements of the origin with the closed contour in the F of s plane have right, in relation to the number of 0s and poles of F of s which is contained in the which are contained in the contour in the s plane ok. So, suppose if the closed contour in the s plane had Z 0s and P poles of F of s the number of clockwise encirclements of the origin in the F of s plane is going to be Z minus P so, that is the mapping theorem ok.

(Refer Slide Time: 01:42)

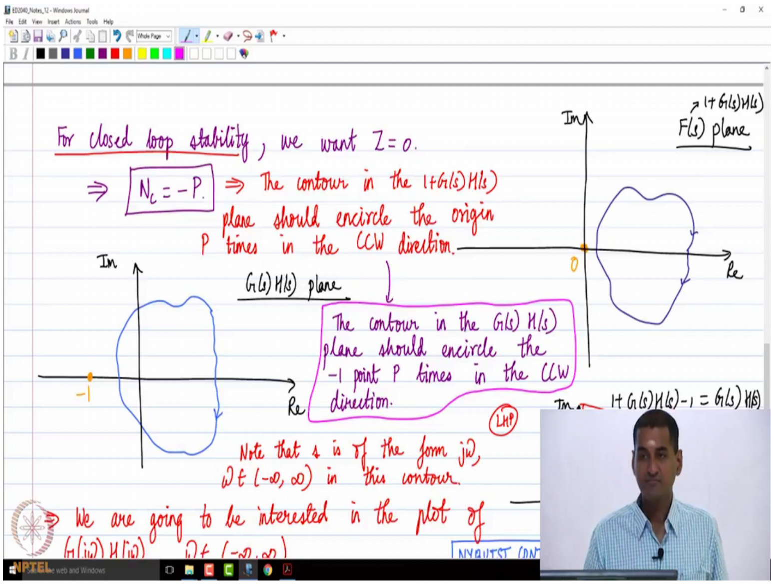


So, and how did we start applying it to our course ok. So, we are going to apply it to stability analysis. So, what we do is that like we consider a contour in the s plane, which essentially goes from minus j infinity to plus j infinity that is traverses the entire what to say imaginary axis and, then takes a what is a semicircular path of infinite radius. So, that it sweeps through the entire right of plane and that contour is typically called as the Nyquist contour ok.

So, essentially this is attributed to Nyquist and essentially it is called as a Nyquist contour ok. So, then if you apply the mapping theorem to this particular problem right so, we looked at the we look at the closed loop characteristic polynomial right. So, if we want closed loop stability we essentially want Z to be 0 right is it not, if we take the mapping theorem right, because the mapping theorem is N C is equal to Z minus P. So, if this Nyquist contour is what I am interested in you know like for closed loop stability, we want Z to be 0 right and in the case where we do not have any open loop poles or 0s on the imaginary axis.

We essentially get a condition saying that N C should be equal to minus P ok. So, in other words if there were K open loop poles, or open loop zeros within this sorry yeah open loop sorry p is a number of open loop poles right. Suppose if there are K open loop poles within this Nyquist contour, the then essentially the contour in the G s H of s plane must encircle the minus 1 point K times in the counterclockwise direction right that is what we studied right.

(Refer Slide Time: 03:36)

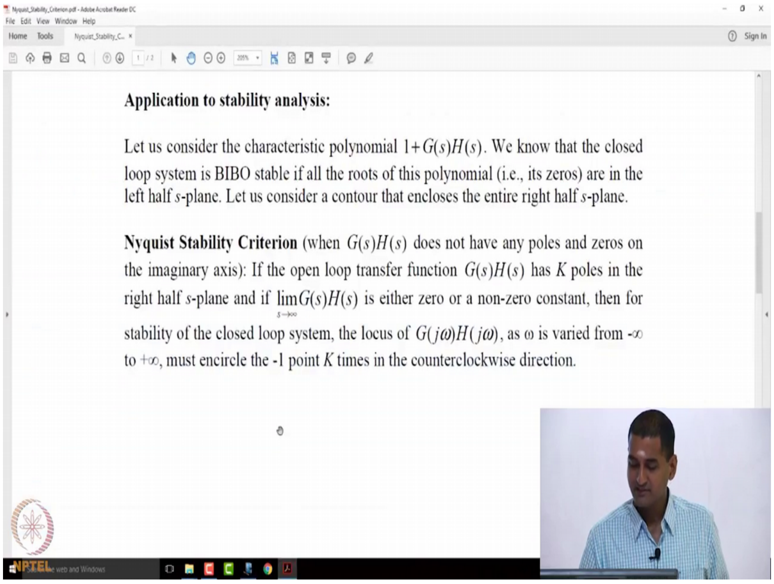


Because we looked at F of s to be one plus G of s H of s, then we shifted everything by minus 1 right that is one the entire plot interior what to say contour was shifted to the left by 1 right. So, how is it like we go to the G of s H of s plane, then minus 1 becomes the critical point. So, that is why the condition for stability you know like becomes this so, so that is what we have ok. So, that is the Nyquist stability criteria right and since we have s of the form G j omega right.

So, when my Nyquist contour is going on the imaginary axis s is of the form j omega right. So, G of s H of s becomes G of j j omega H of j omega right. So, that is essentially the sinusoidal transfer function. So, that is why we are interested in the Nyquist plot. You know that is why the Nyquist plot comes in. So, essentially what we do is that like we look at the Nyquist plot of the open loop transfer function and, then see how many times the minus 1 point encircle is encircled in by the Nyquist plot of the open loop transfer function and we applied it that is how we got the Nyquist stability criteria ok.

So, that is a brief recap of what we did as far as a Nyquist stability criterion is concerned.

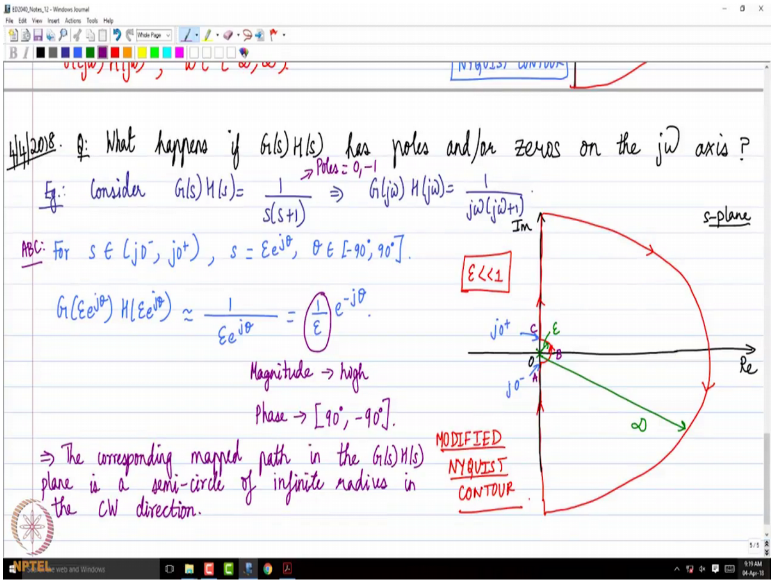
(Refer Slide Time: 04:55)



So, let me read the criteria once again ok, this is a statement of the Nyquist stability criterion when G of s H of s does not have any open loop poles or open loop zeros that of course, G of s H of s does not have any poles or zeros ok, on the imaginary axis. So, if the open loop transfer function G of s H of s has K poles in the right of s plane. And if limit s tending to infinity G of s H of s is either 0 or a non-zero constant, then for stability of the closed loop system the locus of G of j omega H of j omega as omega is varied from minus infinity to plus infinity, must encircle the minus 1 point K times in the counterclockwise direction ok. So, that is the Nyquist stability criterion. So, if it does not; that means that the closed loop system is unstable ok.

So, by looking at the Nyquist plot of the open loop transfer function we can talk about closed loop stability ok. So, that is the advantage of this Nyquist stability criterion is it clear. So, in what context we are going to use the Nyquist plot right ok. So, now, I am just going to pose this question and essentially partially answer it and I want you to figure out the complete answer so, as homework ok.

(Refer Slide Time: 06:08)



So, now a question that naturally follows is that what happens, if the open loop transfer function G of s H of s has poles and zeros and or zeros ok, on the imaginary axis all right it is possible. Because even we have done problems where we have open loop poles in the at the origin for example, right when we did root locus and so, on right we wanted to essentially stabilize it and so, on right. So, then what happens in that case ok. So, let me essentially lead you to the answer ok, then you can also like go back and figure out the complete answer ok.

So, essentially what happens is that like let us say as an example consider the open loop transfer function G of s H of s, to be let us say one divided by S times S plus 1 ok. So, let us say that is my open loop transfer function. So, this implies that G of j omega H of j omega is going to be equal to one divided by j omega divided j omega plus 1 right ok. So, that is what is going to happen.

So, now you see that you know there is a pole at the origin an open loop pole at the origin then what happens ok. So, then what we do is that we construct what is called as the a modified Nyquist contour ok. So, let me explain what this is so, so in the s plane what we do is the following right we essentially start from minus j infinity ok, we start coming closer and closer to the origin ok.

So, this is my origin right. So, what I do is that? When it come closer to the origin, since I have an open loop pole at the origin right so, here I have an open loop pole at 0 ok. So, what will happen if I just make the Nyquist contour pass through the origin; obviously, you will see that the Nyquist plot will anyway tend to infinity right, because the open loop pole come comes as the denominator in G of root of the denominator in G of s H of s so; obviously, as omega tends to 0 you know like the Nyquist plot is anyway going to go to essentially infinity ok, but how that is what we are going to ask ourselves right.

So, then what we do is that we essentially construct a modified Nyquist contour, which essentially takes a semicircular path ok, around the open loop pole of the origin of infinitesimal radius, then we have the usual semicircular path of infinite radius so, that is the Nyquist plot sorry Nyquist path ok.

So, essentially what happens is that like here, we have a path of radius epsilon and anyway my original Nyquist path is of infinite radius ok. So, that is what we do ok. So, near the origin what is going to happen is that like we essentially take a small detour right, we take a very what to say small detour in the form of a semi circular path of radius epsilon ok. So, that is what we do so; obviously, epsilon is very very small ok.

So, this is what is called as a modified Nyquist contour ok. So, this is what is called as a modified Nyquist contour ok. So, consequently what happens here to this one right. So, you see that for S belonging to j 0 minus to j 0 plus, what do I mean by that that is this is like 0 minus right j 0 minus right, because it is very close to origin, but slightly away from the origin, but on the negative imaginary axis.

So, this is going to be j 0 plus right it is very close to the origin, but slightly above the origin on the imaginary axis right. So, S essentially is going from j 0 minus to j 0 plus you see that s s of the form epsilon e power j theta right theta going from minus 90 to plus 90 ok. So, essentially I am representing the, what to say it is not is equal to right. So, actually I am representing in the polar form. So, we have the magnitude to be epsilon because it is a semicircular path and the phase of the complex variable S to be theta right theta goes from minus 90 to plus 90 ok. So, that is what is going to happen so, that is what is going to happen when you have this particular modified Nyquist contour yeah.

So, then what do you think happens you know like to G of epsilon e power j theta H of epsilon e power j theta, let us I plug it in right s s of the form this. So, you see that this will become 1 this will be almost equal to 1 divided by epsilon e power j theta why, because S plus 1 will almost be 1 right, because this complex number is going to be very

very very small right very close to the origin so, S plus 1 is going to be almost equal to 1 right.

So, the term which is going to be left behind is S itself, but S is of the form epsilon e power j theta. So, this open loop transfer function is essentially going to be like this. So, what can you tell about the magnitude of the open loop transfer function very high right why because it is like 1 by epsilon ok. So, please note that the magnitude is like 1 by epsilon right, what I am talking about is only on the it makes sense only on the small path let me call this small small path as A B C you consider the path only A B C ok.

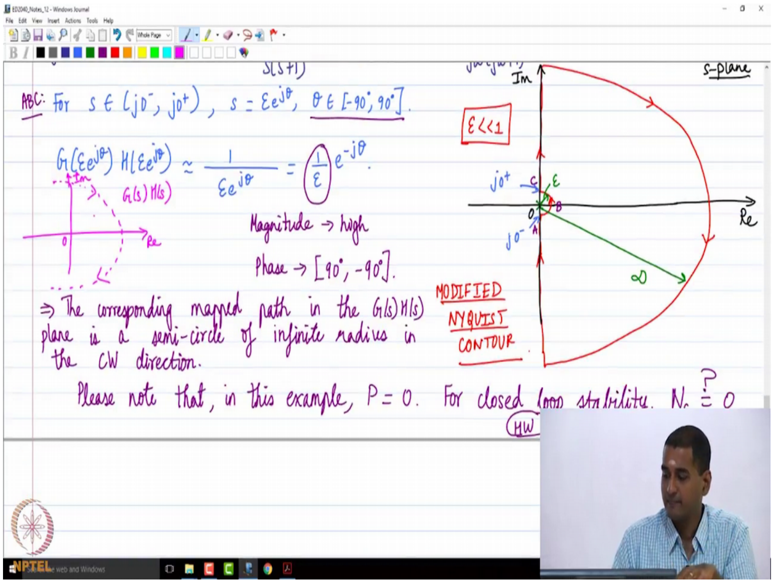
Whatever I am discussing holds true only for that path right. So, so the magnitude is high very very much greater ok. So, it is basically a very high when compared to 1 what about the phase the phase goes from 90 to minus 90 so, what can you conclude see you are having a counterclockwise semicircle of infinitesimal radius in the s plane right, in the modified contour path Nyquist contour, what is going to happen to the Nyquist plot of G of s H of s in this path in this path it is also going to be a semicircle right.

Why because the phase goes from 90 to minus 90, but in which direction clockwise right because the phase goes from plus 90 to minus 90 and you are going to essentially get a semicircle of infinite radius right. So, the conclusion is that the corresponding a path mapped path in the G of s H of s plane is a semicircle of infinite radius in the clockwise direction ok.

That Is what is a conclusion you can draw ok, then you apply the Nyquist stability criteria as it is n C equals Z minus P the modified Nyquist contour ok. You essentially look at number of open loop poles ok, if this is the case what is the value of P in this problem, you see that the poles are at 0 and minus 1. So, what is the value of P here please remember what is P, P is the number of open loop poles within the Nyquist contour or in this case the modified Nyquist contour right.

So, what is the value of P is there any open loop pole within the modified Nyquist contour no ok.

(Refer Slide Time: 15:58)



So, please note that of course, in this example P is going to be equal to 0. So, for closed loop stability what must we have we should have N C is to be equal to P minus P which is 0 right; that means, that the Nyquist plot of G of s H of s should not encircle the minus 1 point at all right. So, check this as homework ok.

So, for closed loop stability N C should be equal to 0 I am putting a question mark, because we do not know right that is what I want you to check ok, please do it as homework is it clear what I want you to do. So, you plot the Nyquist plot of this 1 by S times S plus 1 and then check it right and, then see whether it encircles the minus 1 point yes please.

Student: (Refer Time: 17:05) clockwise (Refer Time: 17:06).

How did I get clockwise ok. So, you see that this path A B C is a semicircle of infinitesimal radius in the counterclockwise direction. So, theta goes from minus 90 to plus 90 do you agree. Now, the phase of the open loop transfer function in that path A B C is going to be minus theta right, because we get if you plug it in you are getting 1 by epsilon e power minus j theta. So, the magnitude is very high it goes like 1 by epsilon the phase becomes minus theta. So, when theta goes from minus 90 to plus 390 minus theta goes from 90 to minus 90 and what is 90 to minus 90.

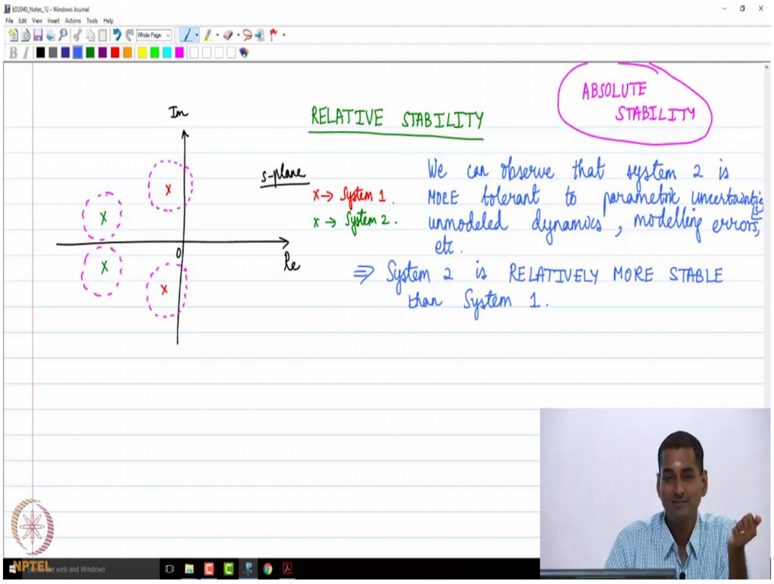
So, let us say yeah to answer that question if I essentially plot the G of s H of s plane ok, what is going to happen is then you may have a circle semicircular path of infinitesimally radius something like this ok. So, this is like plus 90 right and this is minus 90 and that is essentially oops that is essentially clockwise right. So, you are going to go clockwise in the G of s H of s plane ok, that is how we concluded that is it clear. So, please complete this problem ok.

So so, if you have open loop poles on the imaginary axis, what we do is that we just essentially go around it using semicircular paths of infinitesimal radius of course, we are going to have corresponding semicircular paths of sorry a semicircle of paths of infinitesimal radius and, that will be mapped to semicircles of infinite radius in the G of s H of s plane, when you plot the Nyquist plot right and then you go ahead and apply the same criteria right.

So, because now in the modified Nyquist contour you do not have any open loop poles on it right passing there is the modified Nyquist contour does not pass through the any open loop poles. So, you apply the same Nyquist stability theorem ok, that is the modification that we is it clear. So, please complete this problem and then like see for yourself right fine ok.

So, now this completes our discussion on Nyquist stability criteria, the one final concept related to frequency response, which I want to essentially discuss before, we move to control design using frequency response is what is called as relative stability ok.

(Refer Slide Time: 19:41)



So, that is what we want to discuss right. So, let us discuss relative stability and then like, we will see how to quantify it first, let us understand what is it? So, let me give you two scenarios right. So, let us say I take system 1, where in the s plane let us say you know like it has poles you know here ok, let us say we are considering a second order system right.

So, this essentially is system 1 and let us say you know like we have another system whose poles are here, I am I am not putting any numbers here, but then I am just what to say placing them relative to one another so, in a qualitative manner ok. Now, you see that both are under damped second order systems right. So, and then essentially system 1 has poles which are closer to the imaginary axis compared to the compared to system two. So, what can you say about both systems based on whatever we have learnt.

Student: (Refer Time: 20:59) stable path (Refer Time: 21:01).

Both are stable.

Student: (Refer Time: 21:03) stable (Refer Time: 21:03).

Both are stable as far as they are in the since they are all the poles are in the left half complex plane then.

Student: (Refer Time: 21:13).

Why you know.

Student: (Refer Time: 21:16).

Ok. So, we that is a good point right. So, I think we briefly discussed this right, because in our approach whatever we are following is an approximation right whatever approach we have been discussing through in this course right. So, we have been modeling systems as l t i and then like getting transfer functions and, then like defining the notion of poles and, then like relating poles to stability performance and all right.

So, all these are approximations right in real life you know like when we model systems, there are going to be some unmodeled dynamics, there are going to be some errors which will creep in right, there are going to be even if you have a let us say for the for the sake of discussion a perfect model still you know like we are going to have parametric variations right. So, the system will not have the same parameters see for example, if you have a mass spring damper system, there is no what to say no guarantee that the spring constant or the damping coefficient will remain the same forever all right they may change.

So, since the location of these poles are related to those values of those parameters, even these locations can change a little bit right. And in addition to that you know like you can have uncertain unmodeled dynamics coming in modeling errors, you know like all those creeping in so, consequently you know like I may have a band or a zone around which this pole may like right.

So, let us say if just for the sake of argument right, let us say I construct a a region around the nominal value, because it can be part of a little bit right. So, now, you see that you know like in real life we do not know how these perturbations are going to come right so, then if these errors accumulate and become serious enough. So, that the poles which are marked with the red color right corresponding to system one, if suppose they are perturb such that they go closer to the imaginary axis and maybe at some point hit the imaginary axis, then we are going to lose closed loop stability right.

So, in that sense relatively system 2 is more stable ok, that is why this is what is called as a relative stability ok, what we learnt before was absolute stability ok. So, another adjective gets added. So, what was absolute stability both systems are stable as far as absolute stability is concerned, because they are nominal poles are essentially in the left half complex plane see, what we want for stability or absolute stability is that all poles should be the left half complex plane period ok.

For system 1 and system 2 the way we are modeled and analyzed both pole all the poles are in the left half complex plane so, no doubt about it, but based on this argument when there are perturbations, when there are errors when there are parametric variations system 1 is can tolerate those errors to a lower extent than system 2 ok, that is why system 2 is relatively more stable than system 1 ok.

So, let me write that point on so, we can observe that system 2 is more tolerant to parametric uncertainties oops parametric uncertainties unmodeled dynamics, modeling errors etcetera all these can creep in real life right. So, this implies that system 2 is relatively more stable both are stable in the sense of absolute stability ok.

So, it is relatively more stable than system 1 so, that is our conclusion from this particular discussion is it clear ok. So, that is why we are we are talking about relative stability, but of course, there is a cost to pay right. So, see just because I want to make a system more relatively stable, I can keep on pushing all my poles to the left, what will happen, if I keep on pushing the poles to the left let us see what is the advantage of system 1 compared to system 2. So, what do I get if I close two dominant poles are close to the imaginary axis, what will happen to the rise time.

Student: (Refer Time: 26:42).

It will be lower. So, system 1 you will see that is essentially going to be much faster than system 2. So, the dynamic response characteristics of system 1 yes it is going to be more oscillatory ok, but that is why I said there is a trade off system 1 is going to respond faster, as far as having a lower rise time is concerned, but the response will become more oscillatory, because the overshoot maybe would be more ok, no doubt about it right so, but then it is going to be much faster than system 2.

So, there is a trade off in order to make the system more relatively stable yes 1 needs to push the poles more to the left, but at the same time you know we cannot keep on pushing the poles to the left forever, because that will affect the dynamic characteristics right. The system will become more slower right. So, you can immediately see that system 1 is essentially going to be what to say having a much faster response than system 2 that is an advantage for system 1, but it is relatively less stable.