Control Systems
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Lecture – 62
Nyquist Stability Criterion
Part – 2

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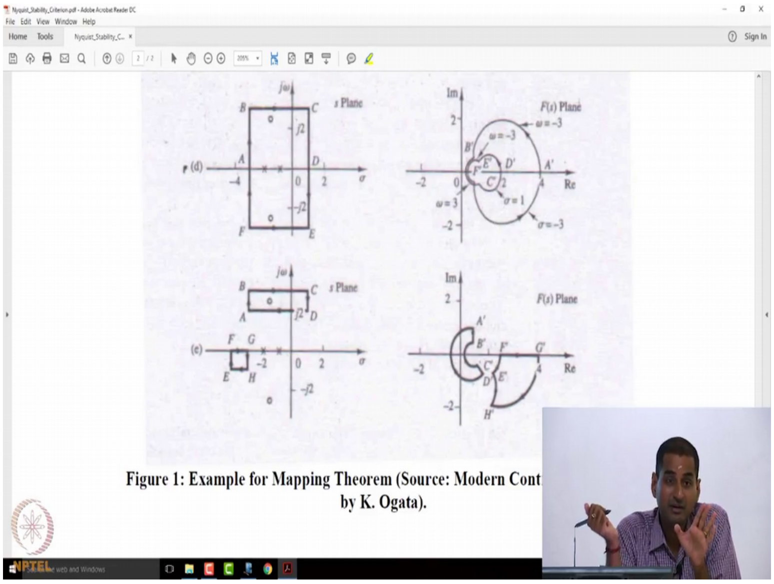

Now, the important question is that how is it useful to us right. So, in what we are going to use it. Yes.

Student: (Refer Time: 00:23).

We are coming to that ok. So, we will discuss all those things right. So, in a certain sense, yes, of course, one can also argue the other way that is why I am coming to this one right, so that is where I am going to say application of mapping theorem to stability analysis that is what we are going to do right.

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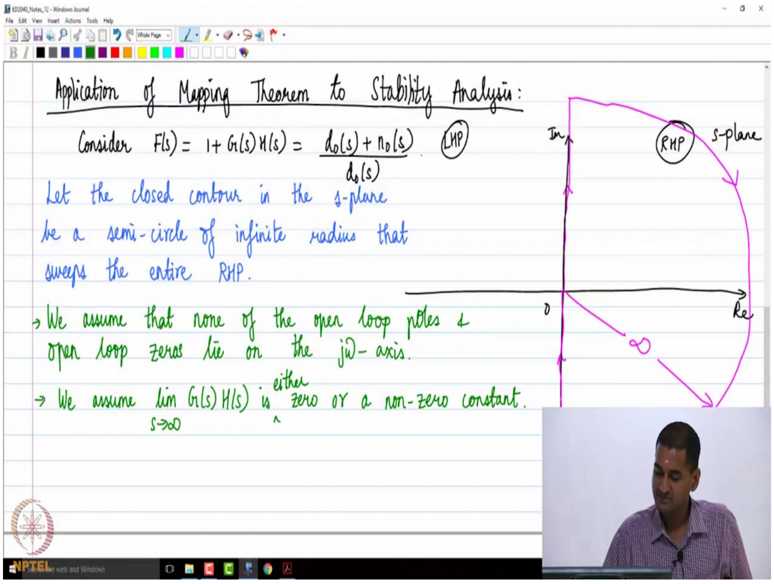

So, application of mapping theorem to stability analysis ok, so that is what we are going to do. So, now, how is it essentially applied let us look at that question right.

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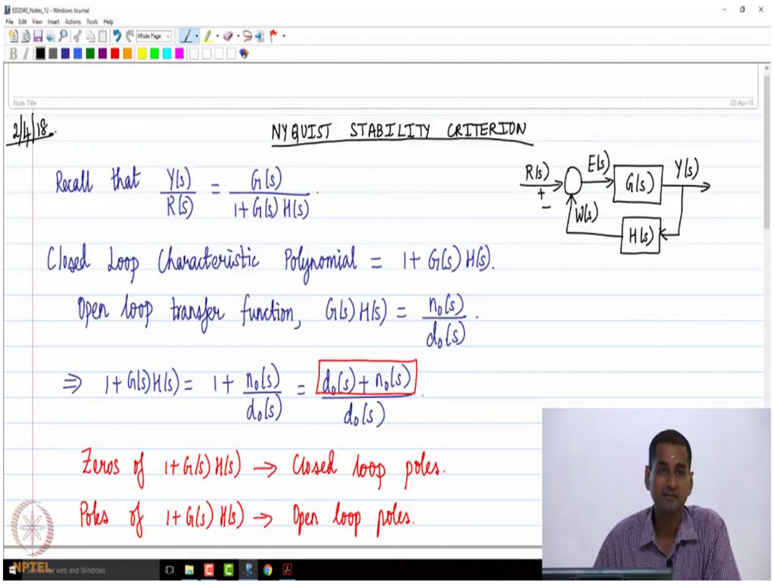

So, the way it is applied is the following ok. Now, we will go back to our closed loop transfer function right and the closed loop characteristic polynomial right which is 1 plus G of s, H of s correct. So, consider F of s to be equal to one plus G of s H of s which was some d o of s plus n o of s divided by d o of s right, so that is what we already had right. Now, the question is that like how do I apply the mapping theorem to this polynomial which is a ratio of two different polynomials right we are interested in the open loop sorry closed loop poles right their location ok. Now, for closed loop stability what is it that we want?

Student: (Refer Time: 02:26).

Which poles?

Student: (Refer Time: 02:29).

Closed loop poles right if I want my closed loop system to be stable I want all the closed loop poles to be in the left of complex that means, that I do not want any closed loop pole to be either on the imaginary axis or the right half plane ok. So, consequently we will see whether we can use them to our advantage ok.

So, let us take the s-plane. Suppose, of course, this is the left half plane; this is a right half plane. Suppose let us say I take a contour where I start from the origin I go along the imaginary axis to infinity along the imaginary axis that sweep the entire right of complex plane like an R in the form of a semicircle of infinite radius and then come back to the come back to the origin ok. Suppose, let us say I consider such a contour in the s-plane. So, my contour the closed contour in the s-plane is a semicircle of infinite radius that essentially covers the entire right half complex plane.

So, first let us discuss it then we will we will sorry first let me convey the concept then we will discuss any questions right. So, let us say this is the contour we are considering in the s-plane ok. So, let the closed contour of course, one can ask look is this contour really closed ? So, do you would you say that look you know like when you take a arc of infinite radius right would you consider it to be really closed or open norms right. So, we will not get into that level of debate here right.

So, we will say that look let the contour in the s-plane be a semicircle of infinite radius that sweeps the entire right half plane ok, so that is the close contour in the s-plane ok. Now, we want essentially no closed loop poles within this contour do you agree see for closed loop stability, we do not want any closed loop poles will to lie within this contour right that is what we want right. We do not want any closed loop poles on the imaginary axis neither do we want any closed loop pole in the right of complex plane right.

So, and if we assume that to as a first step we assume that none of the open loop poles and open loop zeros lie on the j omega axis to begin with ok. Then we will see I will ask I will leave with a question what happens if it, it does not happen right. So, see I can have open loop poles and open loop zeros in the right half complex plane that is allowed right, but I assume that look for the open loop poles an open loop zeroes right none of them lie in the right sorry on the.

Student: (Refer Time: 06:51).

Imaginary axis or the j omega axis ok, so that is an assumption that we are what to say essentially looking at right. Now, further we assume that limit s tending to infinity see this is what I were typed here let me go back to the handout, so that I can explicitly point out which point I am essentially referring to ok. So, these are the assumptions that we are looking at ok.

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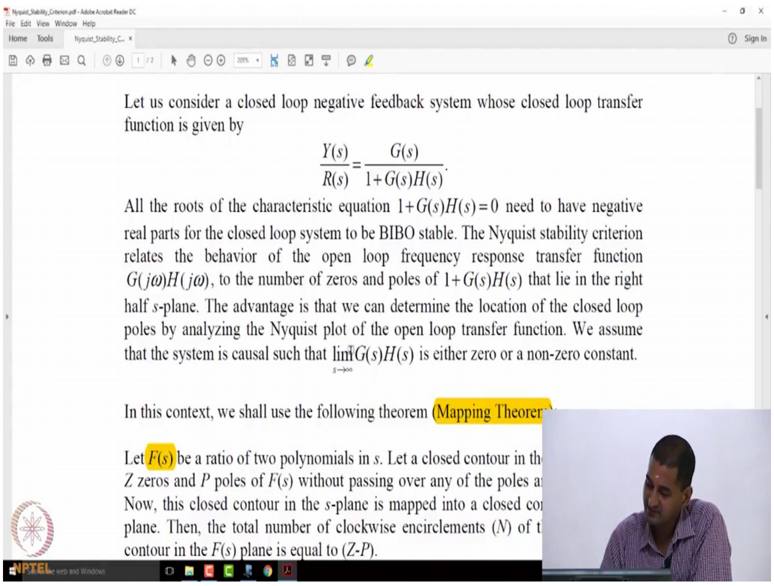

So, I am I am going to write down this assumption ok. So, limit s tending to infinity G of s H of s is either zero or a non-zero constant ok. So, that is what we are looking at. So, essentially we have limit s tending to infinity G of s H of s is a is either zero or a non non-zero constant ok, which is essentially saying that the system is causal ok. We have a proper transfer function to begin with you know that is a class of system anyway we are dealing with.

Why am I looking at this because then this will imply that when I am going on a circle of infinite radius s is tending to infinity right is it not, then that means, that at that value of s the all the points will be mapped to one point in the G of s H of s-plane ok, so that is the implication of the system being causal right ok. So, that these are some assumptions that we took. Now, this mapping is now this contour is now mapped into a corresponding contour in the 1 plus G of s H of s-plane ok.

So, let us say you know like I map this contour in the f of s-plane, but what is the f of s-plane this F of s-plane for stability analysis F of s was 1 plus G of s H of s right ok. So, let us say you know like that gets mapped into some arbitrary contour in the f of s-plane. Now, recall that n c should be equal to z plus p sorry z minus p ok.

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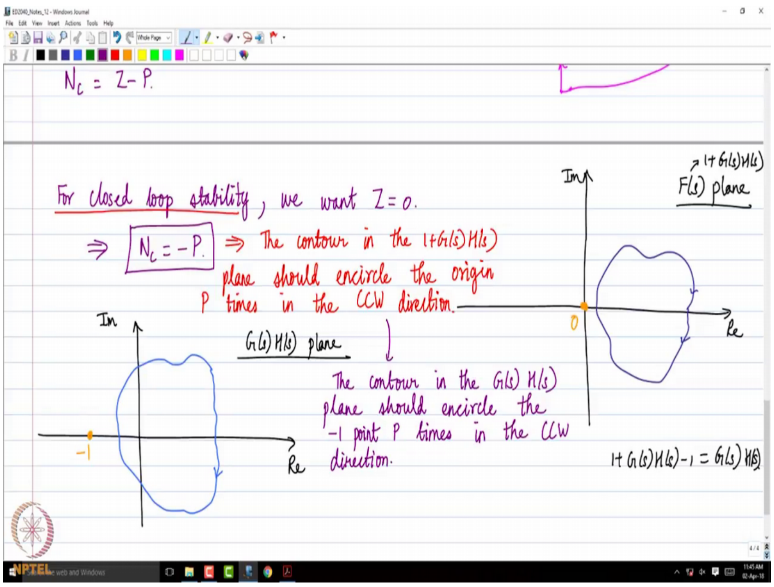

So, once again what is z, z is the number of zeros of f of s within the contour in the s-plane right. Now, for closed loop stability we want z to be.

Student: (Refer Time: 10:06).

0.

Student: (Refer Time: 10:08).

Z, z to be 0. Please remember that is why I let me repeat it again and again right yeah we need to just reflect on this a little bit ok. So, F of s was 1 plus G of s H of s the zeros of this f of s right are the closed loop poles. We do not want any closed loop pole in this contour all right so that that is why we want Z, Z to be 0. So, what should be then N c N c should be minus P. What is P? P is the number of open loop poles right which lies within this contour in the s-plane right.

So, you know the location of open loop poles anyway we are given a open loop transfer function, we know where are the open loop poles in within this contour and I know the value of P right, but of course, there is an important assumption right, so that is where this assumption comes right we assume that none of the open loop poles and zeros lie on the j omega axis because the original mapping theorem states that you consider a contour where none the contour does not go along go on pass through any of the poles and zeros of that function F of s right that is an assumption that we are doing.

We will then see what happens when we see sometimes we may given we may be given you know like open loop transfer functions whether the open loop poles on the imaginary axis can happen. Let us say you take a PI controller you will have a pole at the origin right we have already seen it then can I choose this control we will ask ourselves that question later for the for the time being we will first deal with the initial simpler case where there are no open loop poles on this j omega axis all right, so that is what we are.

So, now what does it say. So, this essentially tells that for closed loop stability the contour in the 1 plus G of s H of s-plane should encircle the origin P times in the counterclockwise direction. Do you agree? That is what is required for stability right. We are not done yet we are only halfway through right. Do you agree? Ok, so that is what this equation tells me right.

Now, now, I am going to essentially go from this 1 plus G of s H of s-plane to the G of s H of s-plane. So, please note you know like we are talking about closed loop stability. So, what it means is that for closed loop stability the contour in the 1 plus G of s H of s-plane should encircle the origin P times in the counterclockwise direction that is the condition. If it does not the closed of a system is unstable that is what we can conclude right yeah.

So, now from the 1 plus G of s H of s-plane, we go to the G of s H of s-plane. Now, how do I draw a contour from the 1 plus G of s H of s-plane to the G of s H of s-plane ? I subtract one see how do I get G of s H of s from 1 plus G of s H of s minus 1 right 1 plus

G of s H of s, you can immediately see that 1 plus G of s H of s minus 1 is going to be equal to G of s H of s. So, say of course, this very simple why am I writing it essentially what it means is that the contour in the 1 plus G of s H of s-plane is shifted to the left by one unit along as far as the real part is concerned. So, you take all the points in the 1 plus G of s H of s contour you just shift it shift the real part of all the points by 1 to the left; that means, you are subtracting 1 then you will get the corresponding contour in the g one plus G of s H of s-plane let us say that contour is something this similar shape ok, but that is something like this ok.

Now, where is the origin shifter see let us say this is my origin which is the critical point right as far as the mapping theorem is concerned what is this critical point shifted to. So, the origin of the 1 plus G of s H of s-plane is shifted to yeah subtracting 1 right what is 0 minus 1.

Student: Minus 1.

Minus 1 right. So, it is shifted to minus 1 in the G of s H of s-plane ok. So, we can say that the number of encirclements of the origin in the 1 plus G of s H of s-plane is the same as the number of encirclements of the origin of the minus 1 point in the G of s H of s-plane. Do you agree? So, let me repeat that statement once again. The number of encirclements of the origin in the 1 plus G of s H of s-plane is the same as the number of encirclements of the minus 1 point in the G of s H of s-plane. I do not change the function. I have just shifted the contour by one to the left all right yeah.

Student: (Refer Time: 16:08) in this case p is a nonzero number.

Student: (Refer Time: 16:12).

P is if P is a nonzero number then that means, the open loop system is unstable. See P indicates open loop poles right within the contour, yeah you can have an unstable open loop system question is that by feedback or you stabilizing it right that is what you are asking you can right ok. So, I hope how the critical point becomes minus 1 in the G of s H of s-plane.

So, the same condition one can rewrite this as the contour that is for closed loop stability, the contour in the G of s H of s-plane should encircle the origin P times sorry should encircle the minus 1 point not origin minus 1 point P times in the counterclockwise direction ok. This becomes the Nyquist stability criteria now ok. So, we are not done yet you know like we are just going step by step this is this is a parallel to next step ok. Let me go. I hope it is clear right.

So, because whatever is the number of encirclements of the contour in the 1 plus G of s H of s-plane, when you map it to G of s H of s-plane, it will be the same as the number of encirclements or the minus 1 point because you are just subtracting one that is it you are just shifting everything to the left by one right. So, the critical point which is origin in the 1 plus G of s H of s-plane becomes the minus 1 point in the G of s H of s-plane right. Is it clear? Now, you are seeing that this contour in the s-plane is also like very very special, it is not an arbitrary contour right like the general example of the mapping theorem that we looked at.

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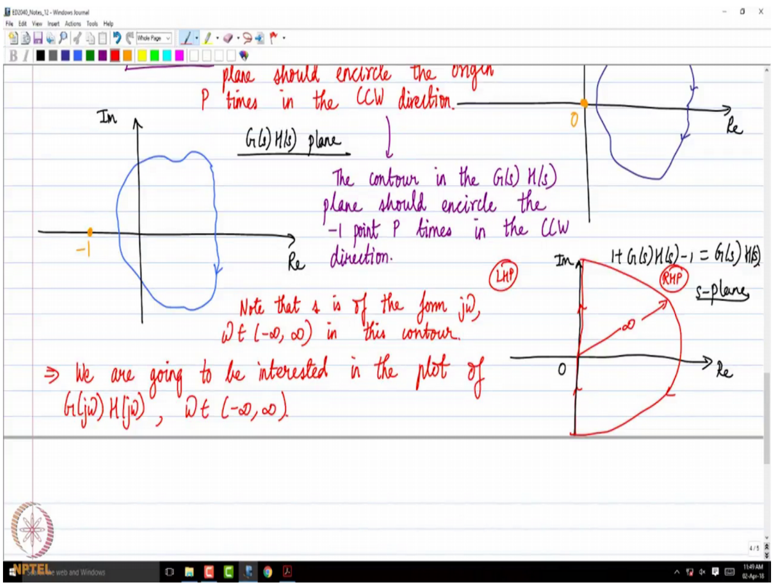

So, if you look at the s-plane, what is the contour that we are considering it is a semicircle of infinite radius that goes from minus j infinity to plus j infinity then sweeps the entire right of plane and then comes back all right. So, this is like infinite radius right LHP and RHP ok. So, this is an semicircle of infinite radius that sweeps the entire right of complex plane is it not ok.

Now, the question that arises is that what sort of form that S takes in this contour. So, note that S is of the form j omega all right omega belonging to minus infinity to plus infinity right in this contour. Is it not, anyway when s sweeps in the arc of infinite radius G of s H of s is a finite number it is it is going to be the same point that is our assumption from that is the consequence of causality right.

We are assuming that the system is causal, so as a result when I go on the arc of infinite radius G of s H of s will be at a point that is it right, it is not going to vary the variation is going to come only in the part where S goes from minus j infinity to plus in j infinity. In that region in that contour s is purely imaginary is it not. So, this implies that we are going to be interested in the plot of rather than saying G of s H of s because s is of the form j omega.

We are going to be interested in the plot of g of omega h of j omega as omega varies from minus infinity to plus infinity right. And what is this plot called as the Nyquist plot of the open loop transfer function right if you give me any transfer function you substitute s equals j omega and see how omega varies from 0 to infinity that is the Nyquist plot of course, here we are treating omega as a parameter.

So, I am making omega to be negative also. And we I asked you a question last class right what happens when omega becomes negative right we figured out that in a certain sense it becomes a mirror image right about the origin right. So, essentially that is what is going to happen here, so that is why we are essentially going to be interested in the Nyquist plot of the open loop transfer function to figure out the closed loop stability, so that is what leads to the statement ok.

So, let me go back we will read this statement. And we will come back and what to say essentially discuss it in the next class and I will allow you to also like think through these details right. We are discussed lot of points you know like we will recap once again right.

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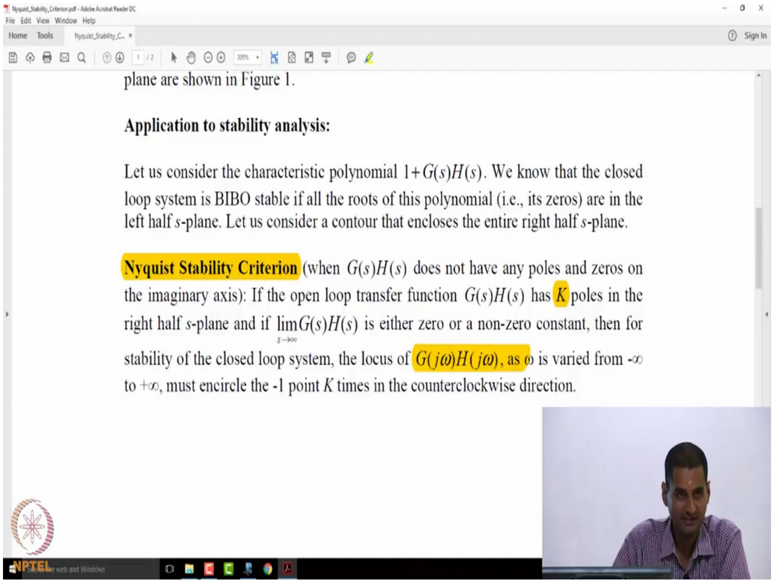

So, if you look at the Nyquist stability criteria that is the Nyquist stability criteria finally, of course, this is a criteria when G of s H of s does not have any poles and zeros on the imaginary axis that is the case we started off it. If the open loop transfer function G of s H of s as k poles in the right of complex plane, and if limit s tending to infinity G of s H of s either zero or a nonzero constant, then for stability of the closed loop system the locus of G of j omega H of j omega as omega is varied from minus infinity to plus infinity must encircle the minus 1 point k times in the clock counter clockwise direction.

So, essentially what we are saying is that we are saying p equals k right. And then we want Z to be 0. So, what should be N C, it should be minus k and that is k counter clockwise encirclements of the origin in the 1 plus G of s H of s-plane and that is going to be k and counter clockwise encirclements of the minus 1 point in the G of s H of s-plane. And we are writing G of j omega H of j omega because like s is of the form j omega ok, so that is why we are looking at this.

Student: (Refer Time: 22:54).

Yeah, that case I will I will discuss what to do in the next class right. So, so essentially that is the first case we are dealing with right. Essentially what happens when no open loop pole or open of zero lies on the imaginary axis that is the case we are still that. Now, we will then see what happens if it does what should be changed right, so that is something which we will discuss.