Control Systems
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Lecture – 58
Nyquist Plot 1
Part 2

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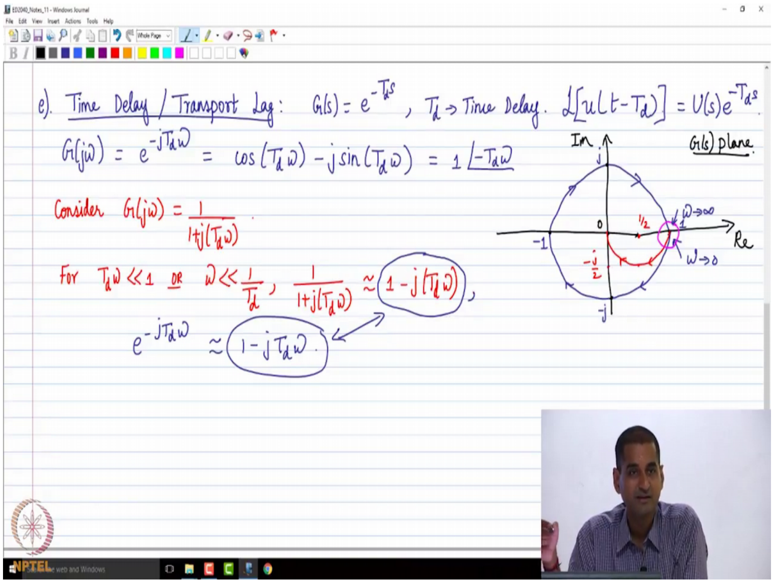

Before we go to second order factors, I am just going to do another important factor which becomes a very useful for engineering applications right. Because if you recall when we did transient response analysis, I talk to you about time delay right. In many physical systems, you are always going to have a time delay. So, what is meant by a time delay, I give an input now, the output starts increasing from 0 after a finite time interval right which I cannot neglect. So, then there is time delay.

So, for example, you know like I can have friction resulting in a time delay right. So, let us say I take this desk, I give a force, the desk does not start to move immediately right. So, I may need to overcome the static friction before the desk starts to move right that is the time delay. In let us say you know like in a fluid systems; sometimes I may give an input you know like I you will have some transmission delay before the fluid pressure starts to build up right. Then what happens is that like we have a finite time delay and we need to incorporate that correct.

So, if you recall what was the transfer function of the time delay some people call it as a transport lag. So, because if you look at process control and so on, they will call it as transport lag, because in process control they are include a interested in you know like some fluid systems, fertilizer plants, chemical plans and so on right, they call it as a transport lag right. So, essentially it is a time delay right.

So, what is the transfer function of a time delay, e raise to the power minus T s right of course, T d s or T s right. So, what did I put T d for the time delay?

Student: T d.

T subscript d right so, T subscript d was a time delay right. So, this is the transfer function of the time delay. How did we get it? Because if you recall we have if you had a time delay my input excitation becomes something like this right u of t minus T d. Then if you take the Laplace of this, what is going to happen you are going to get U of s times e power minus T d s that is why the transfer function of the time delay is taken as e power minus T d s right.

Let us construct the Nyquist plot for this right. So, what will happen to G of j omega then this will become e power minus G T d omega right. And what is this, how can I rewrite this; this is cos T d omega minus j sin T d omega right that is the real part and the imaginary part correct. I am just using the Euler's relation right. And then like what can you say about its magnitude?

Student: 1.

One Phase?

Student: (Refer Time: 03:28) T d omega (Refer Time: 03:32).

Minus T d omega right, correct. Do you agree right? So, anyway I need not have even written the imaginary and real part, I can just observe the exponential function this is just a complex exponential. And the exponent does not have any real part right pure imaginary exponent right. So, you can immediately get this right.

So, what do you think is going to be the shape of this particular sinusoidal transfer function. If I want to plot the real part and imaginary part right, what do you think is going to be the shape as omega goes from 0 to infinity. I think this thing should give you the answer, the magnitude is always one. So, what can you say?

Student: (Refer Time: 04:27) circle.

Circle right, it is a unit circle right, is not it. The magnitude is always 1 right and the phase goes like minus T d omega right. So, if you plot it, so I am just going to try my best to draw a circle freehand. So, let us see so how it goes. So, essentially I am going to get a circle ok, which essentially goes like this that is that is you have a this one. This is omega tending to 0 as omega tends to infinity, once again I am going to come back to the same point right, so that is that is what is going to happen to my circle all right. This is the a Nyquist plot of this transport lab.

Now, let us on top of this, let me superimpose the bode diagram sorry the Nyquist plot of this first order factor. So, let us consider the first order factor 1 divided by 1 plus j times T omega right which we have already drawn. What was that that was a semi circle of radius 1 by 2 right which was centred at oh sorry right. So, this is 1 by 2 right, and let us say this is sorry that is j by 2 it is a minus j by 2 this is minus j this is let maybe a careful let me use the same colour, so that this is 1, right this is minus j, this is minus 1, and this is plus j right.

So, so essentially we had a semicircle as the plot of 1 divided by 1 plus T j omega right that is what we have right. Why am I making this comparison you can see that immediately at low frequencies, you see that the two curves approximately are the same. Do you agree?

So, you see that at low frequencies, you know like the Nyquist plot of e power minus T d j omega and 1 plus 1 1 divided by 1 plus j T omega or almost the same. Of course, I am assuming to be very if I want to be very careful I should say 1 plus T d omega oh sorry this should be yeah because T d is a time delay so that is what right I make capital T equals T d right.

So, at low frequencies you know like I am going to have what to say this thing to be true. So, you can immediately see that for T d omega very much less than 1 or omega which is less than 1 by T d, you can immediately see that 1 divided by 1 plus j T d omega the magnitude is nothing but sorry not the magnitude the function itself, I can approximate it as 1 minus j times T d omega right, if T d omega is very small.

Similarly, what can I say about e power minus j T d omega e power minus j T d omega can also be approximated as 1 minus j T d omega for low frequencies right, so that is what we have correct. So, you can immediately see that at low frequencies you know like the functions become the same. And why did I what to say point this what to say essentially highlight this point here, I told you that you know like the function e power minus T d s in fact, an infinite order factor right. If you recall our discussion then right so then we have to do some finite order approximations right; so, that we can get proper transfer functions if you recall when we want to analyze systems with time delay.

What was what were the couple of approximations that we discussed, one was this another was the bode approximation right. So, I hope now it is clear how why this first order approximation make sense right. And it is also very clear when you can apply them. See suppose if I make this first order approximation, I approximate e power minus T d s as 1 minus T d s right. I can use it as long as I am subjecting the system to free inputs whose frequencies are much lower than 1 divided by T d ok, so that is something which we need to analyze.

See, for example, let us say T d is let us say you know like point naught one then I need to ensure that you know like I am essentially subjecting the system to frequencies which are lower than 1 by point naught 1 alright, so that something which I need to be careful about; So, then that approximations very very reasonable ok.

So, why am I pointing this because we will see that in many engineering systems, we may need to approximate time delay and that is where this discussion helps us right. It helps us in taking a call as to physically when you can approximate use these approximations, is it clear. So, I just use Nyquist plot to show you that you know like when this approximation is reasonable. So, of course, the at high frequencies two curves deviate from one another as you can clearly observe fine. Any questions here, yeah.

Student: Sir, taking to omega tending to 0.

Omega tending to 0 where?

Student: (Refer Time: 10:59).

On the top ok, no omega tending to 0 it starts from see at omega equals 0 it starts from 1 0, it comes into the fourth quadrant; as omega tends to infinity once again it goes and finishes at 1 comma 0. So, for this transport lag, the starting point and the ending point are the same.

Student: (Refer Time: 11:23).

Student: You get this phase angle plus minus 9.

Which one?

Student: Sir, (Refer Time: 11:29).

No, see that is where you know like see you need to essentially use this visualization along with this mathematical block ok, because this stand inverse function is also periodic right. We need to look at it from that perspective. If you plot the what to say if you keep on plotting the values of the real part and the imaginary part, this is what you will get ok, so that is where we end up with ok.

So, as far as the Nyquist plot of the transfer function which corresponds to the transport lag or time delay is concerned, it will be a unit circle so that is what will happen fine yeah. So, now I am going to just briefly discuss the second order factor, but I am going to leave it to you as homework ok.

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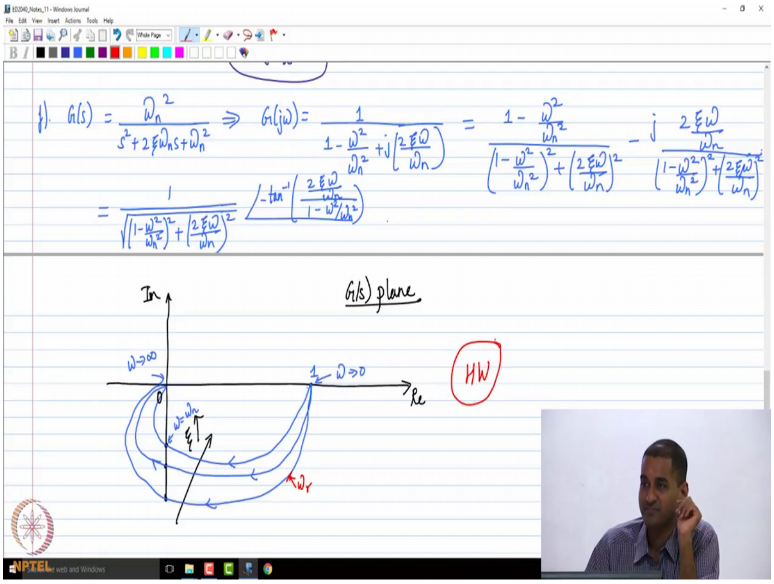

So, the detailed plot so let us go and plot for G of s to be equal to omega n square divided by s square plus 2 zeta omega n s plus omega n square. So, this will be 1 divided by sorry this will we have already done this when we did bode plot. So, G of j omega is going to be 1 divided by or 1 minus omega square by omega n square plus j times 2 zeta omega by omega n right. So, this is something which we which we have already know.

So, what are we going to get for the real part and the imaginary part the real part is going to be 1 minus omega square divided by omega n square the whole thing divided by 1 minus I am just multiplying by the conjugate of the denominator right. So, I am going to get this factor in the denominator that is the real part. And then I will get minus j times 2 zeta omega by omega n divided by 1 minus omega square by omega n square whole square plus 2 zeta omega by omega n whole square ok, so that is we will get right.

So, in terms of magnitude and phase are the same things I can rewrite it as 1 divided by square root of 1 the real part square plus the imaginary part square right. The real part square is this; and then the imaginary part square is 2 zeta by 2 zeta omega by omega n whole square right. And the phase is going to be minus tan inverse of 2 zeta omega by omega n that will be 1 minus omega square divided by omega n square that is going to be the phase.

So, this part I am going to leave it to you as homework at as to how we got this Nyquist plot for the second order factor. So, I am just going to plot the Nyquist plot you know like you just need to complete it based on what we have discussed now ok. I just adopt a similar approach like just plot the sorry this is not that omega, this is real part ok, this is the imaginary part. And this is the G of s plane, this is the origin.

So, you will observe that you know like all curves are going to start at 1, when omega tends to 0. And what happens is that the Nyquist plot takes a takes the shape and then like comes like this and ends like this so that is what happens for a second order system. And of course, we are going to have a family of Nyquist plots for different values of zeta ok, zeta keeps on increasing it comes closer and closer ok.

So, this is the trend as zeta increases. And the, it cuts the imaginary axis when omega equals omega n because that you can immediately see that omega equals omega n right. So, you can immediately see that the, what to say the real part is 0 right. If you look at the real part for any value of zeta at omega equals omega n, the real part is 0. So, it cuts all curves cut the imaginary axis at omega equals omega n ok. So, this is what happens at omega tends to infinity, but these are the points where omega is equal to omega n omega equals omega n is the corner frequency if you remember in the bode plot right.

So, what I suggest you to do is I essentially construct a similar table as omega tends to 0, omega tends to omega n, omega tends to infinity, and also omega tends to omega r. See omega r is the point where the, what to say how do you get omega r oops graphically I can get omega r, what is omega r is the frequency at which the magnitude is the maximum. Magnitude is a maximum means the point should be the farthest from the origin.

So, for example, here this will be omega r right, so that is where it is farthest from the origin right, you get the maximum magnitude. So, you can also identify resonant frequency from here right. So, you please look at the resonance frequency also on a cone for it ok. Of course, please remember the resonant frequency is what to say reasonable one is holds only when there is damping ratio is between 0 and 1 by root 2. So, this generating this plot I leave it as a homework ok. So, I have told you the answer I have also told you the process right. I given you the real part imaginary part phase magnitude you know you just need to construct the table and draw this right. It just takes the shape.

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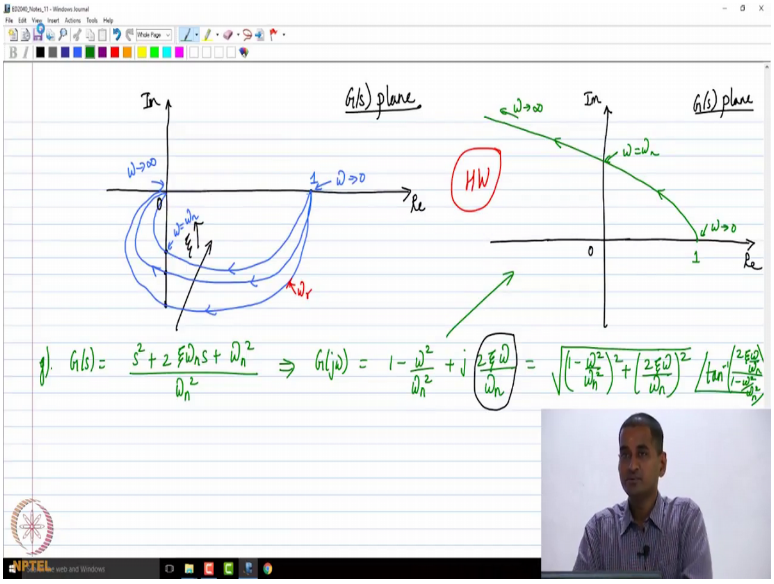

So, then you also need to do the reciprocal so that is g. What is a reciprocal it is going to be s square plus 2 zeta omega n s plus omega n square divided by omega n square. So, this implies that G of j omega is going to be 1 minus omega square by omega n square plus j times 2 zeta omega by omega n that is the real part and the imaginary part.

So, the phase the magnitude and phase is going to be 1 square root of 1 minus omega squared by omega n squared whole square plus 2 zeta omega by omega n whole square. Now, the phase is going to be tan inverse of two zeta omega by omega n divided by one minus omega square divided by omega n square. So, that is what we would get as the phase.

So, so once again if you plot this you can immediately observe that if you look at the real part and the imaginary part what can you observe as far as this particular factor is concerned. What can you say about the imaginary part for any zeta?

Student: (Refer Time: 19:26).

It is always non-negative. See, the imaginary part is plus 2 zeta by 2 zeta omega by omega n right. So, it is always going to be non negative. What can you say about the real part?

Student: (Refer Time: 19:42).

It is first going to be positive, and then it becomes 0 at omega n then becomes negative. But you can see that the real part monotonically becomes more and more negative after that ok; and the imaginary part becomes more and more positive after that. So, you will see that the Nyquist plot of this will look something like this.

So, this will go like this. So, this is what will happen as omega tends to 0; this is what happens as omega tends to infinity at this point omega is equal to omega m. So, I am just giving you the structure of the Nyquist plot. So, please complete once again for both these factors draw the table and then observe so that is what I want you to do. Is it clear?

So, you will see that once again you know like the Nyquist plot of the factor and its reciprocal right does not have a correlation as was the case in the bode plot ok. So, please remember that that is an important observation here. So, this is the Nyquist plot. So, I just a given you an exposure to introduction to Nyquist plot. So, a Nyquist plot is also a visualization of the sinusoidal transfer function. So, where we plot the real part of G of j omega this is the imaginary part of G of j omega in the complex plane right.

So, and important thing to notice is that like unlike the bode plot, I cannot split the transfer function into individual blocks and then draw the individual Nyquist plots and just add them in a very straightforward manner you know, we cannot do that ok. So, essentially I just wanted to expose you to this. What we are going to do tomorrow is that we are going to see how one could use this Nyquist plot to our advantage in stability analysis which is relevant to this particular course ok.

So, I am going to discuss what is called as a Nyquist stability criteria; And we will see how that is going to be used in stability analysis of closed loop systems right, so that is another technique ok. So, I am just going to discuss that technique with you tomorrow before we go to control design based on frequency response methods right, so that is that is going to be the action plant ok. So, we will meet in the next class and continue this discussion.