Control Systems

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Lecture – 57
Nyquist Plot 1
Part – 1

So, let me get started with today's class. So, today what we are going to do is that like we are going to look at what is called as a Nyquist plot right. So, if you recall where what we have been discussing we have been looking at frequency response. And what was frequency response. It is the response of the class of systems under steady to sinusoidal inputs right. And we saw that if we have stable LTA systems, if you give a sinusoidal input of frequency omega, the steady state output was also a sinusoid of the same frequency, but scaled in amplitude, and shifted in phase right.

And the entity that becomes extremely important in that case is what is called as a sinusoidal transfer function right G of j omega right, so that is what we have been looking at. And what we have been doing is that like we learned about the bode plot, you know like where we figure out how to graphically illustrate, or represent the sinusoidal transfer function in terms of the magnitude and phase right as omega varies from very low frequency to very high frequencies right, so that is what we did in the bode diagram. So, the Nyquist plot is once again a visualization of the sinusoidal transfer function, but a slightly different visualization ok. We will see what this is.

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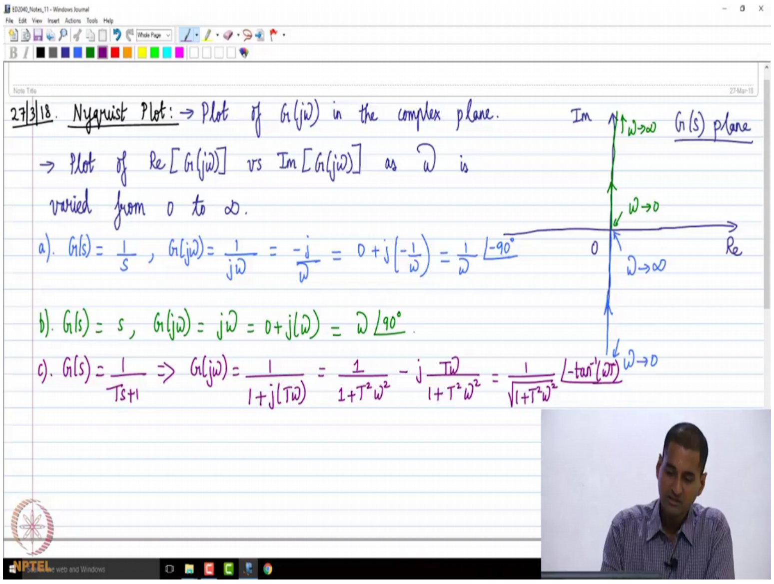

So, what is this Nyquist plot, ok, so a Nyquist plot is nothing but the plot of a G of j omega, which is a sinusoidal transfer function ok, in the complex plane that is it, you know like, so it is pretty straightforward to understand. So, what we do is that like, we essentially plot the real part of the sinusoidal transfer function, which is the imaginary part of that sinusoidal transfer function, in the complex plane ok, so that is the Nyquist plot ok.

So, we some people will call it as the G of s plane to indicate that it is no longer the s-plane as its that is we are not tracing the variable s, but rather we are tracing the transfer function right G of s right, so that is also a complex valued function right. So, people will call it as the G of s plane, or G of j omega plane, and so on.

So, it is essentially the plot of the real part of G of j omega versus the imaginary part of G of j omega, as omega is vary from 0 to infinity ok, so that is what happens here right. So, as we vary omega from low frequencies to high frequencies, what happens to a real part and the imaginary part.

So, may ask the question, you know like see what is the application of this Nyquist plot right, how do you use it in practice, we are going to look at that right. And we are going to see how it the Nyquist plot can be used for even like a stability analysis right, and even control design ok, so that is something, which we are going to look at as we go along.

So, what we will do is that we will once again construct the Nyquist plot of individual blocks that we have been learning right. And then we will identify some difference between the Nyquist plot, and bode plot ok, as we go along ok.

So, let us start with you know like a of course, I am not starting with a constant, because a constant is this plot around the real axis right. So, if I take my first building block as K, like what we did in the bode diagram, so that K is going to be either, plotted on the positive real axis, or the negative real axis, depending on its sign right, so that is pretty straightforward.

So, what happens from the other factors on it. Suppose let us say I consider 1 by s right, which was another factor that we consider, so then what happens, G of j omega is going to be equal to 1 divided by j omega, and this can be rewritten as minus j by omega, do you agree, correct.

So, I can I just multiply by multiply and divide by j, I just write it as minus 1. So, this I can write it as 0 plus j times minus 1 over omega. So, essentially what I am doing is that I am just writing as the real part, and the imaginary part right. And even, I can follow an alternative representation of a complex valued functions right, I can write their magnitude and phase right, correct. So, I can write this as, what is the magnitude of this function now?

Student: 1 by omega.

1 by omega, right. What is its phase?

Student: (Refer Time: 05:05).

Student: Minus 90.

It is going to be minus 90, so that is what we are going to get, because so you see that the function is always going to be on the negative imaginary axis right. So the plot of this particular factor right.

So, consequently I can rewrite like this. And so, if I want to plot this right, so how will this look like, so let us say, I plot this on the G of s plane right, so what will happen, as omega starts from 0. Where are we going to start from?

Student: (Refer Time: 05:38).

So, essentially I am going to start from minus j infinity right. And as omega tends to infinity, where am I going to end up with, I am going to end up at the origin right, so that is what is going to happen. So, essentially what is going to happen is it, the Nyquist plot of this particular factor is going to be along the negative imaginary axis. This is omega tending to 0, and this is omega tending to infinity right, so that is the Nyquist plot of 1 by s ok, so that is what happens here right.

So, now let us essentially plot for a few more factors then we will discuss a few points right. So, now let us plot it for the derivative factor G of s is s, so then G of j omega becomes j times omega. So, this I can rewrite it as once again 0 plus j times omega. So, 0 is the real part, omega j omega is the imaginary part right. So, so in terms of magnitude and phase, what will happen, I can rewrite this as omega times omega n plus 90 degrees right, so that is what is going to happen.

So, what is going to happen to the Nyquist plot of this particular factor, at very low frequencies as omega tends to 0, where am I going to start from, I am going to start from the origin right, because the value becomes 0. And as omega tends to infinity, I go to infinity along the imaginary axis ok, so that is what is going to happen. So, for this, the Nyquist plot is given in green right. So, you see that it is just along the positive imaginary axis ok, so that is what happens for this derivative term right ok.

So, now and let us go forward, and then look at other factors right. So, what is the next factor that we consider?

Student: (Refer Time: 07:32).

G of s is 1 divided by T s plus 1 right.

Student: T s plus 1.

So, this I can rewrite if I want to figure out that sinusoidal transfer function, this can be written as 1 divided by 1 plus j times T omega right, correct. So, if I multiply and divide by the conjugate and then simplify, what will I get, I if multiply and dividing by the conjugate, which is going to be 1 minus j times T omega, I am going to get, 1 divided by 1 plus T squared omega square minus j times T omega divided by 1 plus T squared omega squared do you agree that is going to be the real part, and the imaginary part. So, in terms of magnitude and phase, how can this be rewritten?

Student: (Refer Time: 08:31).

What do you think will be the magnitude of this transfer function, it is going to be just 1 divided by square root of 1 plus T square omega square.

Student: Square root.

See by now, I think I am sure all of us are getting more and more comfortable right. So, you look at the function itself right. So, if you have this complex valued function, you want to find the magnitude. It is just the basically you just take the you have a ratio, you just take the ratio of the individual magnitudes right, so that is what you have right. So, you just do that.

And what is going to be the phase once again, it is going to be the algebraic sum right. So, what is going to be the phase here, the phase is going to be minus tan inverse of omega T right, so that is what we are going to have for this particular factor right, so that is what we get, is it clear.

So, so now I am I am just going to essentially help you with something you know like, which is a good practice in plotting Nyquist plot. So, you will see that say as suppose to a bode diagram, you know like for even you want to plot the Nyquist plot of a general transfer function, it is a little bit more challenging, because in the bode plot, what we did, you give me any transfer function, I break it down into those individual blocks. And then essentially plot the magnitude and phase plot of the individual blocks, add them that is it right. So, the technique was pretty what to say straightforward right.

So, but we need to of course, be careful in plotting the individual factors of bode diagram right. But, in Nyquist plot, we do not have what to say, luxury right. So, you give me a higher order transfer function, I need to essentially plot it, as it is right. So, it becomes a little bit more challenging to do it by hand.

So, I am just going to essentially share a process, which has helped me, you know like do these things right. So, we are not going to look at Nyquist plots of very complex factors at least, you know like when we discuss things analytically, because I am not going to ask you to draw the Nyquist plot of you know like very high order factors by hand, in your exams ok, you can be assured of them. So, if you want you to analyse, I would ask you to use MATLAB, but we should know what the concept is right, so that is why I am teaching you the theory behind it right.

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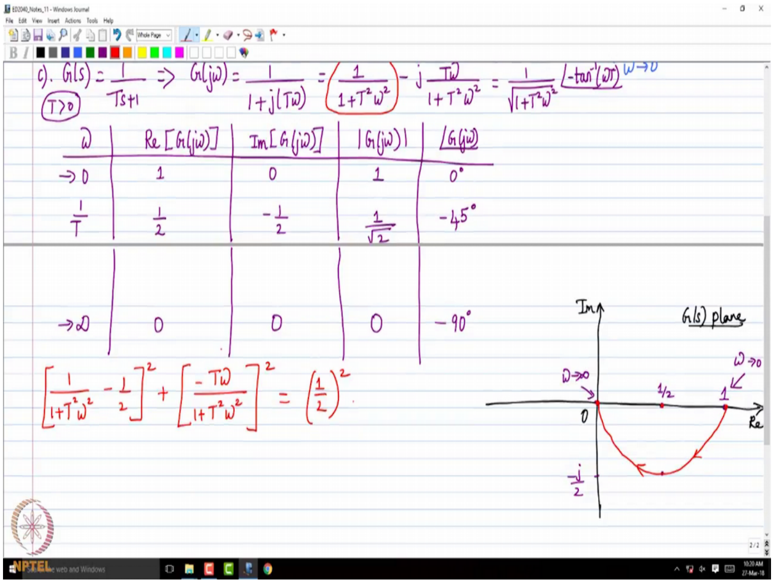

So, one process, which helps is to figure out what happens at critical frequencies right, and the limiting frequencies, and even some frequencies in between right. So, let us do that. So, typically it is useful to construct a table, you know like as to what happens to the real part of G of j omega, and the imaginary part of G of j omega, as we vary at various critical points critical values of omega, and also it is magnitude and phase ok.

So, let us say, you would be construct a table like this ok. So, as omega tends to 0, what do you think will happen to the real part and the imaginary part? Look at the real part, what is the real part?

Student: 1 (Refer Time: 11:55).

1 divided by 1 plus T squared omega square right. As omega tends to 0, what do you think will going to happen to this?

Student: (Refer Time: 12:02).

It is going to go to?

Student: 1

1 right. What can you say about the imaginary part?

Student: 0.

0 right. Please note that, the imaginary part is always going to be negative right, and the real part is going to be positive. So, in the complex plane, in which quadrant is this Nyquist plot going to lie?

Student: (Refer Time: 12:25).

See you can see that for all omega between 0 and infinity, because capital T obviously, we take it to be of course, we take capital T be to be greater than 0, for this discussion right. So, so you can immediately observe that the real part is always a what to say, I would say non-negative, and the imaginary part is always non-positive right, for all values of omega between 0 and infinity right. So, which quadrant do you think the Nyquist plot of this factor would lie in, in the complex plane?

Student: (Refer Time: 12:58).

In the?

Student: (Refer Time: 13:01).

This quadrant alright. What is this quadrant called as, 1, 2, 3, or 4?

Student: (Refer Time: 13:08).

4th quadrant right, so that is what it is right.

So, essentially you will see that the Nyquist plot will lie in this quadrant right, so that is what will happen. So, now, you tell me right, what happens to the magnitude of G of j omega as omega tends to 0, obviously, it is real path square plus imaginary path square, and then take the square root. So, the magnitude is going to be 1. What about the phase, it is going to be 0 degrees right, as you go to 0. Now, is there any other critical frequency, you know like having a drawn the bode diagram, can you think of any other critical frequency for first order factors?

Student: (Refer Time: 13:47).

So, it is the corner frequency right 1 by capital T. What happens is 1 by capital T to the real part and the imaginary part?

Student: (Refer Time: 13:57).

You see that the real part becomes 1 by 2. The imaginary part becomes minus?

Student: (Refer Time: 14:03).

1 by 2; of course, I am I am not writing j, if you are very particular, you can write minus j by 2, but I am just writing the component alright. So, j is it is understood that we have the j right. So, then what happens to the magnitude on the phase?

Student: 1 by 4.

Magnitude is?

Student: 1 by 4 1 by 4 1 by 4.

1 by?

Student: Root 2.

Root 2 alright. What about the phase?

Student: Minus 45.

It is going to be minus 45 that is what else going to happen. So, let us continue this table ok. So, now, as omega tends to infinity, what do you think is going to happen to the real part and the imaginary part? Both will go to?

Student: 0.

0 right 0. And what do you think will happen to the phase?

Student: 8.

It will essentially go to minus 90, we will see why ok. So, so we will we are going to interpret it in a different way ok, like we will see why that happens, it happens like this ok, so that is something, which we will figure out, when we draw the Nyquist plot. So, let us construct the Nyquist plot right, so and then see what happens here. So, I have the real axis, the imaginary axis right, this is the G of s plane, and we want to plot the Nyquist plot right.

So, now if I want to plot the Nyquist plot of this function right, I know I know that there are certain critical points right. So, 1 is one critical point, 1 by 2 is one critical point right. So, because at 1 by when at frequency of capital 1 by T, you know like we are having the real part as 1 by 2. And the imaginary part as?

Student: (Refer Time: 16:09).

Minus?

Student: (Refer Time: 16:11).

J by 2 alright. And then like 0 and 0 is also important, because that is what happens when omega tends to infinity all right. So, this is omega tending to 0.

Now, what happens to the Nyquist plot of this particular factor ok. We have identified three points right. Now, I need to draw a curve, which passes through these three points right. So, of course, you know like given these three points, I can draw so many curves right. So, we will see that this Nyquist plot has one specific structure right. How do we get that right, so let us let us try it out?

So, what we will do is it like you evaluate and tell me, what you get when you take the real part, which is 1 divided by 1 plus T squared omega square subtract half from and take the square. And then like you take the imaginary part square, which is minus T omega divided by 1 plus T squared omega square, and square it. And then like if you equate it, what do you get, can you quickly solve and tell me.

Student: (Refer Time: 17:15).

Yeah.

Student: Here, imaginary part is minus T omega (Refer Time: 17:20) the imaginary part is equal is equal to minus T omega times the real part.

Minus T omega times a real part: yes.

Student: So, if you plotting, imaginary part verses real part. It just be set of lines. Let us look minus T omega then omega keeps varying (Refer Time: 17:41).

So, you are see please note that, the real part and the imaginary part are parameterize with respect to omega. So, in fact, what is going to happen is that; what you are saying is correct, if I plot for a particular omega, and I vary the real part, but does not work like that.

See what you are saying is that you are saying imaginary part is minus T omega times a real part ok, if you plot real and imaginary, if you take the real part on the abscissa, and the imaginary part on the ordinate, you will essentially get a series of straight lines, you know like for any for a for a family of omega. But, then we are not varying real part. See real part is not the independent parameter here, omega s. So, for a given omega, the real part is not under our what to say control, you know we get we get what it is, is it clear.

Student: (Refer Time: 18:35).

So, please you will understand; why I did that right. I am asking you to evaluate it kindly simplify this, and tell me what answer you get. See I have given you this, because I a priori know, what is the answer right. So, I am just asking you to do it, and tell me.

Student: 1 (Refer Time: 19:31).

1 by 4 right. I have rewritten it as 1 by 2 whole square. So, what is this equation correspond to. So, if you recall our geometry right. So, what is such an equation?

Student: (Refer Time: 19:46).

It this essentially this equation will trace out a circle right, which is centred at?

Student: (Refer Time: 19:55).

1 by 2 comma 0 right. And of radius?

Student: 1 by 2.

1 by 2 right and that is what is the Nyquist plot here ok. So, this starts like this ok. Of course, I am drawing a free hand diagram. So please pardon if it does not look like a semicircle, you know like, so essentially it goes something like this right. So, it is so the Nyquist plot of this factor is going to be a semi-circle, you know like which is essentially going to be centred at 1 by 2, and starts at 1, 0, and ends at 0 comma 0 ok, so that is going to be the Nyquist plot.

So, the I am going to draw an arrow like this, to indicate what happens as omega increases from 0 to infinity right, so that is what is the Nyquist plot of this particular factor 1 by T j omega plus 1 right, so that is the Nyquist plot ok, is it clear, how we got it.

Of course, you may ask the question, hey how did you get this in the first place right, how did you what to say, pull out you know this equation, and write. You know like of course, that comes with the experience ok. So, there is no harden straight answer to that, that is why, you know plotting Nyquist plot is both art and science right.

As you as you keep on doing more examples, you get an intuitive feel of what would happen, you know how that factor would look like. And sometimes it is going to be very non-intuitive you know, I will tell you why right. So, it that will become clear as soon as we plot the next factor ok, so that is something, which I which we will discuss shortly right. So, it so happens that the Nyquist plot of this 1 by T s plus 1 is going to be in the form of a semi-circle of radius 1 by 2 ok, so that is that is the Nyquist plot.

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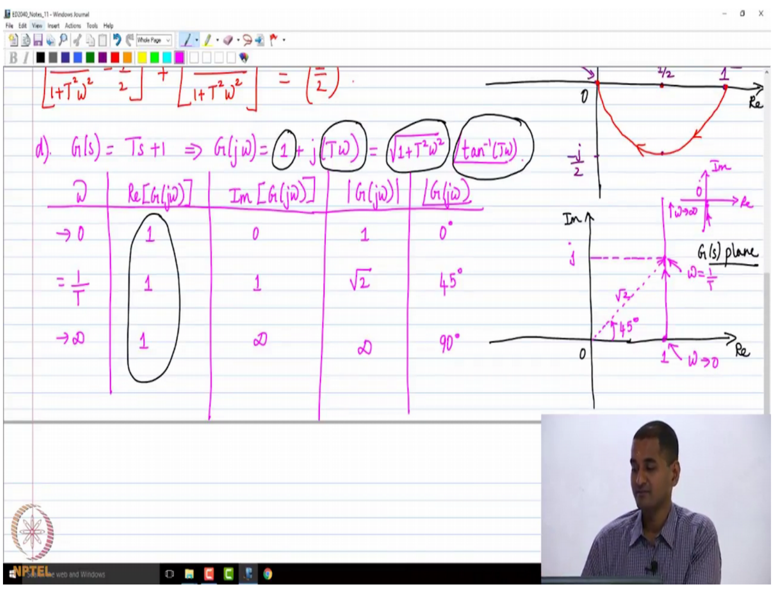

So, let us we will gain some insight into this process. And why Nyquist plots, you know like are a little bit more challenging. You know like once we do the next factor, which is the reciprocal of this 1 by T s plus 1 right. See when we plot a bode diagrams right, what would we observe, if we have two factors, which are the reciprocals of one another, what happened to the bode diagram?

Student: (Refer Time: 22:04).

They happen to be just mirror images or reflections above the 0 decibel line right. So, based on how we took it right. So, essentially our low frequencies values, where the 0 decibel lines. So, about that line, it was just a mirror image. Of course, we need to be careful there right. So, of course, if I took 1 by s and s, it is also a reflection about that what to say 0 decibels, and omega equals 1 point right. So, it just flips gets flipped about that right. About the 0 double line which passes through that point right ok.

So, now let us see what happens in the Nyquist plot right. So, this is G of s, so what happens to G of j omega, it is essentially 1 plus j times T omega. So, this will give me, the magnitude as 1 plus T squared omega square, and the phase is going to be just tan inverse of T omega.

See by the way before we go any further, I hope it is clear, why this phase was minus 90 right. So, you look at how this curve comes to 0. So, if you look at how this Nyquist plot comes to 0, approaches the origin, you see that it approaches origin, if you zoom in at the origin, it comes in like this right.

So, if you zoom in further and further, if you draw a vector from the origin to that curve that is going to be at an angle of minus 90 ok, so that is how we put that is why I put minus 90 there, is it clear ok. Why this happened right, so that is a reason for the phase becoming minus 90 as omega tends to infinity for the factor 1 by T s plus 1 ok. So, this is what happens here right.

And now, let us construct a similar table right, omega real part of G of j omega, the imaginary part of G of j omega, the magnitude of G of j omega, and the phase of G of j omega ok. So, let us construct a table in this manner. So, let us start filling in this table, using a similar approach ok. So, let us say, as omega tends to 0, what happens to the real part and the imaginary part?

Student: (Refer Time: 24:43).

1, 0, 1, 0 degrees right. The magnitude is 1, phase is 0 degrees. So, at omega equals now 1 by T what happens?

Student: 1.

You will get 1.

Student: (Refer Time: 25:01).

1 right, root 2, and 45 correct. Then as omega tends to infinity, what do you get?

Student: (Refer Time: 25:15).

1.

Student: (Refer Time: 25:18).

Infinity.

Student: (Refer Time: 25:22)

Infinity.

Student: 90.

90 right; that is what do you will have, am I correct ok. So, I am just looking at the equation about, you know that is all we are doing right. See this is the real part. See if you look at it, the real part is always 1 for this particular factor. The imaginary factor, a imaginary part is T times omega all right. The magnitude is this, and the phase is this ok. So, you see that I am just calculating for various omega, you know I am just writing the values wrong that is all we are doing. Nothing extraordinary here right, so just a pretty straightforward.

So, now if I plot the real part verses the imaginary part in the G of s plane, what are we going to get, what will happen, at omega equals 0, let us say we start from 1. So, please note that, the real part is always 1, you could what you say really observe that right, so because you can see that the real part of the sinusoidal this factor is always 1. So, what can you say about this curve, if the real part is 1, what can you say?

Student: (Refer Time: 26:40).

And the imaginary part is always non-negative, so what can we immediately say about this 1, this is going to be a vertical line right starting at 1 comma 0, and then just going up, you know which is parallel to the imaginary axis. This is what happens as omega tends to infinity right. At omega equals 1 by T, it is going to be j right, so that is what is going to happen.

Student: (Refer Time: 27:10).

Sorry. Is this correct, at omega equals 1 by T, the real part is 1, and the imaginary part is j right. And so, the magnitude is going to be root 2 ok, then the phase is going to be 45 degrees right. So, I am just drawing it on the same curve right; so just to convince ourselves that that is the case.

So, you see that this is what to say, Nyquist plot of the factor T s plus 1. Now, you look at the Nyquist plot of the factor 1 by T s plus 1 and T s plus 1, can you draw some conclusions like what we did in the bode plot, no right. One is a semi-circle, another is a straight line.

So, now you understand, you know like why progressively, we slowly understanding right. Why in the Nyquist plot is a drawing a Nyquist plot is a different ballgame right, and sometimes it is a bit of a challenge right. So, in bode diagram, you know like if you had like reciprocals, you know like we immediately flip the curve right. And we could immediately process it, and go on. But here, it is not so straightforward, because the shape itself is different.