Control Systems
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Lecture – 52
Bode Plot 2
Part – 2

So, now let us come to the phase plot.

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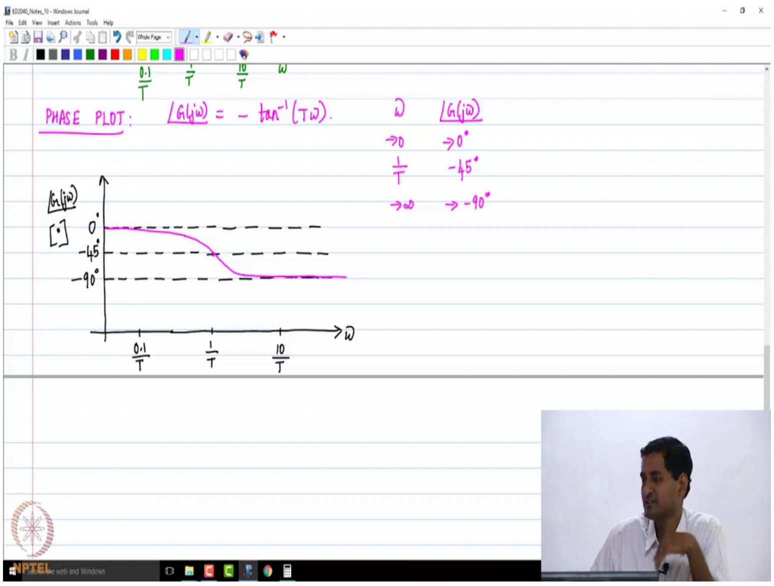

So, what do you think will happen to the phase? So, please note that the phase of this particular factor is going to be minus tan inverse of T omega right. So, what do you think happens to the phase? So, if you plot if you calculate the phase of omega versus G of j omega, what is going to happen? As omega tends to 0, the phase tends to 0 degrees.

When omega is 1 by T which is the corner frequency, what is the phase? It is going to be minus 45 degrees right. So, you are going to get minus tan inverse of 1 right. So, and then like as omega tends to infinity, the phase will tend to minus 90 degrees ok. So, it is a smooth curve right. So, that is what is going to happen.

So, if you plot the phase plot, this is what is going to happen ok. So, let us say we plot against omega or the phase of G of j omega in degrees, what are we going to get, right. So, let us let us do that. So, once again I take 1 by capital T which is a corner frequency.

Let us say I take 10 by capital T and 0.1 by capital T. And say typically, you will see that in most bode plots, you know like we essentially plot the bode plot around the frequency range of interest right and by and large you know like you choose a frequency range and you go one or two decades either side. You know that is typically the range of interest right, that that covers a pretty broad range ok. So, that is why I am taking 1 decade above and 1 decade below a particular frequency ok.

So, that is what is happening. So, essentially, we are going to have 0 degrees right. And let us say this is my minus 90 degree line and let us say in between I have minus 45. So, all of us know that the how the tan inverse plot looks like. So, I just need to make sure that you know like I passed through 1 by T and minus 45 right that is important.

So, when I start from low frequencies you know like I am going to have values which are very close to 0, then what is going to happen? I am going to have an inflection point at 1 by T and minus 45 then the curve just goes to minus right. So, that is how the phase curve is going to look like right, correct, right. So, that that is the phase plot right of this first order factor ok, yeah.

So, what we have done is it like, we have plotted both the magnitude plot on the phase plot for the factor of the form 1 by capital T s plus 1 right ok. So, now, the question becomes you know like what happens to the reciprocal; same story you know like let us quickly run through what happens through the reciprocal, but before we do that, let us look at this magnitude plot right.

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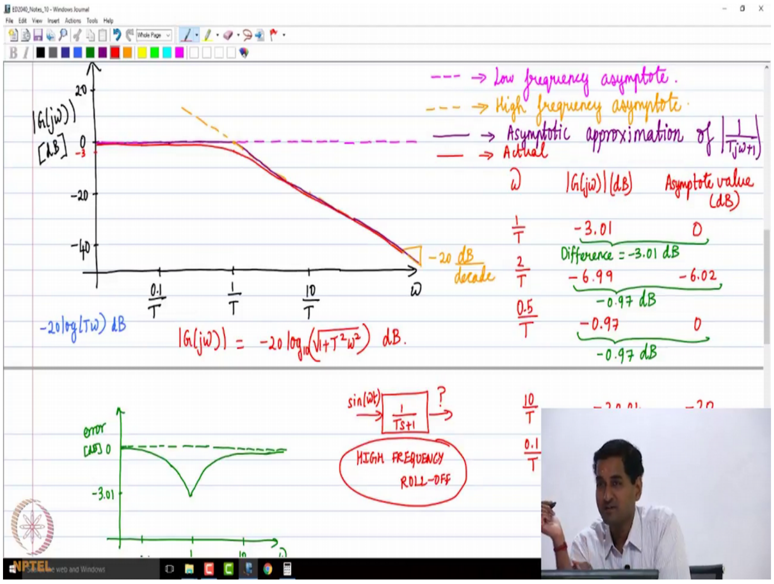

So, and on this plot let me plot the actual one right. So, the actual one is going to be, we are going to have an error of what minus 3 at the oops around like let us say minus 3 decibels at the corner frequency. So, the actual plot is going to be something like this, right. I am just drawing an approximate curve just to illustrate the point ok.

So, the actual curve is going to be something like a smooth curve like this which will actual. So, the red one is the actual right. So, that is just illustration that right. So, if you look at the magnitude plot of this factor 1 by T s plus 1 right. What can you immediately say about it is characteristic now? So, does it appear familiar with something which you already learnt you know?

So, you can observe that it has the characteristics of a low pass filter right. So, because what it does for if you transmit signals? Ok, let us say you know like you consider this 1 by T s plus 1 as a block and you provide sinusoidal inputs, right sin omega t or cos omega t and then like you ask yourself the question what am I going to get right as the output.

So, you see that if as long as my frequency is within 1 by capital T which is the corner frequency the by and large the magnitude of the input signal is not attenuated right. So, it allows it to pass; of course, there is a phase shift all right, but beyond a frequency sorry for a frequency beyond 1 by capital T, what is going to happen there is going to be an attenuation right. And higher the frequency, higher is the roll off ok.

So, what people call as roll off or high frequency roll off typically, there is just a term which one may encounter in frequency domain analysis ok, what is called as high frequency roll off it is just to indicate you know what I got what rate is my magnitude curve dropping at high frequency. For example, here the high frequency roll off is minus 20 decibels per decade ok, so, for the first order factor that we have plotted right.

So, because as frequency keeps on increasing, the essentially, the rate at which the curve is going is that minus 20 decibels per decade. That is a slope right, that is what is called as high frequency roll off.

So, you see that as frequency increases you know like I am going to have lower and lower magnitude. So, what is going to happen if I give a sinusoidal input whose frequency is let us say, 10 by capital T right. The magnitude is going to be scaled by minus 20 decibels what is minus 20 decibels in the, what you say absolute unit that is going to be one tenth right.

So, the magnitude of the sinusoidal signal with a frequency of 10 by capital T is going to be one tenth at the output. So, you can immediately see that you are having the low pass action right, low pass filter action. It is attenuating high frequency signals right. So, 1 by T capital T being the cutoff frequency you know like as far as the raw pass filter is our concerned, right ok.

So, you can immediately see that the definition of bandwidth also applies right. So, what is bandwidth? What is cutoff frequency? See you take any factor; cutoff frequency is a frequency at which the magnitude becomes lower by minus 3 decibels right when you compare it at the low frequency constant value.

Here the low frequency constant values 0 decibels, at what value does it becomes 3 decibels lower than the low frequency value? 1 by capital T, right. What is 3 decibels lower than the low frequency value here? Here, the low frequency values 0 decibel right. What is 3 decibels below 0 decibel, minus 3 decibel right?

So, you see that the transfer function of this form achieves 1 minus 3 decibels at capital sorry 1 by capital T which is the corner frequency. So, in terms of filtering terminology, that is called as a cutoff frequency right. And the region, frequency range between 0 and the cutoff frequency is what is called as a bandwidth.

So, if you map this term to the transfer function of let us say a sensor or an actuator right, let us say in practice you model a sensor or an actuator using this a first order term which is very common by the way in practice ok, what is going to happen is then this if you give me the transfer function, I can immediately tell you the bandwidth. Let us say I give you a transfer function for an actuator right as 1 by 0.1 s plus 1. I immediately know that T is equal to 0.1 by comparing with this.

So, what is going to be 1 by T, 10? So, cutoff frequency is going to be 10 radians per second right. So, the bandwidth of that actuator is going to be 10 radians per second right. So, you can immediately talk about bandwidth and cutoff frequencies right once you have the structure right. So, I am just talking from a practical perspective, you know like all these mathematics is fine. But from an engineering perspective you know like this these concepts are commonly used. So, please remember these ideas right, fine ok.

So, that is the actual curve. So, and that is the phase plot right. So, now, what is going to happen, when we have a factor of the form T s plus 1?

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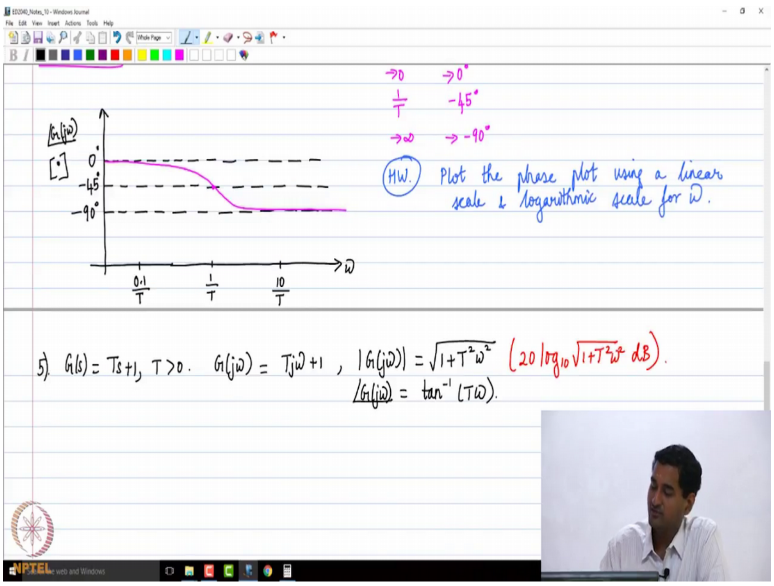

So, we consider 1 by capital T s plus 1. So, let us now consider the reciprocal right, so, T s plus 1. So, t is greater than 0. So, immediately we see that, G of j omega is going to be T j omega plus 1. So, this implies that the magnitude of G of j omega is going to be 1 plus t square omega square square.

Student: (Refer Time: 10:26).

Ok.

Student: Now this scale was.

Ah why do not you plot and check ok, good question ok. So, please go to MATLAB plot using a linear scale and a logarithmic scale and check ok, please ok. So, overlap on the same plot and then like, so, I will just write homework here just to make sure that you remember you know, all of us remember. I will also do that ok. Plot the phase plot using a linear scale and logarithmic scale for omega ok, good fine please do.

You assume some values for capital T and then take ok. So, let us come back to the reciprocal. So, if G of s t s plus 1, G of j omega is T j omega plus 1 and the magnitude is going to be square root of 1 plus T squared omega squared; so obviously, in terms of decibels, it is just going to be 20 log square root of t squared omega squared right.

So, what about the phase of G of j omega it is going to be just tan inverse of t omega. So, now, you are going to help me with it. So, it is pretty straightforward right to essentially conclude these things.

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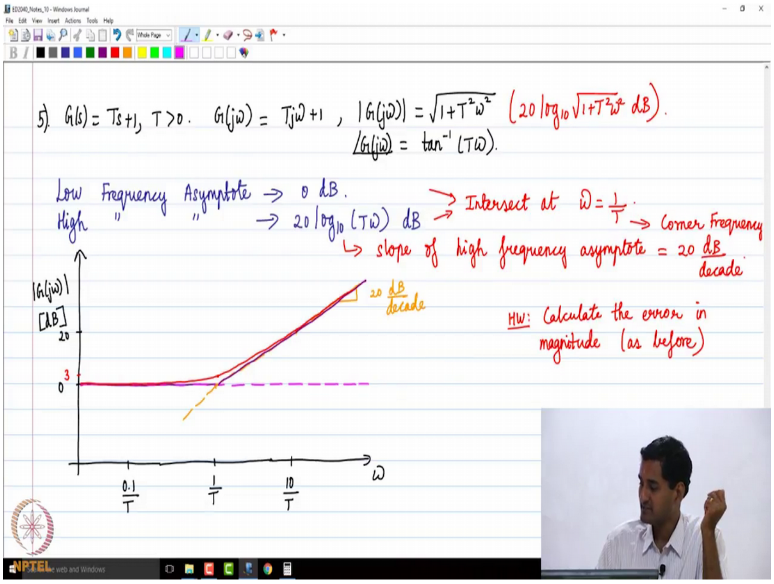

So, what is going to be the low frequency asymptote, for of the magnitude plot by the way. It is going to be the 0 decibel line right same as omega is lower than 1 by capital T. It is going to be 0 decibel right. It is going to tend to 0 decibel. What is the high frequency asymptote as a T omega becomes much larger than 1. It is going to be 20 log of T omega, see what was minus 20 now became plus 20; same concept right.

So, now when omega is much lower greater than a 1 by capital T, 1 plus T squared omega squared can be approximated as T squared omega squared. Take the square root you get to a T omega that is it right. So, you get 20 log of t omega ok. So, that is what you get in decibels. Now, where do they intersect? They intersect at omega equals 1 by capital T.

So, omega equals 1 by capital T remains the corner frequency ok. So, no change in that, right. So, this remains the same ok. Now, what is the slope of the high frequency asymptote? Slope of the high frequency asymptote is going to be plus 20 decibels per decade right. So, that is also pretty standard right. So, you repeat the same calculation, you substitute down in terms of what to say 10 times omega, you will get 20 plus 20 log to the base 10 t omega.

So, between omega and from omega to 10 times omega, you have an increase of 20 decibels right. So, that is why the slope is 20 decibels per decade right. So, then you know like, how would this magnitude plot look like in this case? So, once again if I plot the magnitude curve for this in decibels, what would happen? Let us say if this is 0, this is 20 right. So, let us say this is 1 by capital T, 10 by capital T and 0.1 by capital T. Let me just use the same colors as before ah. So, that we are consistent right. I used pink for low frequency, yellow for high frequency and let us use the same color.

So, let us say I use pink for the low frequency asymptote. So, the legend remains the same right as before. So, how do I draw the high frequency asymptote please know that, this is just an illustration; low frequency asymptote makes sense only at low frequencies; obviously, right. So, how do I draw the high frequency asymptote, 1 by T 0 is one point and 10 by at 10 by T is going to be 20 decibels right.

So, the high frequency asymptote is going to be this. So, which has a slope of plus 20 decibels per decade and the approximation is going to be this, that color and as homework, calculate the errors, the error in magnitude as before ok, but you will see that it is a very simple process, but I want you to do it right.

So, you will see that now at the corner frequency, you will have an error of plus 3 decibel and one octave above and below, you will have a, what you say error of 0.97 decibels whatever was minus becomes plus ok. And one decayed above and below, the error will be 0.04 ok, almost close to 0. So, the actual curve will be something like this ok. So, now, you see that the high frequency amplitude at high frequencies, the amplitude is increasing at the rate of the magnitude is increasing at the rate of 20 decibels per decade right.

So in fact, like, so, the factor T s plus 1 is in fact like amplifying high frequency signals ok. It is not attenuating right. So, this is like a high pass filter right. So, the higher the frequency, the greater is a scaling right; the amplification of the input signal ok. So, that is the characteristic of capital T s plus 1 ok. So, the color legend remains the same ok, that is why I am not rewriting it here, but you understand right.

So, immediately, you see that this is going to be a reflection. That is what we discussed in the last class. So, you see that the magnitude plot of T s plus 1 is just a reflection above the horizontal line passing through the 0 decibel point right. So, you see that it is just a mirror image right. You just flip it, you will get the bode plot of the magnitude plot of T s plus 1. What about the phase plot? Let me let me draw it in the next page ok.

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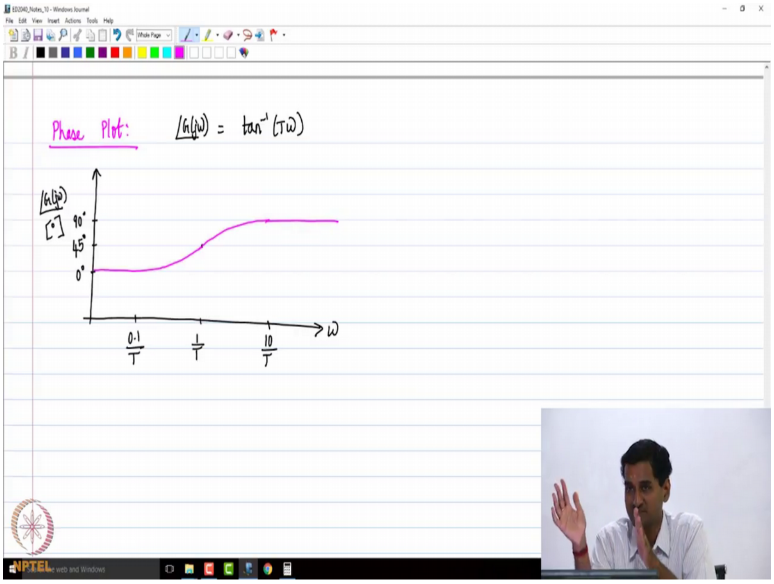

So, what happens to the phase plot? So, once again it is going to be a reflection right. So, what is going to happen? So,, so, what would the phase tend to as omega tends to infinity? It will tend to plus 90, right. At omega equals 1 by capital T, it will be 45 now. Why, because please note that the phase of G of j omega now is tan inverse of t omega, not minus; it is just tan inverse right plus.

So, essentially if this is my 0 degrees and this is my 90 degrees and this is 45. So, what is going to happen is it at a 1 by capital T and let us say 10 by capital T and 0.1 by capital T 1 by T, the values is 45 right. So, what is going to happen to the phase plot is it? It is going to start from 0 and then, pass through 1 by capital T and 45 and then settle to 90, asymptotically ok. So, that is what will happen to the phase of this factor 1 by capital T s plus 1.

So, you see that it is just the reflection, once again about the, what you say 0 degrees line, right the horizontal line passing through the 0 degree ok. So, the this is the, these are the bode plots you know the magnitude on the phase plot of the first order factor, know, please remember why we are doing it because, if you recall where we stopped in the last class you know you give me any transfer function what I am going to do, I am just going to write the transfer function as a product of these individual building blocks right.

Once I do that, what is going to happen I plot the magnitude plot on the phase plot of the individual blocks and then I just add them that, is it right. So, I then I get the magnitude and phase plot of the transfer function which I am given ok. And that is what we are going to do. So, today, we learnt about first order factors right of the form 1 by capital T s plus 1 and T s plus 1, right. Tomorrow, class you know we are going to look at second order factors ok. We will see how to plot the bode plots of second order flutters, then we will do an example ok. So, that is what, that is that is our action, fine.