Control Systems
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Lecture - 50
Bode Plot 1
Part - 2

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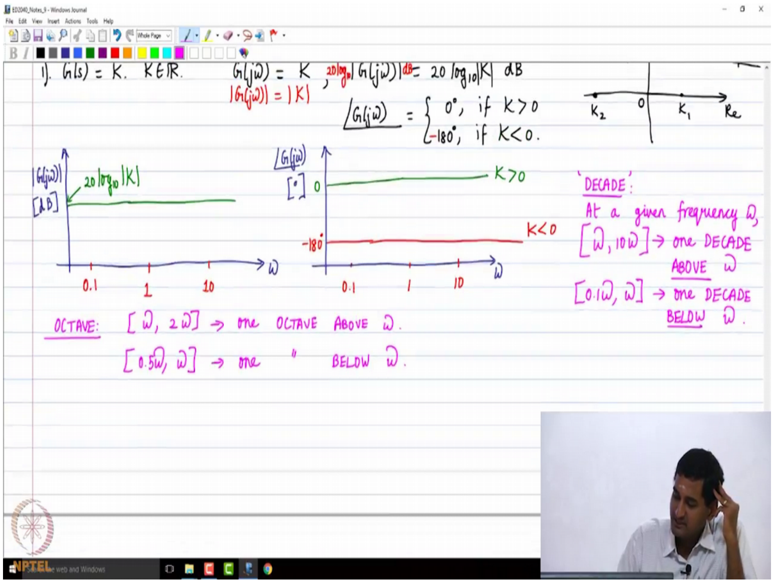

So, why is it that I should consider in this particular unit what advantage do I get? Now, see please note that we are using a logarithmic scale, right. So, if I have log omega log of 10 omega, what is log of 10 omega?

Student: 10 plus 1 plus.

What will I get if I go one decade essentially above or below omega?

Student: 1 plus omega.

1 plus omega 1 plus log omega. So, what do we get out of it now we will see, ok. So, when we plot the bode diagram, we will figure out what are we going to essentially get out of it, right. So, please wait for maybe some more time, maybe next class we will be able to figure out why it is done, ok.

There is another region which is banned which is used which is called as an octave, ok. What is an octave? Let us say you give a frequency omega. The region what you say omega to 2 omega is what is called as 1, the region 1 octave above omega.

Similarly, from 0.5 omega to omega is what is called as 1 octave below omega, ok. So, these are all some terms which are typically used, ok. That is a decayed and then, octave ok. So, please remember that and you will see that you know like by and large you know like we express slopes in these curves in terms of decades or octaves, ok. So, because we want to typically see how the magnitude curve or the frakes phase curve in all these changes vary frequency from by a factor of 10 or 2, ok.

So, that is the convention and that is what we will see, as we go later, right. So, why we are essentially varying it by 10 times? So, that is the first factor you know like where we consider G of D to be just a constant. Got it, ok. So, please remember this octave and decayed and so on, right.

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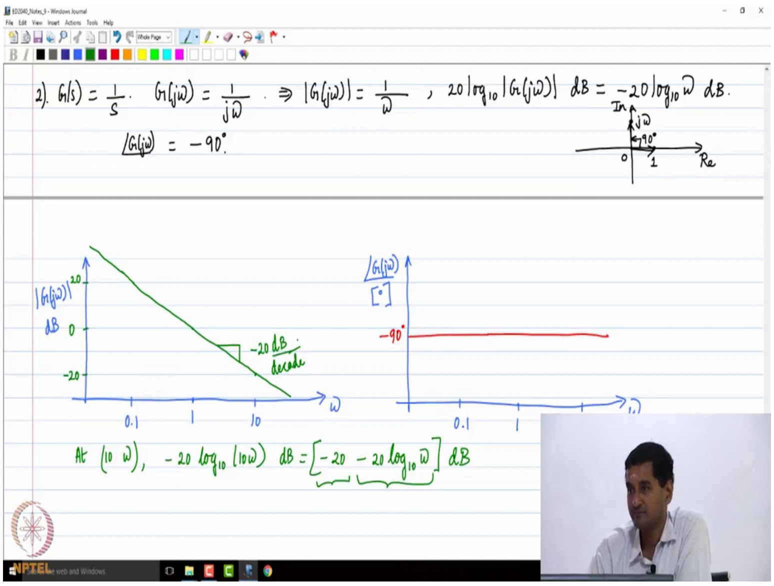

So, now let us plot the bode diagram for 1 base, right. So, what is going to be the bode diagram for 1 bias? So, G of j omega is going to be 1 by j omega, right. So, this implies that the magnitude of G of j omega is going to be we discussed this yesterday, right. So, if you have a complex function is a ratio of individual factors, the magnitude is going to be just the ratio of the individual magnitudes.

So, here it is going to be 1 divided by what is the magnitude of j omega omega. That is it, ok. So, this implies 20 log to the base 10 magnitude of G of j omega in decibels is going to be what 20 log to the base 10, 1 by omega. What is log of 1 by omega to the base 10?

Student: Minus log omega.

It is minus log omega, right. So, we are essentially going to get minus 20 log to the base 10 omega. Of course, omega is positive, right. So, it is frequency, it is positive. So, I am not putting an absolute value here, right. So, I hope it is clear how we got this, right. So, because 20 log to the base 10 of 1 by omega is going to be minus 20 log of omega, right so that is what we have an decibels measure, right ok.

So, that is the magnitude. What about the phase? Phase is once again going to be the algebraic sum of the numerator and the denominator factors. Numerator is 1, the phase is 0 degrees, the denominator is j omega and the phase is going to be 90 degrees. So, the phase of 1 by j omega is going to be minus 90 degrees, right. Do you agree? Ok because if you go to the complex plane, one is going to be here, right.

So, the vector 1 is going to make an angle of 0 degrees with the positive real axis where as j omega, j omega is somewhere here. So, this phase is going to be 90 degrees, right correct. So, totally the net phase is going to be 0 minus 90. So, we are going to get it as minus 90 degrees. You know that is the phase contribution of 1 by j omega.

So, immediately you see that you know like this independent omega, right it is constant with respect to omega whereas, the magnitude is a function of omega, right. So, that is minus 20 log to the base 10 omega decibels. Now, the question is that how do I plot them, right. So, let us plot them and then, see how we get those values, ok.

So, let us say you know I start with 1, I want I go 10, then I go at 0.1 and this is the magnitude of G of j omega in decibels, right. Similarly, we have the phase diagram. Phase diagram is pretty easy to plot here. So, the phase of G of j omega is in degrees, ok. So, once again let us say I have 0.11, sorry 10, sorry 1.1 at 10, right. So, the phase diagram is pretty easy.

So, this is going to be a straight line, a horizontal line at minus 90, right. So, that is what I have right for the phase, for this particular factor, but what can I say about the, how can you plot the magnitude plot. So, that is going to be minus 20 log omega, right. So, essentially when omega is equal to 1, what is the value? Let us say you substitute omega equals 1 radians per second. In this magnitude value right what do you get? What is log of 1 to the base 10? 0, right.

So, you get let us say this is my zero decibel value, ok. So, let us say you substitute omega equals 10, what do we get as decibel measure?

Student: Minus 20.

We get minus 20, right. So, let us say omega equals 10, I get the value as minus 20 and let us say I have substituted omega equals 0.1, what do I get? Plus 20, right. So, at omega equals 0.1, I get plus 20. So, immediately you see that the magnitude curve is going to be a straight line because it is a non-linear function of omega, but then you can see that it is a linear function of log of omega, right. See if you call log of omega, log of omega to the base 10 as x, this is like minus 20 x, right. So, that is it is a straight line once you have the abscissa in the logarithmic scale, ok.

So, that is why we get a straight line, right and you can immediately see that the slope of this line is negative, right. So, because as omega increases the value is decreasing right now, what do you think should be the slope of this line; the slope of this line as where we calculate it as the following right because the function under consideration is minus 20 log omega.

Let us say 10 omega, what are we going to get? We are going to get minus 20 log to the base 10 10 omega decibels. So, this is going to be minus 20 and minus 20 log to the base 10 omega, right. So, you take any omega, the value is going to be minus 20 log omega decibels, right. You go to 10 omega, what is going to happen? The value is going to decrease by 20 decibels, right. That is what we can have, right.

At any omega the magnitude is going to be minus 20 log to the base 10 omega. You go 10 omega from that omega if the value is going to decrease by minus 20 decibels. So, for that reason the slope is indicated as minus 20 decibels by decade, ok. So, in one decade of frequency, the magnitude essentially drops by 20 decibels, ok. That is the slope of this magnitude curve, ok. I hope it is clear how we got this our slope and one can also illustrate this in octave.

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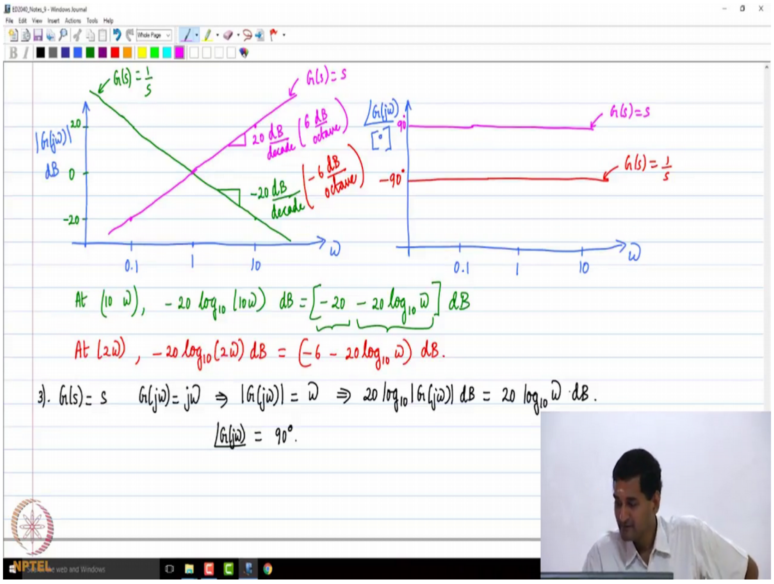

Let us say at 2 omega, what is going to be the value of this magnitude? It is going to be minus 20 log to the base 10 2 omega decibels. So, that is going to be what is log of 2 to the base of 2? I think it is 0.3010, right. So, you multiply it by minus 20, you are going to get approximately minus 6. Am I correct? So, this is going to be approximately minus 6 minus 20 log to the base 10 omega decibels, right.

So, consequently people will also say that the slope is minus 6 decibels per octave, right. That means that in one octave from a frequency omega u, the magnitude will drop by minus 6, ok. So, that is the other way people will write the slope, ok. I hope it is clear how we calculated the slope, right. There is a straight line. So, we are just calculating in this one, ok. Is it clear?

So, that is what we have, as the bode diagram for the factor 1 bias, right. So, that is in general what we call as an integral term, right 1 base is an integral term. So, now if I want to consider G of s to be s, what do you think will happen? Let us say G of s, I take it to be s, ok. Let us repeat the same things. So, it is pretty straightforward, ok. Once we know the process, this is going to be G of j omega is going to be j omega. So, this implies then the magnitude of G of j omega is going to be just omega, right. So, this implies that 20 log to the base 10 magnitude of G of j omega in decibels is going to be equal to 20 log to the base 10 omega in decibels.

So, previously we had minus 20 log to the base 10 omega. Now, we are having plus 20 log to the base 10 omega, right. So, what about the phase or G of j omega? It is going to be 90 degrees, ok. So, that is what we have, right. So, the phase is going to be 90 degrees, right. So, that is what we will have, correct because j omega is on the positive. What you say imaginary axis, the phase is going to be 920 degrees, sorry 90 degrees, right. So, that is what we are going to have.

So, now you can immediately plot the magnitude plot on the phase plot. I am just going to draw on the same diagram, ok. You kindly draw it in a different diagram, because I am drawing it on the same diagram to just show you a point, right. So, let me let us first draw the phase, ok. The phase is going to be plus 90, ok. So, this is for the factor G of s is equal to s, ok. This was for the factor G of s is equal to 1 by s, right.

So, let me write down this for G of s equals 1 by s. Now, how will the magnitude plot look like for s? So, that is 20 log omega, right and omega equals 0. Once again we are going to have 20 log 1 to be 0 and omega equals 10. What are we going to have? It is going to be plus 20 decibels, oops ok.

So, at omega equals 0.1, we will immediately see that it is going to be minus 20 decibels, correct. So, now if I join this, this is going to it be in this way, ok. So, this is the magnitude plot for A the factor s. So, what do you think is the slope of this curve; once again a straight line?

Student: 20 decibels.

It is going to be plus 20 decibels per decade or 6 decibels per octave and that is going to be the slope of this curve, ok. So, that is the difference, right. So, can you observe something from here? I hope it is clear how we plotted it, right. It is pretty straightforward once we get a hang of what is happening, right. So, how did we plot this using just the formula, right and what can you observe from these plots, any observations?

See please note that 1 by s and s are reciprocals of one another. So, what can you infer from this plot? In a certain sense, you know it is going to be a mirror image, right correct.; the phase plot on the magnitude plot, but mirror image about what axis?

Student: Omega.

Omega equals 1.

Student: Y equals 0.

Y equals 0.

Sorry over there.

Student: At point 0.

At point what you say? 1 common 0, is it ok? So, we will please what to say keep this question in mind, ok. When we go to the first order factor with a constant term and second order factors, I will ask you this question once again, then you will be able to generalize, and give a general answer, what happens to the bode diagram.

When you have two factors which are reciprocals of one another, see at the end of the day think about it, ok. If you have let us say a reciprocal and the factor itself in a transfer function, what will happen is, they will cancel off each other, right. So, what should be the net magnitude and phase?

Student: 0.

It should be 0, that is what is happening, right because we are going to logarithmic scale, you add the magnitude at any frequency, do you get 0? Obviously, you would. Similarly, you add the phase, you will get the answer as 0, right. So, that should give you the hint, right as to what should be the line about which it should have be a mirror image, alright. That is the horizontal line passing through 0, right

So, about that it will be a mirror image, right because obviously you want the sum to be 0, right because there are reciprocal factors of one another, ok. So, what I am going to do is that like I am going to essentially, so what we have done till now is that we have done three factors, right. So, we have done k, we have done 1 by s, we have done s, right. So, what we will do is it like we will do the first order factor in the next class, but before that of course once again you know one exists, so that you remember. So, what do you observe it observe about factors; there are reciprocals of each other, ok.

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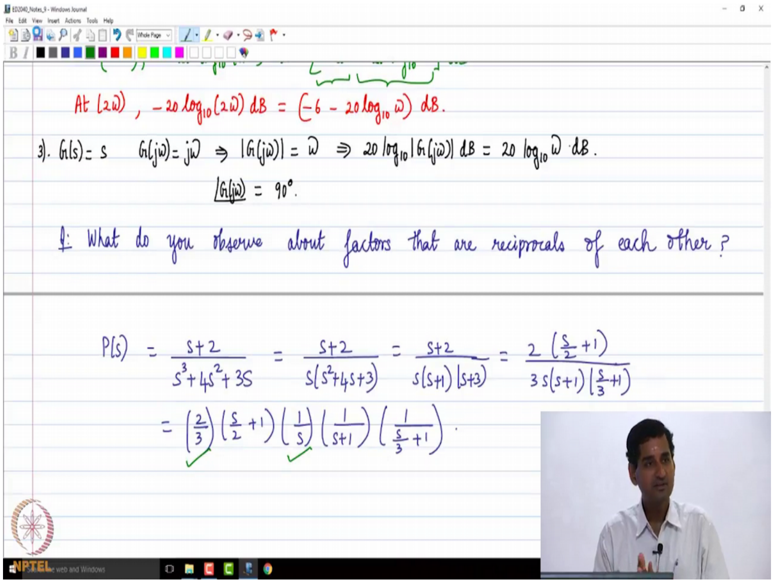

So, that is a question which we just answered, right. So, I am just phrasing the question, so that you remember right that we discussed this, right. So, just to basically convince ourselves that you know on a given any particular transfer function, we will be able to rewrite in this manner, just to essentially to say look at that. Let us say we consider some function like you know like some arbitrary function s plus 2, let us say s q plus, that is 4 s squared plus 3 something 3 s, ok. So, something like this, right.

So, what can you have? What can I do you know like if I have a plant transfer function like this, I can rewrite this as s squared plus plus 4 s plus 3. What is s squared plus 4 s plus 3? It is going to be s plus 1 times s plus 3, correct. So, this I can rewrite this as two times s by 2 plus 1, correct and three times s times s plus one times s by 3 plus 1, ok. So, this I can plant transfer function I can write as 2 by 3, correct times s by 2 plus 1. I am rewriting in this way times 1 by s times 1 by s plus one times 1 divided by s by 3 plus 1. Is it clear? How we broke it down into the factors that we have learnt or we are going to learn, right.

I hope it is clear, right. So, you can have s by a term, you divide by a right pull a out of the term, right so that you make the constant one and then, your factor is all the constants. Anyway you will get some constant outside. That is ok, right. So, you can see that we have learned how to plot bode diagram for the constant and 1 by s term.

What we are going to do in the next class is, we will learn how to plot bode diagram for 1 by t s plus 1 and t s plus 1 and then, the second order factor, ok. So, that is what we are well, then we will see how to put everything together and plot bode diagram for any given transfer function, ok.

So, that is what we will do in the next class, fine ok; any questions? So, if not I will just stop here and then we would meet and then continue next class.

Thank you.