Control Systems
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Lecture – 44
Case Study-Control Design
Part - 2

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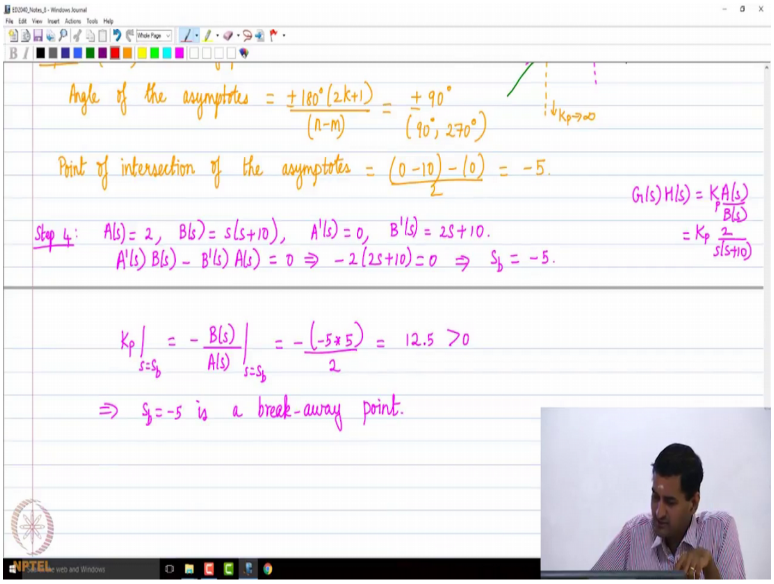

So, S b equals minus 5 is a breakaway point right. So, that is what we have done.

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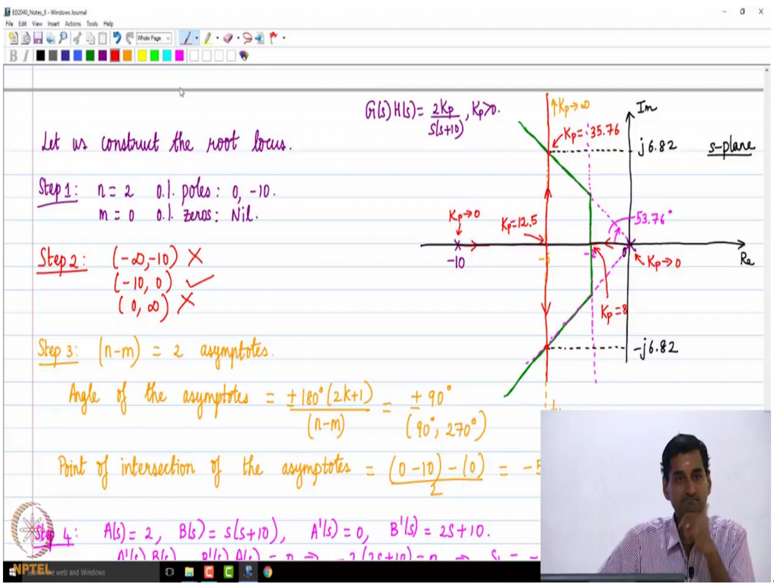
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So, that is why it breaks away and anyway it goes along asymptotes right, but anyway we will come and plot it as we go along right.

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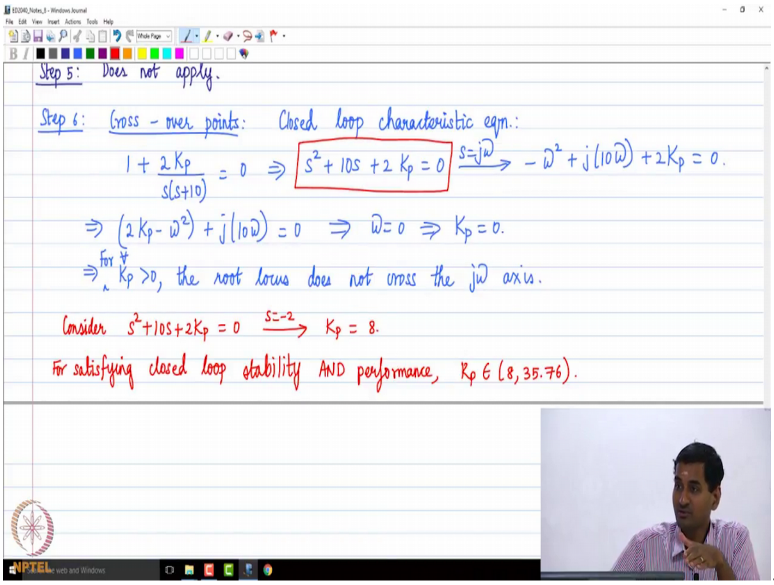

So, what is Step 5? So, we will go step by step right. So, step 5 was to.

Student: (Refer Time: 00:38).

Get the angle of departure or angle of arrival right, but it does not apply here right because why there are no complex open loop poles or complex open loop zeros here right. So, step 5 does not apply right for this problem right.

So, let us look at Step 6. So, step 6 is about getting the crossover points. So, for the crawl of determination of the crossover points please note that you need to have a look at the closed loop characteristic equation right. So, what is a close loop characteristic equation? It is going to be 1 plus G of S, F of S equals 0, right. So, that is what we need to look at. So, that is going to be in this particular87 case 1 plus 2 K P divided by S times S plus 10. So, that is so, we are going to get S square plus 10 S plus 2 K P is equal to 0.

Here what we do? We are looking for solutions of the form S equal j omega right, but whether this is a crossover point right crossover point at some point on the imaginary axis where a root locus branch may cross from the left of plane to the right of plane or vice versa. So, that is a crossover point right. So, we substitute S equal j omega. So, what do we get? We get minus omega square plus j times 10 omega plus 2 K P is equal to 0.

So, this will give us 2 K P minus omega square that is the real part plus j times 10 omega equals 0 right.

So, this immediately implies it from the imaginary part omega is equal to 0 is the only possibility; that means, the origin right, and but already we know that the open loop pole is at the origin alright. So, you from the if omega is 0 that is what we get from the imaginary part immediately see that the real part will give us K P is equal to 0 correct, see 2 K P minus omega square is equal to 0, you substitute omega is equal to 0 what will you get; 2 K P is equal to 0 or in other words K P should be 0.

So, omega equals 0 K P equals 0 is the only crossover point; that means, that the root locus does not cross the imaginary axis except when it starts at the open loop pole that is it. So, that is what we can interpret from this calculation. So, there are no further crossover points.

So, it starts at the origin when K is 0, when K tends to 0 because that is an open loop pole right. So, for any K P greater than 0, so, the conclusion we can draw is that for all K P greater than 0 the root locus does not cross the j omega axis. So, that is the interpretation we can have, yes.

Student: Sir that 0 does not lie on the root of locus can directly say that there would not be any (Refer Time: 04:18) or not.

Zero does not lie on the root locus.

Student: Second step we are deciding which part of the (Refer Time: 04:26). So its 0 does not lie on the root of step j omega.

Yes absolutely, but we need to be careful right. So, I got your question.

So, I think we might have discussed this point because you are right if 0 does not lie on the root locus from step number 2, we can straight away say that the origin is not a crossover point, but I can have a complex conjugate crossover point once again see imagine this you know like we can have two branches which are cutting the imaginary axis.

In fact, which happened if you recall last week right we considered another example where we had a open loop poles at plus or minus j it was not a crossover point, but the 2 branches started on the imaginary axis you are correct, but my what I am adding to a statement is that, that does not that only says that 0 is not a crossover point it does not exclude the fact that we can have crossover points on the imaginary axis right.

So, let us go back and complete the root locus. So, what is going to happen is it like we are going to have 2 branches starting from a one starting from minus 10, one starting from 0, they break away at minus 5 and then they just go to infinity along the asymptotes. So, that is what is going to happen.

So, now, the question is for what values of K P this is the root locus right we have plotted the root locus. So, now, you can see the advantage of using this method right. So, we have a visual characterization right or visualization of the two branches of the closed loop poles right. So, 2 branches of the root locus each one corresponding to one closed loop pole right.

So, you can immediately see that for all K P greater than 0 both branches lie in the left off plane. So, as we already know K P greater than 0 implies closed loop stability, now we want closed loop performance right. So, the question is for what range of K P would both branches lie in that open trapezoidal region right, that is the question we need to ask ourselves. So, how can I determine that see where is the open trapezoidal region let me redraw it highlight it in green once again. So, it is this right. So, for what values of K P would the 2 branches live within this region, how do I find it.

First I need to find the value of K P at minus 2 right, because K one branch of the root locus starts from 0 keeps on increasing sorry keeps on coming to the left as K P keeps on increasing and it comes with in this performance region only when it crosses minus 2 right S equals minus 2 right. So, how do I find the value of K P at s equals minus 2; obviously, I substitute S equals minus 2 in the close loop characteristic equation, what does the closed loop characteristic equation, it is this.

So, if you substitute S equals minus 2 here what do you get for K P, K P should be.

Student: 8.

8 right, you substitute S equals minus 2 right. So, what we are going to do is, that so, consider the closed loop characteristic equation right S square plus 10 S plus 2 K P is equal to 0. So, here if you substitute S equals minus 2 we will get K P to be 8 alright. So, that is pretty straightforward algebra correct.

So, K P is going to be 8 here. So, now, you can see what we have done right, for stability we wanted K P to be greater than 0 right, for performance K P should be greater than 8 right only then both branches will enter into this trapezoid, but not only that we have to find out what is the value of K P when the root locus hits this point and this point right because beyond this the two branches of the root locus go outside the desired region right. So, how can I find the value of K P at this point?

First I need to figure out you know like what is that point all right, how do I find out the coordinates of that point, I hope it is clear what I am doing right I am figuring out for what range of K P would the two branches or root locus lie in this green what to say the region open trapezoidal region which is bounded by this green colored lines right. So, how do I calculate this point of point I can use angle right. So, can you tell me what is the coordinate of course, it is going to be my real part is minus 5 what is going to be the imaginary part yeah, what is going what is that going to be 5 10 53.76 right.

So, what do you think you will get. So, what is this value and similarly what is this one, what is 5 times tan of 53.76, please take tan 53.76 multiplied by 5 what do you get?

Student: 6.82.

6.82, so, this is going to be j times 6.82 and this is going to be minus j times 6.82, now how do you find the value of k at that point or K P at that point? Once again we substitute the corresponding value of S in the closed loop characteristic equation right alright. See what is the value of S here? Minus 5 plus 6.82 j please plug it into closed loop characteristic equation, if you do that you know like please check it you know like I got the answer as 35.76.

So, that is what I got the answers like, so, if you substitute it in that closed loop characteristic equation 35.76. So, what does this imply? This implies that for performance I need to essentially have my proportional gain to be between 8 and 35.76

that is it right. So, what I can conclude is that, for satisfying closed loop stability and performance K P should be in the region 8 and 35.76.

So, please note that see initially to begin with K P was a real number right between minus infinity to plus infinity stability essentially reduced that region into half only from 0 to infinity, but that is that is that is once again a big region right. So, through this analysis what we have done, we have figured out that look you know like 0 to infinity will give me closed loop stability, but then 8 to 35.76 will give me performance also the way I desire it right. So, do not you think this is a very useful tool to know, you know like we. So, please note that what we have done we have progressively reduced a huge region of proportional gain minus infinity plus infinity first we reduce to 0 to infinity, then 0 to infinity now has been reduced to 8 to 35.76 right.

And even if I go back to the root locus and look at this in fact, I would design be for some K P between 12.5 and 35.76 right, because by and large we would want our closed loop system or control systems to be slightly under damped right. So, if I want to satisfy the performance specifications close and close loop stability and still the design and under damped second order system right I would choose K P between 12.5 and 35.76.

I can also choose K P between 8 and 12.5 that will also satisfy performed specification, but the system will be over damped, see if you have an ordnance system where is maximum peak overshoot. So, maximum peak overshoot anyway is not there right and yet I can satisfy the condition on setting time right so, that is essentially one this one.

Of course, it is going to be slightly sluggish right, because the settling time expression assumes that you know like you are going to have an underarm second order system. So, in a certain sense you know like what I would do is that like I would pick K P between 12.5 and 35.76. So, that it is consistent with whatever analysis we have done, because please remember how did we get the desired region of closed loop poles to begin with by using the step response of an under damped second order system right.

For an under damped second order systems the poles should be complex conjugates right, when are the poles complex conjugates as for the root locus here when K P is greater than 12.5 right. So, I would choose K P between 12.5 and 35.76 and tune the final system. So, this way the analysis theoretical analysis can help us figure out a range of controller gains, then we need to go to the actual experimental demand find you. So, that is what ultimately we would end up doing, is it clear?

Student: (Refer Time: 16:59).

Because why is 12.5 the transition point right, because please note that that 2 branches or root locus are starting from minus 10 and 0 all right and when K P increases from 0 you can immediately see what is happening right. So, one branch is going to come on this real axis right in this manner, another branch is going to start from 0 and travel to the left.

So, what do these branches indicate they are the closed loop poles? So, till K P equals 12.5 you can mentally see that both branches are on the real axis. So, then what can you say about the closed loop poles they are real. If you have two real distinct poles for a second order system what is it called, over damped right. So, at 12.5 they intersect they become critically damped and then beyond 12.5 you see that the two branches go into the move away from the real axis. So, you have complex conjugate closed loop poles right. So, the system becomes under damped yeah.

Student: Now I want to calculate this (Refer Time: 18:15) while KP is negative (Refer Time: 18:20).

Because we are dealing with a symmetric root locus right. So, because if you are have going to have a what to say two closed loop poles you know like at the same value of K P think about it this way right K P is a parameter right. So, let us say you substitute K P equals 35.76 you are going to have one closed loop characteristic equation right you are going to have some constants as parameters.

So, when you calculate the roots you know since we are dealing with real polinom that is polynomials with real coefficients, if you have a complex root the conjugate also must be root. So, conversely if you have if you parameterize the polynomial using one parameter you choose any two complex conjugate roots which are solutions to the polynomial the value of the parameter should be the same right otherwise there will be a dicot in right is it not.

That is why K P is the same right. So, it does not matter whether it is minus 5 plus 6.8 to j or minus 5 minus 6.82 j fine so, I hope it is clear right. So, what we have done is that like we have just ran through one study where we can generate models and do this process right so, of control design right. So, that is just to give a flavor of what we could do with whatever we are learn.

And you can immediately see that root locus is a very handy tool to know right because like you can have a graphical visualization. See if I had asked you know like without a root locus how do you satisfy these conditions performance conditions would have been very difficult right it would have been more challenging to get these bounds right on K P with root locus it was quite straight forward and also very intuitive right because we can see the graphical visualization right.

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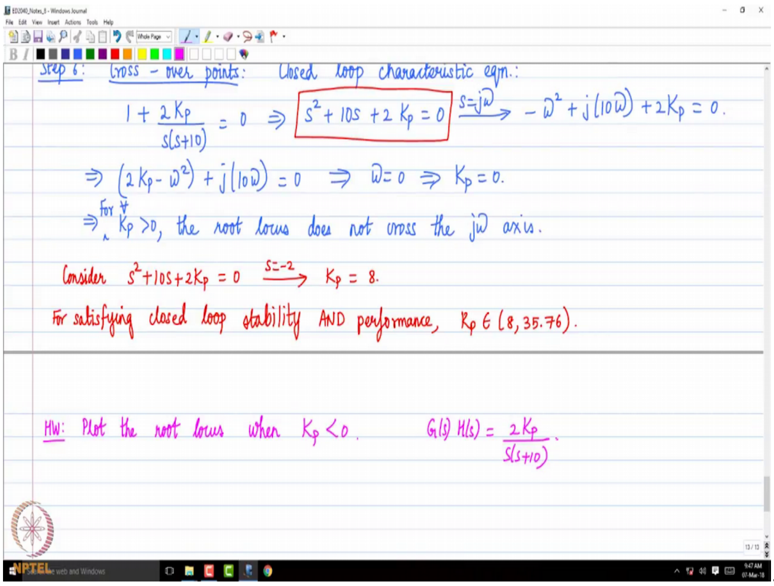

So, what I am going to leave you is a homework problem where this is just for your own learning process right. So, plot the root locus for the same system when K P is negative. So, this is just an exercise for you. So, that like you get familiar with the other ways of plotting the root locus also. So, you immediately; obviously, you will see that when K P is negative, already we know that you know like closed loop system is not going to be stable, you will see that one branch will always go in the right of way. So, that is what will happen when K P is negative.

But the reason I am asking you to do it is because see many times people would want the root locus to be plotted for a parameter that is varied from minus infinity to plus infinity, then you cannot essentially plot at one go what you need to do is it if you give me a transfer function like this and you ask me to plot the root locus for K P going from minus infinity to plus infinity right, because why what was the open loop transfer function, it was 2 K P divided by S times S plus 10.

Now, you asked me plot the root locus you know for K P for all real values of K P right. So, I need to go from minus infinity plus infinity right. So, what I do is, that I divide that region into two sub regions minus infinity to 0, 0 to infinity then I plot the root locus for one region let us say minus infinity to zero; that means, K P is negative and then 0 to infinity; that means, K P is positive, then I get the complete root locus. So, that is what I want you to do right, we have plotted the root locus for a K P greater than 0 as homework, what I want you to do is, I want you to plot the root locus or for K P less than 0. So, there you get the complete picture; obviously, you will realize that the closed loop system is not going to be stable for K P less than 0, right.

So, we will just a confirm that or support that result you know like using the root locus fine so, please do this. So, this completes our discussion on root locus. So, what we are going to do is it like from the next class you know like I am just going to go to frequency response, but before we go to frequency response in the next class I am going to give you a brief introduction to what is called as the state phase representation right, which forms the basis for what is called as modern control theory you know which you will learn maybe as the next course and controls if you chose to go into control systems.

And in fact, we will use those tools also in or advanced control courses like whichever courses you take right on advanced controls. So, I will just introduce that to you and show you the equivalence between the two approaches, but this course is going to be about transfer function right. But since we are at the halfway point I thought that is a good time to introduce state space to you right then we will go to what is called as frequency response analysis, we will do frequency response that is a response of this class of systems to sinusoidal inputs we will study those characteristics and then we learn how to analyze systems for the frequency response and then like design, controllers using frequency response. So, that is going to be the second half of our course fine.

So, I will stop here and then like we will meet on Monday.

Thank you.