

Control Systems
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Lecture - 42
Case Study-Modelling
Part-2

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$$(2) + (3) \Rightarrow \left[J_1 \left(\frac{N_2}{N_1} \right)^2 + J_2 \right] \ddot{\theta}_2(t) + \left[f_1 \left(\frac{N_2}{N_1} \right)^2 + f_2 \right] \dot{\theta}_2(t) = \left(\frac{N_2}{N_1} \right) T_m(t) - T_L(t).$$

Transfer Function: Take Laplace transform and apply zero IC:

$$\downarrow [\ddot{\theta}_2(t)] = s^2 \theta_2(s) - s \theta_2(0) - \dot{\theta}_2(0).$$

$$\downarrow [\dot{\theta}_2(t)] = s \theta_2(s) - \theta_2(0).$$

$$\Rightarrow \theta_2(s) = \left(\frac{N_2/N_1}{\left[J_1 \left(\frac{N_2}{N_1} \right)^2 + J_2 \right] s^2 + \left[f_1 \left(\frac{N_2}{N_1} \right)^2 + f_2 \right] s} \right) T_m(s) - \left(\frac{1}{\left[J_1 \left(\frac{N_2}{N_1} \right)^2 + J_2 \right] s^2 + \left[f_1 \left(\frac{N_2}{N_1} \right)^2 + f_2 \right] s} \right) T_L(s).$$

Now, to get the transfer function; so, let us get the system, transfer function ok. So, to get the system transfer function; obviously, what should we do? You know like take Laplace transform on both sides and apply 0 initial conditions right. So, this is a very important step and since we have done this step many times you know like, I am just, going to skip and go directly to the final answer, but I suggest that you do it completely right. So, as I keep on telling you, you know like you should always write the Laplace transform completely, you know like. So, let me do it once more.

So, please know that the Laplace transform of the second derivative of theta 2 is going to be S square theta 2 S minus S times theta 2 0 minus theta 2 dot 0 of course, it is obvious from the context that the second two terms are initial conditions ok, where theta 2 is evaluated in the time domain right. So, and the Laplace transform of theta 2 dot is going to be equal to S times theta 2 S minus theta 2 0 ok. So, we need to keep in to keep in mind all those things right.

So, once we essentially apply the Laplace transform and substitute initial conditions to be 0 and collect the terms and so on. What are we going to get? We are going to get the following ok, $\theta_2(s)$ is going to be N_2 divided by N_1 , the whole thing divided by $J_1 N_2$ by N_1 whole square plus J_2 times S square plus $f_1 N_2$ by N_1 whole square plus f_2 times S ok.

This multiplying T_m of S right correct, that is what the first term is right ok, on the right hand side then what we have then? We have another term right. The load torque minus 1 divided by the same denominator $J_1 N_2$ by N_1 whole square plus $J_2 S$ square plus $f_1 N_2$ by N_1 whole square plus $f_2 S$ ok, the whole thing multiplying T_L of S ok. So, this is what we will give right. Is this correct? Did I miss with something? I think it is fine right ok. So, you see that the finally, the output of the system θ_2 has two components right. So, this component is the 1 due to the input.

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Neglect the load torque T_L .

$$\Rightarrow P(s) = \frac{B_2(s)}{T_m(s)} = \frac{(N_2/N_1)}{[J_1(N_2/N_1)^2 + J_2]s^2 + [f_1(N_2/N_1) + f_2]s} = \frac{a}{s^2 + bs} = \frac{a}{s(s+b)}$$

$$a = \frac{(N_2/N_1)}{J_1(N_2/N_1)^2 + J_2}, \quad b = \frac{f_1(N_2/N_1) + f_2}{J_1(N_2/N_1)^2 + J_2}$$

Let us take $a=2$, $b=10$.

$$\Rightarrow P(s) = \frac{2}{s(s+10)}$$

So, the first term is the 1 due to the input ok. The second term is due to the load torque right. So, please note that you know like if I want to move the, output shaft to any position right. You can immediately see that the, the load torque is going to play a role all right. So, as far as the output responses, the response of the system is concerned right.

So, we need to keep that in mind. So, let us say, you give a step voltage as sorry, step input torque 40 m. The final θ_2 is going to be dependent on the load torque right; obviously, ok, but the plant transfer function per says the, function multiplying T_m of s

right that is the plant transfer function right, because the 1 relating the input and the output ok.

So, just for, what to say for the, for further analysis and design momentarily, we will neglect the load torque ok. So, that like we can carry on with the analysis, then you revisit towards the end and see how our design is affected by the load torque ok. That is something we will come back and look at it ok. So, for the time being, let us neglect the load torque T_L ok. This implies that the plant transfer function P of s , which is going to be equal to θ_2 of S divided by T_m of S .

It is going to be equal to N_2 by N_1 divided by $J_1 N_2$ by N_1 whole square plus $J_2 S$ square plus $f_1 N_2$ by N_1 whole square plus f_2 times S ok. This, this I can rewrite it, as a divided by just for the sake of, notational simplicity right. So, if I have to write all the parameters in the general form again and again right, that is going to be very cumbersome ok.

So, let us, let us write it as a divided by some S squared plus b s right. So; obviously, you know like, what are a and b . So, here what I am doing is then I am just saying that the parameter a is going to be, N_2 by N_1 divided by $J_1 N_2$ by N_1 whole squared plus J_2 right, that is what I am essentially taking S and b is essentially $f_1 N_2$ by N_1 whole square plus f_2 divided by $J_1 N_2$ by N_1 whole square plus J_2 ok. So, that is what we are considering right.

So, immediately we can see that this is a second order system, whose transfer function is of the form a divided by S times S plus b ; obviously, a and b are positive right, one can immediately note that the parameters a and b are positive and what are the system or plant poles. We have a pole at the origin, we have pole at minus b right. So, is this plan going to be stable no right, because of course, it depends on how we look at it as we discussed some people will call as critically or marginally stable, but we know that when we have a non repeating pole of the origin. You give a step input, the output will become unbounded right.

So, you can immediately picture is it, let us say you have a motor shaft and a gear transmission. You give a step voltage, you will see the output shaft will keep on rotating you might have observed it, even in your experiment on D C motor that you some of you might have done right. So, essentially the output shaft might have just kept

on a rotating right. So, if you do not apply any control to the system right. So, that is what is going to happen here. So, now, the question is, what do we do right.

So, for the, for essentially once again, the sake of simplicity, you know like let us take. So, that we work at some numbers rather than parameters. Let us say we take a 2 b 2 b 2 b 10 ok. So, this implies that the plant transfer function is let us say 2 divided by S times S plus 10 ok. So, that is what just I am just considering some number right. So, essentially, so that like we work with some, numbers right.

So, now the question is for us to design a closed loop feedback system and then you know like ensure that the closed loop system is stable and then ensure that it satisfies some performance, requirements ok. So, that is the second task which I am going to pose ok.

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The screenshot shows a digital whiteboard interface with a toolbar at the top. The main content area contains handwritten text in blue ink:

let us take $a=2, b=10$.
 $\Rightarrow P(s) = \frac{2}{s(s+10)}$

TASK 2: Design a stable unity negative feedback control system that satisfies:
Settling time < 2 s, $M_p < 10\%$.

In the bottom right corner, there is a small video inset showing a man in a white shirt speaking. The NPTEL logo is visible in the bottom left corner of the whiteboard area.

So, task 2. So, I hope, I call the first one as task 1 yeah. Task 1 right. So, task 2 oops, yeah task 2 is a design, a stable unity, negative feedback control system that satisfies the following ok. Settling time, less than, let us say 2 seconds peak overshoot M_p less than let us say 10 percent ok. So, the task for us is to design not only a stable closed loop feedback system, but also one that will stabilize sorry, that will satisfy this performance requirements right of settling time less than 2 seconds and peak should less than 10 percent ok.

So, So, what I am going to do is that like I am going to stop here ok. So, I will let you think about it and then like we will complete this task tomorrow ok. So, that we go through, the entire control design process once right. As I told you we are at the halfway mark before going into, the next chunk of topics.

Now, I thought let us work out a problem from start to finish right, based on whatever we have learnt. So, that we understand. So, just stay out of proportional control, you know like, you will see that in this case even a proportional controller would stabilize the closed loop system and then think about how you would, you can use root locus to ensure that you can satisfy these performance requirements ok. Please, think about that and then like we would continue in the next class.