Control Systems
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Lecture - 40
Root Locus 4
Part-2

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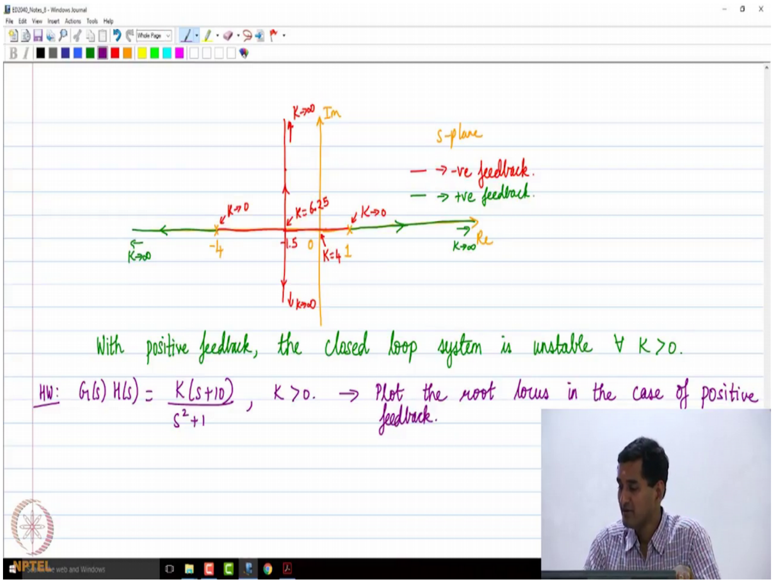

So, if we scratch the root locus, now what do we see? So, let us say this is the real axis, imaginary axis, ok. Explain this and the open loop poles were at minus 4 N plus 1, right. Let us say I use a red color to construct the root locus with negative feedback, right. If you recall what happened with negative feedback, the root locus was something like this, right. So, I think the point of intersection was at minus 1.5 and then, it broke out and it went in two branches, right. This was K tending to 0, right and this happened as K tends to infinity, right. This was with negative feedback, right. So, the red line indicates with negative feedback, right.

Now, let us look at what happens with positive feedback, right. Based on the root locus construction that we did, we can immediately see that with positive feedback, I have one branch. It just goes along which starts from plus 1 and then, in goes along the positive real axis as K tends to infinity. That is it, right. Another branch with just starts at minus 4 and goes along the negative real axis as K tends to infinity. That is about it, right. So, the green line indicates the root locus with positive feedback, ok. So, of course I think here the value of K was 6.25, right. So, please check this. I think from the previous example, right you have to indicate the corresponding points and what was the value of K at the crossover point? K was 4. All right, previously correct.

So, you can immediately observe that once you have positive feedback, what can you say about the stability of the closed loop system? We can immediately see that with positive feedback, the closed loop system is it stable or unstable? For all K greater than 0 is unstable, all right. For all K greater than 0, we can immediately observe this, right because with positive feedback, you see that one branch of the root locus that starts from plus 1 is always in the right of plane, right no matter what we do with the value of K, right.

So, one branch is on the left half plane, but then we need both to be in the left half plane, right for closed loop stability, correct. So, consequently we have a problem, right. So, there with positive feedback we are not even able to stabilize the closed loop system with a positive value of K, all right. So, you conveniently see that how the root locus has changed drastically, right. Once we change the nature of the feedback that we introduce ok, but I hope the steps are clear and how we constructed, what to say a root locus for this particular case is clear, right ok.

So, now what I want you to do is that I want you to go and repeat the same exercise for the second problem that we did, right. Do you remember what was the open loop transfer function? I think it was K times S plus 10 divided by S square plus 1. Am I correct? Was it the second example that we considered in last class K greater than 0, right ok. So, plot the root locus in the case of positive feedback, ok. So, that is an exercise. I leave it to your swamp, ok. Once again notice the differences between what happens when we have a negative feedback and positive feedback in this example also, right. That is something which you have to do as an exercise. Is it clear? Any questions on this?

We are coming. Good point. You are preemptively and I want to say gone into the next discussion. That is good, ok; very good point. So, I am going to ask you that question next, but any questions on these steps? I am sure everything is pretty clear, right.

Now, we will come to our friends point, right. What happens when K is negative, right. So, see till now I have been sticking to a case, where k is positive, but he did not be right because say let us say K is a controller gain. See like in the example that we did last class, right with the vibrating system, let us say K be pulled out of the proportional derivative controller and basically it was a controller gain, right. So, now the question is you know like it is a real number, right. So, it can have both positive and negative values, but till now we have only plotted the root locus only for positive values of the parameter K. You know what happens when you have negative values, ok; so that something which you are going to ask ourselves now, right.

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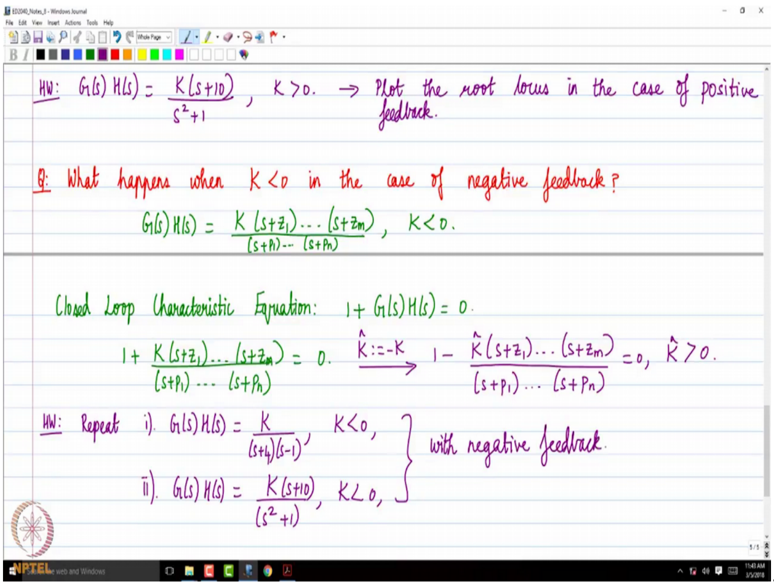

So, the question that we need to ask is what happens when K is negative in the case of let us say first we consider negative feedback, ok. That is what we are asking.

So, this means that G of SH of S can be written as K times S plus Z1 all the way till S plus ZM divided by S plus P 1 all the way till S plus PN, but the only change this K is now negative, ok. So, that is what is a change that we have. So, then what happens to the closed loop characteristic equation? What happens to the closed loop characteristic equation negative feedback, right. With negative feedback loop closed loop characteristic equation is 1 plus G of SH of S equal 0, right.

Please remember that. So, what happens to this one? If I substitute this structure, it will become as 1 plus KS plus Z1 S plus ZM divided by S plus P1 S plus PN equals 0. Now, what do I do? What I can do is following, all right. I can say let a parameter K hat be defined as minus K. I can do that, all right. So, I can essentially define a variable K hat which is the negative of K. So, what do you think happens to the range of K hat? So, the same closed loop characteristic equation now becomes 1 minus K hat S plus Z 1 till S plus ZM divided by S plus P1 S plus PN equal 0, but K hat is now positive.

Now, the answer is obvious, right. So, what do we need to do? We need to follow the steps that we discussed today. See this looks like a closed loop characteristic equation of a pseudo positive feedback system with a positive parameter K hat, ok. Physically the original system is negative feedback, but this is basically a pseudo positive feedback system with a positive gain K hat. You follow the same steps that we learnt today, ok.

In the case of positive feedback and positive gain, you repeat the same stuff, determine the answers of K hat, then your root locus will be the same ok, but then what you need to do is, your answer whatever the answers you give for stability and power, once you multiply by you change the sign. So, when you caught us in terms of values of K, it should be minus of K hat. That is all you need to do, when you have, when you want to go the other way, ok. So, is it clear?

So, now using this I want you to do the following as homework, right. Repeat what to say problem 1 and 2, all right. G of SH of S equals K times; sorry K divided by S plus 4 times S minus 1. Of course, K being now less than 0 and second problem is G of SH of S is K times S plus 10 divided by S squared plus 1 K being less than 0 of course with negative feedback, all right. Plot the root locus in this case, ok. Is it clear? Steps once you start doing it, you know you will become an expert in constructing the steps. As I discussed just follow the steps methodically you know, then you will be fine right, yeah. Is it clear?

Now, let me ask you the follow up question, ok. Well I think the answer would become obvious to you by now, right.

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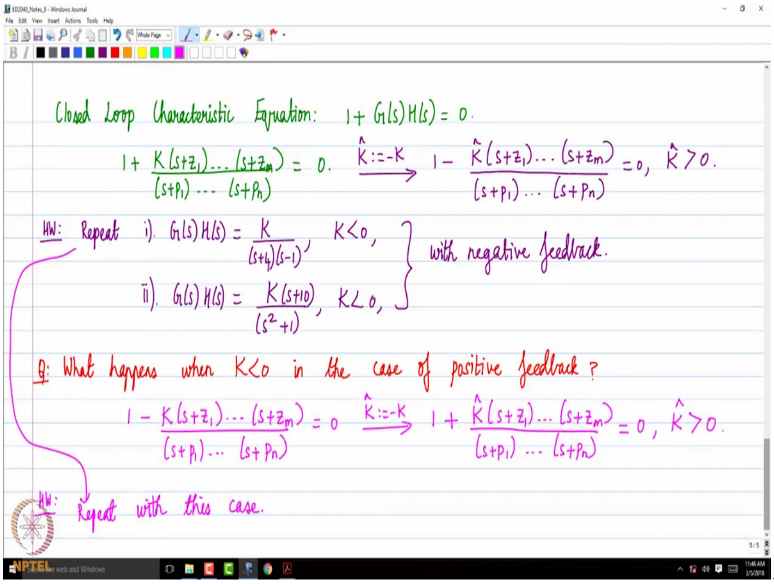

So, the follow up question is that I am going to ask is that what happens when K is less than 0 in the case of positive feedback? We can, answer is pretty obvious, right. If you have positive feedback, what do you think will happen to the closed loop characteristic equation? It will become 1 minus KS plus Z1 all the way till S plus ZM divided by S plus P1 all the way till S plus PN is equal to 0.

So, once again we define K hat which is negative of K, all right. So, what do you think will happen here? I will get 1 plus K hat S plus Z1 all the way till S plus ZM and S plus B 1 all the way till S plus PN is equal to 0, K hat being now greater than zero. So, one can immediately observe that this is like a pseudo negative feedback system with a positive gain K hat.

So, you repeat the steps with the; what to say odd multiple of pi, right. What we learnt originally construct the root locus, find the values of K hat, then in order to get the value of K multiplied by minus 1, ok. Is it clear? So, your homework is once again repeat in this case, ok. So, that like you practice all the four cases, right there is what are the four cases?

Positive feedback, negative feedback was one attribute, positive K negative K was one attribute, right. So, then you essentially need to use a correct set of steps, fine. So, if I call the set of steps that we discussed in the previous discussion as the first set and what we discussed today as a second set, there is let us say you know we call set 1.

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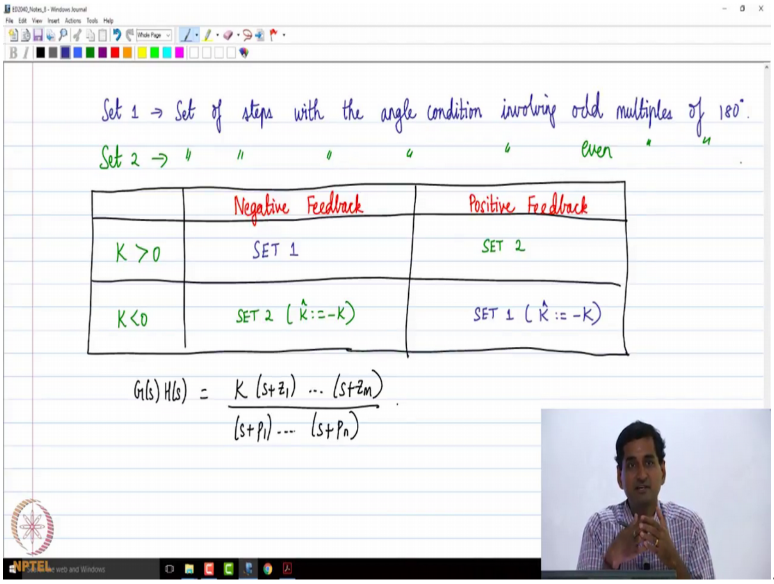

As the set of steps with the angle condition involving odd multiples of pi or multiples of 180 degrees and let us say you know like set 2 is the set of steps, the angle condition involving even multiples of 180 degrees, right we can immediately construct a grid like this, right.

So, let us say we have a negative feedback, right. Positive feedback just to have a birds eye view, right of how to, what steps to follow to construct the root locus and then, like we have K greater than 0 and K less than 0, we can form a grid like this. So, what would be used for the case when we have negative feedback and positive K of course? Please remember you know like what K is. We should be in a position to write the open loop transfer function as K times S plus Z1 all the way till S plus ZM divided by S plus P1 all the way till S plus PM, right.

We should be able to basically express the open loop transfer function in this structure, right. Only then we can do all these steps, ok. So, if we have let us say negative feedback and K greater than 0, what do we do with set of steps? Would I use set 1? Set 1, all right. That is pretty obvious, right. So, then if you have positive feedback and K greater than 0, use set 2, right.

That is what we learnt at the beginning of today's class, right. Set 1 was what we learned till the last class and set 2 was what we learned today, right. With K greater than 0 now, then we discussed K less than 0. If K is less than 0 and we have negative feedback, what did we do? We used set 2, but of course with the K hat definition as minus K, right. That is the difference and when we have positive feedback with negative K, you set 1 with an equivalent definition being K hat being defined as negative K, ok. So, that is how we need to construct, ok.

So, depending on your design choice, you know like as far as see sometimes, many times what will happen is that we will do a case study tomorrow, like that we will essentially run through the entire process, ok.

Now, that we have done with root locus design, I will take a physical problem. We will start from the beginning you know. We will derive a mathematical model, get that plant transfer function, then we will try to design controllers, then we will try to find out what ranges of controller gains will stabilize, the closed loop system, then how to construct the root locus to satisfy certain performance requirements, right. As we order, we already realized the root locus provides us with a graphical representation, right.

So, we can immediately see how easily we can get the ranges of controller gains that would satisfy performance requirements if you recall performance requirements were expressed or visualized as an open trapezoid, right. For example, in the S plane left off plane, right. So, with the root locus, we can immediately get a what the value of K. If at all they exist would place my closed loop poles in the desired region, right so things become much easier to visualize and swap, right.

So, that is something which is the entire process. I will run through once tomorrow and maybe in the next class, right so that you know like we are almost halfway through the course. You know at this point, it is better to pause and take a stock of what we have done and how it can be used for in practical design, right. That is my motivation for doing a case study. Tomorrow we will do a case study, then we will go and will have a brief introduction to state space representation and then, we will go to frequency response which should keep us engaged to the rest of the semester.

So, what we have been doing is broadly what is called as Time Domain Analysis. Although like we are analyzing a frequency domain, but by and large you know like the performance specifications that we are putting forth is in the time domain, right like settling time, rise time you know and peak overshoot. That is based on transient response, right unit step response in the time domain, ok. So, some people will call it as you know like control design using time domain based specific agents, all right. We look at equivalent frequency domain based specifications when you do frequency response, ok. I will stop here and then, like we will continue tomorrow.