Control Systems
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Lecture - 39
Root Locus 4
Part-1

Let us get started at today's class, right. So, we are looking at Root Locus. So, today I am just going to complete a few more concepts related to the root locus, and then me move on with the other topics from tomorrow right. So, so, if you recall you know like what we considered in the root locus just to recap what we are doing.

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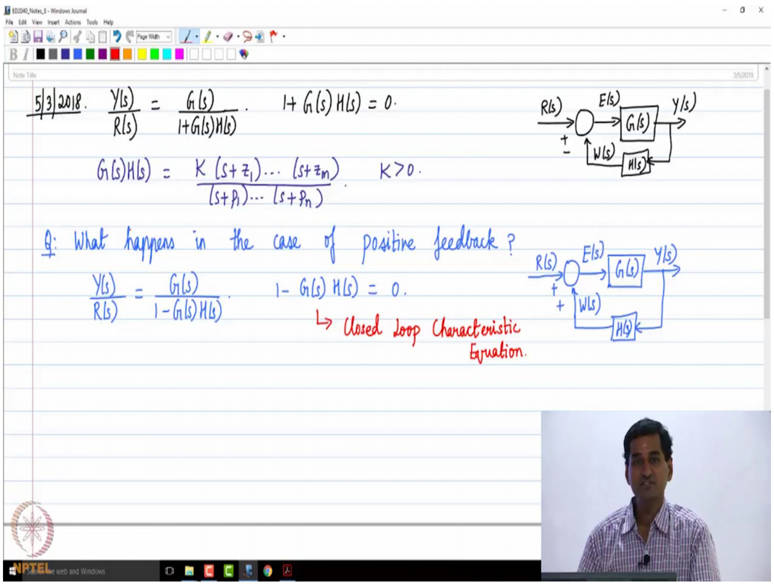

You know, like if we consider the basic block diagram, which has been our focus. So, G of s being the forward path transfer function H of s being the feedback path transfer function, and let us say we have negative feedback, right.

So, we were looking at you know what happens when we have one parameter in the open loop transfer function being varied, right. So, the closed loop transfer function was G of s divided by 1 plus G of s H of s, and the closed loop characteristic equation was 1 plus G of s H of s equal 0, right? This was a negative feedback. So, consequently what we looked at was what happens to the roots of this equation, right.

So, as we vary a parameter and what we did was it we considered the open loop transfer function G of s H of s to be some k times s plus z 1, all the way till s plus z m divided by s plus p 1 all the way till s plus p n, all right. Of course, first we considered k to be greater than 0, right. So, that is what we have looked at right.

Now, the question that we are going to ask ourselves today to begin with is what happens in the case of positive feedback. Suppose let us say we had positive feedback with everything else remaining the same, ok. The structure of G of s H of s being k times as plus z 1 tool s plus z m divided by s plus p 1 times s plus p n, right and k being positive right.

So, if we had the same a structure, and the only difference being the fact that instead of a negative feedback we had positive feedback. So, how is this block diagram going to change? So, everything else remains the same, other than the fact that are the summing junction I am going to essentially sum the 2 signals, right rather than subtracting them as we did before, ok.

So, let us say due to some reason you know like we had a structure like this right. So, this was our w of s right. So, that was a signal w of s. So, let us say we had such a case right positive feedback, then what happens to these steps involved in the construction of the root locus, right. So, let us answer that question and let us do an example today. So, in this case the closed loop transfer function becomes G of s divided by 1 minus G of s H of s, right. So, this will be the closed loop transfer function. So, consequently the closed loop characteristic polynomial and the closed loop characteristic equation would be 1 minus G of s H of s equal 0, right. So, this would be the closed loop characteristic equation in this case.

So, any value of s that satisfies 1 minus G of s H of s equal 0 would now be a closed loop for us.

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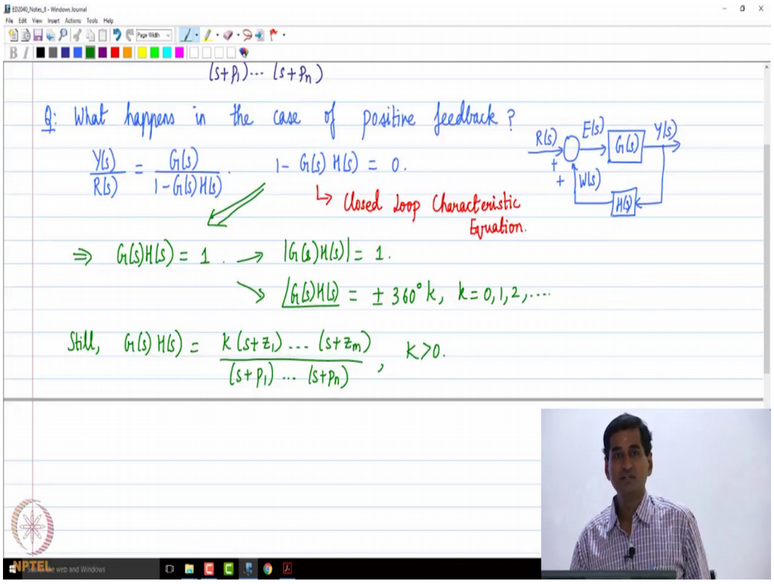

This implies is that what happens as a result this implies that, any value of s which makes the open loop transfer function to be equal to 1 will be a closed loop pole, all right. So, what will this give us this will once again give us 2 conditions the magnitude condition being g of the magnitude of G of s H of s being equal to 1, the magnitude remains the same right, but the main difference is in the phase. Now we see that plus 1 is on the positive real axis, right. So, the phase of G of s H of s is going to be an even multiple of pi 0 included in it, right. So, one can say it is 0 degrees or 360 degrees or 720 degrees and so on right depends on how we count it, right.

So now rather than an odd multiple of pi, the angle condition is going to become an even multiple of pi ok. So, that is what is going to happen right. So, that is the main change that comes about when we have negative feedback right. So, still you know like we are considering G of s H of s to be of the form k times s plus z 1 all the way till s plus z m divided by s plus p 1 all the way to s plus p n of course, k being positive right. So, that is the condition that that is a scenario we are still considering, even in the case of a positive feedback, ok.

Now, the question arises as to how the steps involved in the construction of the root locus change, right with this modification, so, that is what we are going to learn now. And we will repeat the same example once again to understand how the root locus. In fact, changes right once we changed the nature of feedback from negative feedback to positive feedback, is it clear? Ok, what is the change that occurred.

So, let us go to the document where I had listed the steps involved in the construction of route locus and figure out what really happens, right. So, it is pretty straightforward to figure out that since the angle condition changed whatever steps that we learnt previously in the that were involved in the construction of the root locus, and those that use the angle condition are going to change, right.

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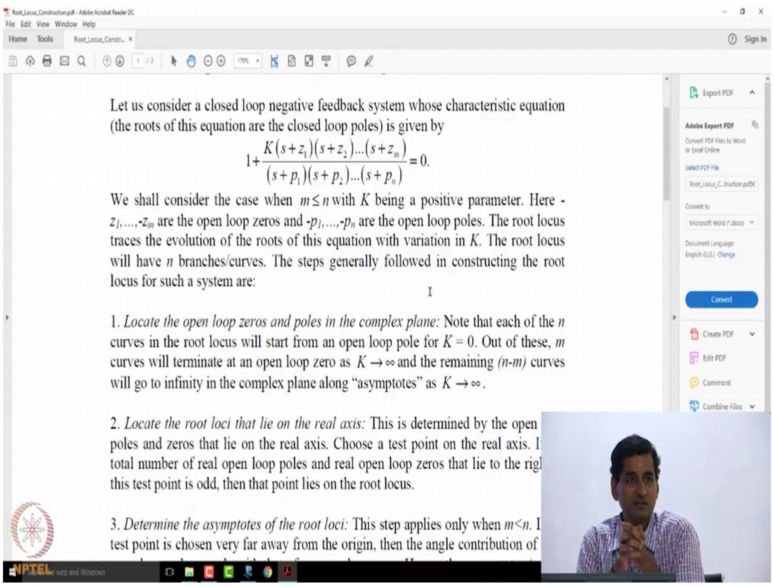

So, let us go through each step. So, the first step was to locate the open loop 0s and open loop poles in the complex plane, right that pretty much remains the same, right. So, because it doesn't matter whether it is negative feedback or positive feedback this step remains the same, right. You locate the open loop poles and open loop zeros in the complex plane, as k tends to 0 we are going to have the open the root locus starting from an open loop pole, and they go to the open loop 0s as k tends to infinity, and along asymptotes if n is greater than m, right. So, that step remains the same.

Now, locating the root locus portion of the root low size that lie on the real axis; obviously, this step is going to change right, because if you recall how we constructed this step it depended on the angle condition, right. So, previously in order to get an odd multiple of pi, right? We looked at a test point on the real axis and then we look to the right of the test point right, and we argued that if the number of open loop poles and open loop 0's on the real axis to the right of the test point is odd, then we get an odd multiple of pi because each would contribute pi, right? And then that test point lied on the root locus. Now we need an even multiple of pi.

So, what do you think will happen here? Now you take a test point, you look to the right the number of real open loop poles and real open loop 0s to the right of it should be even, right? Or it can be 0, because even we can have 0 degrees satisfying the angle condition, right. So, consequently step 2, I have typed the corresponding changes in the same document if you scroll down to page number 2. So, with negative feed with positive feedback you see that step 2 is now changed, right.

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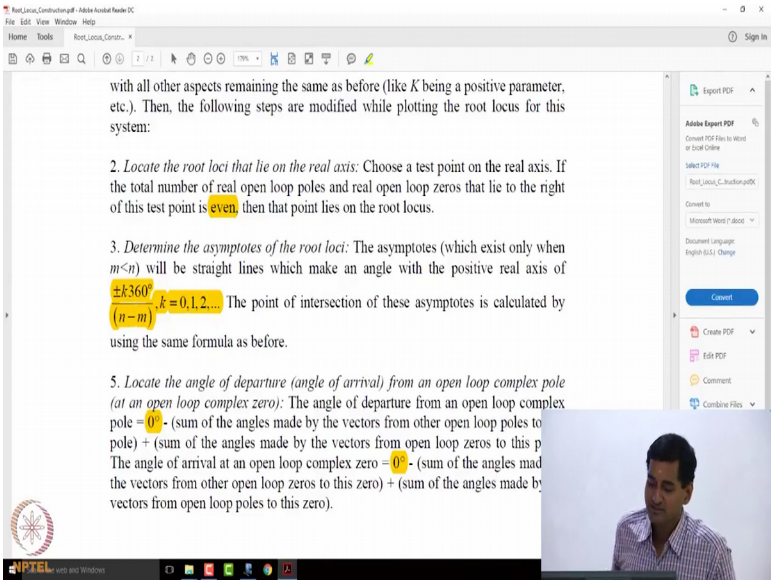

So, you see that the main change that comes about is the fact that the criteria is changed from odd to even, ok. So, you take a test point on the real axis, and then you count the number of real open loop poles and real open loop 0s to the right of it should be even or 0, all right. So, then that test point lies on the root locus, that is how the second step changes, right.

Now, let us go to the third step. Third step is to determine the asymptotes; obviously, the asymptotes angle depends on the angle condition, right. So, what was the angle of asymptotes plus or minus 180 times 2 k plus 1 divided by n minus m that now changes to plus or minus 360 k, ok.

So, that is the change which comes about right. So, because we see that what was an odd multiple of pi, now has to be an even multiple of pi right. So, consequently we get this change, right. So, 2 plus or minus 360 right times k divided by n minus m. So, once again we are going to have n minus m asymptotes the a point of intersection of the asymptotes on the real axis that formula remains the same, ok. Sum of open loop poles minus some of open loop 0 is loaded by n minus m that does not change, ok.

Let us come to step 4 step 4 remains the same because it is just a matter of figuring out potential breakaway break in points and then determining whether they lie on the root locus, right..

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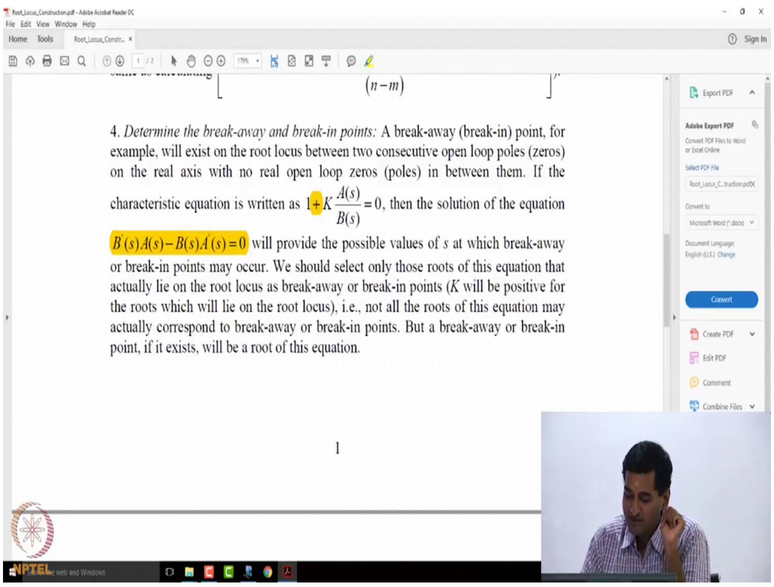

But then we need to be a bit careful now, why? Because the characteristic equation can now be written as 1 minus k times A of s divided by B of s this equation remains the same, because you take the derivative for repeated roots that condition remains the same, but then the characteristic equation is now 1 minus k A of s divided by B of s, right.

So, what we need to be careful about is that although potential breakaway points and breaking points are solutions of this equation. The way the formula for determining the corresponding k will now become k equals B of s divided by A of s, rather than minus B of s divided by A of s, ok. So, that is one change which is going to happen, why? Let me write it down here in the journal notes file, ok. So, please note that in step 5, step 4.

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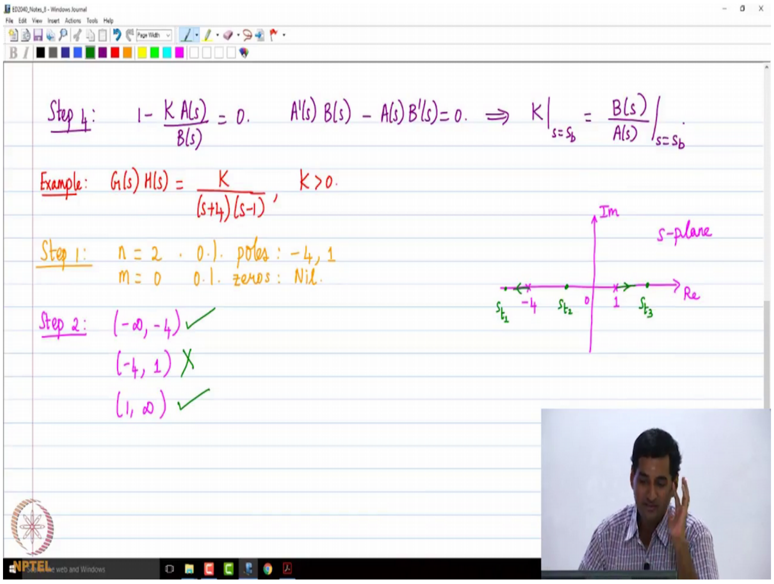

The closed loop characteristic equation would be this, ok.

Now, write with negative feedback because that is going to be 1 minus G of s H of s equal 0. So, the condition for finding the breakaway point remains the same, potential breakaway or break in points a remain the same, as before, right? But once you get this what you need to figure out is that the values of k at potential breakaway break in points would now be determined by this formula, right this closed loop characteristic equation. So, consequently it will become B of s divided by A of s right evaluated at s equals s b rather than minus B of s divided by A of s right which was the case with the negative feedback ok. So, that is a change that happens here, ok.

So, that that is a change which one should be aware of in step number 4, but conceptually the step remains the same, right yeah ok. Now step 5 does not change; obviously, does right because that depends on the angle condition the angle of departure and angle of arrival uses the angle condition. So, instead of 180 degree degrees the condition becomes you know like the formula becomes 0 degrees or 360 degrees, does not matter whether you put 0 degrees or plus or minus 360 degrees. You know, it should be an even multiple of pi that is what we want that right. So, you see that the angle of departure and angle of arrival step has changed, right because to a incorporate the change in the angle condition, right that has come about due to the fact that we have no positive feedback right instead of negative feedback.

So, what about step 6, step 6 remains the same because crossover point you substitute s equal g omega the closed loop characteristic equation. Of course, we need to be aware that now the closed loop characteristic equation is slightly changed because it becomes 1 minus G of s H of s equal 0. So, we need to be careful with that processing, ok.

But otherwise you just substitute is equal 0 omega in the same equation and proceed as before, you equate the real part to be 0 and imaginary part to be 0 and find potential solutions for omega and the corresponding value of k, all right? So, and then like we go forward, right so, that is the 6th ok. So, we see that you know these are the changes that are brought about due to the fact that we have positive feedback rather than negative feedback, ok.

So now let us go and repeat the exercise that we have already done, right. So, and then like we will check out what are the changes that we got, ok. So, as an example let us repeat the same exercise, and you please help me in doing this right. So, I think the first exercise that we considered was the open loop transfer function was k divided by s plus 4 times s minus 1, am I correct? All right, with k greater than 0 was it right correct yeah, ok. So, let us let us do all the steps right in the construction of the root locus, right. So, the first step remains the same. So, there are 2 open loop poles, no open loop 0's. So, the 2 open loop poles are at minus 4 and plus 1, there are no open loop 0's right?

So, consequently we are going to have 2 branches starting from minus 4 and 1 as k tends to 0, and the 2 branches of the root locus will go to infinity along asymptotes as k tends to infinity, right. So, that is what is going to happen, all right? So, let us look at the second step. So, second step is to locate the part of the real axis that lie on the root locus, right? So, let us draw the s plane, and then let us figure out what happens right. So, the 2 poles where at 1 and minus 4 so, we immediately see that the real axis is divided once again into 3 regions minus infinity to minus 4 minus 4 to plus 1, and 1 to infinity.

So now if we take a test point here, s t 1, and then like look to the right, how many open loop poles and open loop 0s do we have? 2 and that is even, all right. So now, minus infinity to minus 4 lies on the root locus. So now, what about minus 4 to plus 1? No right, because if you take a test point s t 2, and then in this region and then look towards the right you see that there is one real open loop pole to the right that is odd right. So, that criteria is not satisfied so, it does not lie on the root locus.

So now let us take a test point between one and infinity, you can immediately see that there are no real open loop poles and open loop 0s to the right. So, the region one to infinity lies on the root locus. So, immediately you can see a change, right in there in the way the root locus is going to evolve, right. So, consequently what is going to happen is that one branch is going to start from minus 4 in this direction, ok.

And the other branch is going to start from plus 1 in this direction. You know, with in the previous instance with negative feedback the situation was the reverse right the branch from minus 4 started towards the right, correct? And branch from plus 1 started towards the left no the situation is just flipped ok, and that is due to the fact that you know the scenario has changed, right. So, that is something which we need to observe, ok.

So, let us go to the next step, let us go to step 3.

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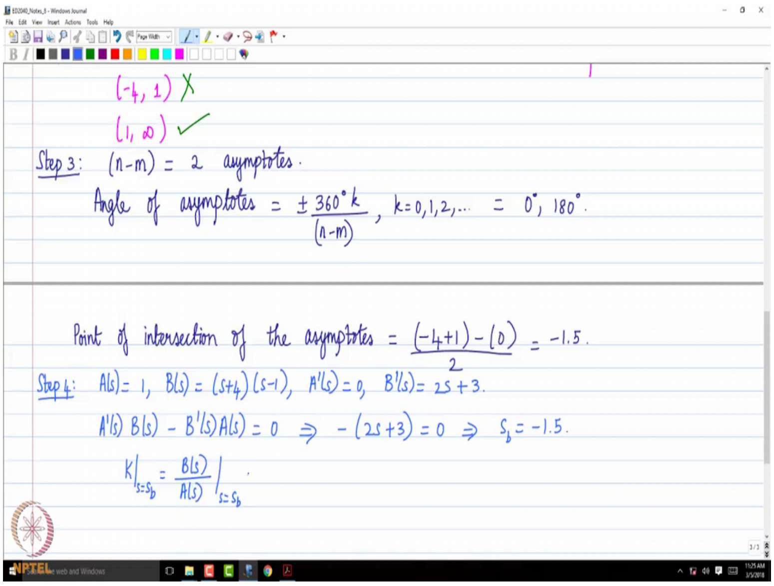

So, step 3 means you know like we need to figure out the asymptotes. So, n minus m, that is going to be equal to 2 asymptotes, right. So, what is the angle or made by the asymptotes, it is going to be equal to plus or minus 360 degrees times k divided by n minus m k being 0 1 2 and so on, right.

So, in this case you will immediately see that since n minus m is 2, I can take it as 0 and 180 degrees right. So, it becomes you substitute n minus m equals to you know like you can write the solution as 0 degrees and 180 degrees right. So, those are the angle made by the asymptotes with the a real axis, right? Now the question is it like what is the point of intersection of the asymptotes, right .

So, the point of intersection of the asymptotes that formula still remains the same. So, that is nothing but sum of open loop poles minus sum of open loop 0s divided by n minus m, right? What is the sum of open loop poles? It is going to be minus 4 plus 1 right minus 3, then there are no open loop 0s divided by n minus m that is going to be 2. So, it is still going to be minus 1.5, right. That is the same thing which we observe which we obtained previously, right with negative feedback.

So, what this really means is the following, right. So, let us say I have minus 1.5 the 2 asymptotes are just going to be along the one along the negative real axis, one along the one going towards the positive real axis, that is those are the 2 asymptotes ok, that is what this calculation tells us. Right because there are 2 asymptotes that are supposed to intersect at minus 1.5, and supposed to make an angle of 0 degrees and 180 degrees with the positive real axis. So, obviously, they have to travel along the positive real axis and the negative real axis, ok. So, that is going to be the asymptotes, right those 2 are going to be the asymptotes, right. So, as far as the, this particular root locus is concerned, ok.

Now, let us go to step 4, step 4 is once again going to be breakaway break in points. So, the structure is still the same A of is this, B of is going to be s plus 4 times s minus 1 A prime s is going to be 0. A B prime s is going to be equal to what did we get last time ? It was 2 s plus 2 s plus 3, right that is what we got, correct. So, if you if we process that equation A prime s, B of s minus B prime s A of s here prime indicates a derivative with respect to s please remember that, right?

So, this implies that we will get the finals simplified equation as this. So, once again we get s s b, ok, I am just putting a subscript B at the end to indicate that there is a potential break away or break in point, ok. So, once again we get the same solution right as before, what we did in negative feedback, now there is an important difference, ok.

So, as we discussed if I want to figure out the value of k at s equals s b, now I need to do B of s divided by A of s at s equals is s b. Now can you calculate and tell me what you get.

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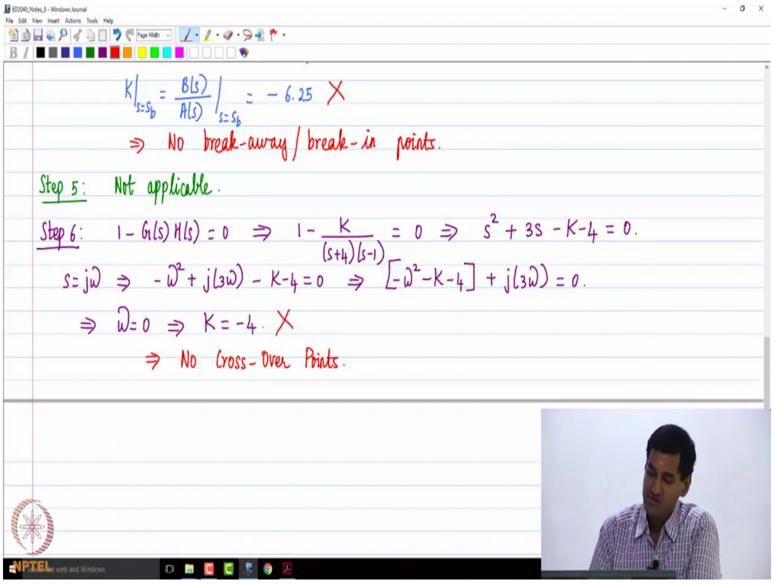

Just substitute minus 1.5 in B of s because A of s is anyway, 1 right what do you get? Minus 6.25 and consequently this is not ok, all right, but anyway we could have figured it out right I did not even have calculated k, but I am teaching you a good process to follow, ok.

In this case, in this example, I need not even have calculated k ok, but I am I am doing it for our general case, you give me any general open loop transfer function, how should I construct the root? Locus ok, that is what we are learning, right why I need not even have calculated k in this example, because look at s b minus 1.5. So, where is minus 1.5 minus 1.5 lies between minus 4 and plus 1 does my the region minus 4 between minus 4 and plus 1 lie on the root locus, know from step 2 we know that minus 1.5 cannot be on the root locus.

So, that that could have be that is enough to show us that there is no breakaway break in point. But still it is better it is see things will not turn out this way all that that is why we need to calculate k and that is a better way to process this information, right. So, you see that of k is negative. So, consequently there are no breakaway or break in points, right. So, this implies that no breakaway break in points for this particular case, yeah.

So now let us go to step 5, what is step 5? Step 5 angle of departure from an complex open loop pole and angle of arrival at a complex open loop 0. Does that step apply to the example? No, all right so, it still does not apply. Because we have only real open loop poles in this example right so, step 5 is not applicable ok. So, as far as this particular transfer function is concerned, right.

So, step 6 we need to figure out the crossover points, right. So, what is the closed loop characteristic equation, it is going to be 1 minus G of s H of s equal 0, ok, please remember we are talking about positive feedback, right. So, it, obviously, becomes 1 minus G of s H of s equal 0. So, consequently what happens to this one? We get the closed loop characteristic equation as this. So, if we simplify this, what am I going to get we are going to get s square plus 3 s minus k minus 4 equals 0, am I correct, right?

Now, in this example in this closed loop characteristic equation, you substitute s equals j omega, right? Why do we substitute s equal j omega, because on the imaginary axis the structure of s s of the form j omega right because the real part is 0, that is why we are searching for whether there are solutions of this structure, right as far as the closed loop characteristic equations is concerned. So, if you substitute a s equal j omega we immediately get minus omega square plus j times 3 omega minus k minus 4 equals 0. So, this gives us a minus omega square minus k minus 4, this is the real part plus j times 3 omega equals 0.

So, we can immediately see that from the imaginary part of omega is 0, right that is the only possible solution from the imaginary part. So, this immediately implies that k should be minus 4, ok. So, this is not possible once again, right. So, because k is negative of course, I need not even have done these calculations once again because step 2 immediately tells me that there is no crossover point, because omega basically is 0 is not a cross 4 point or the s equals 0 is not a crossover point.

But why should we still do this because I have to double check. See step 2 only tells me that the root locus will not cross over at the origin, but I can always have cross over at some other point on the imaginary axis, right? Can you know all right? So, in this example it turns out that there are no other points like that right, but it is always better to do these steps methodically, ok. So, this means that no crossover points.