Control Systems
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Lecture - 37
Root Locus 3
Part-1

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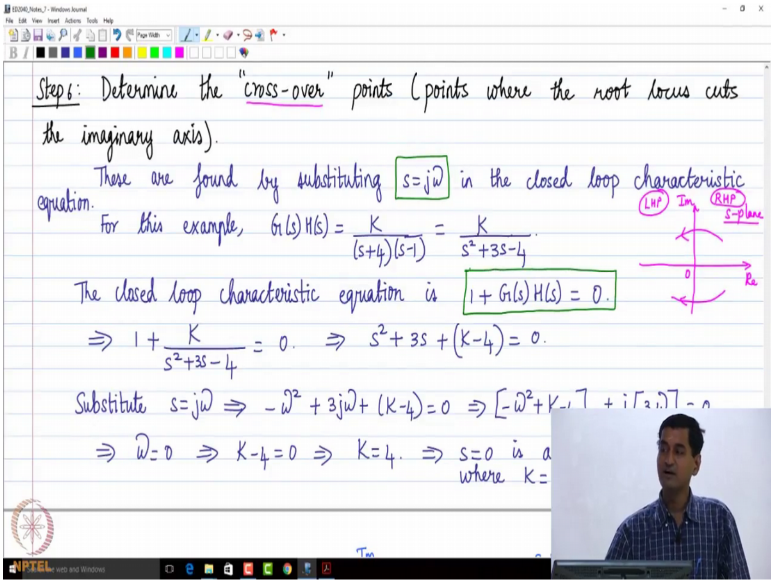

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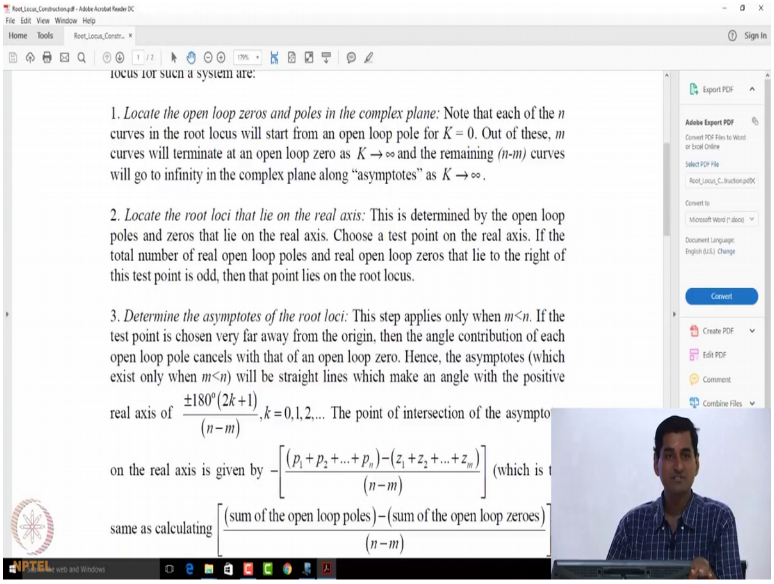

So, we are looking at the Root Locus. So, let us go back and recap, what we did right. So, let me go back to the stock 1, and then I will come back to this notes file. So, if you recall, we did 5 steps right. So, the 1st step was given an open loop transfer function. The 1st step was to locate the open loop poles and zeros in the s-plane right. So, and we learned that you know the root locus will have n branches in general, each of those n branches will start from an open loop pole, and they will end at an open loop 0, if it exists ok. And the remaining n minus m branches will go to infinity along asymptotes right.

Then we figured out how to get the parts of the root locus that lies on the real axis, you know like we use the angle condition in that regard right. So, we only looked at the real axis, and we divided the real axis into sub regions based on the location of open loop poles and zeros. In each sub region you take a test point, and you look to the right of the test point of the number of real open loop poles and real open loop zeros is odd, then that point that region lies on the root locus right that is the 2nd step.

The 3rd step was to determine the asymptotes of the root loci. And using the angle condition, once again we figure out that the angle made by the asymptotes with the real axis is going to be plus or minus 180 degrees times 2 K plus 1 divided by n minus m right. So, what are asymptotes, asymptotes essentially indicate the locus of the closed loop poles as essentially the what to say K tends to infinity right. In case, there are no open loop zeroes, where we can terminate that branch right. So, and the point of intersection of the asymptotes on the real axis is essentially given by sum of open loop poles minus some of open loop zeroes divided by n minus m right that was step 3.

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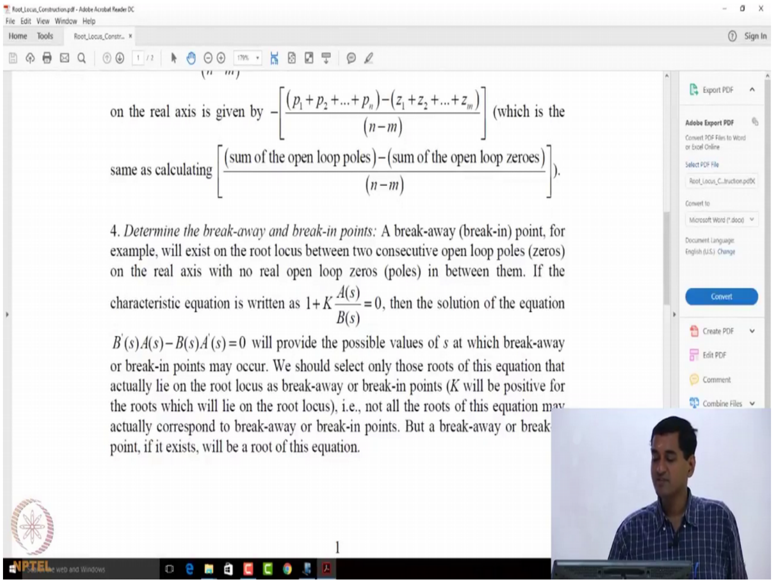

Step 4 was to figure out potential break-away or break-in points, we discussed when break-away and break-in points could occur. And we figured out that you know at a break-away or break-in point, we are going to have a repeated pole right. So, if a polynomial has a repeated pole, you know like then what we do is that, we take the first derivative, even that value or root you know like satisfy is the first derivative equals 0 that is the concept we use to figure out how we get break-away and break-in points.

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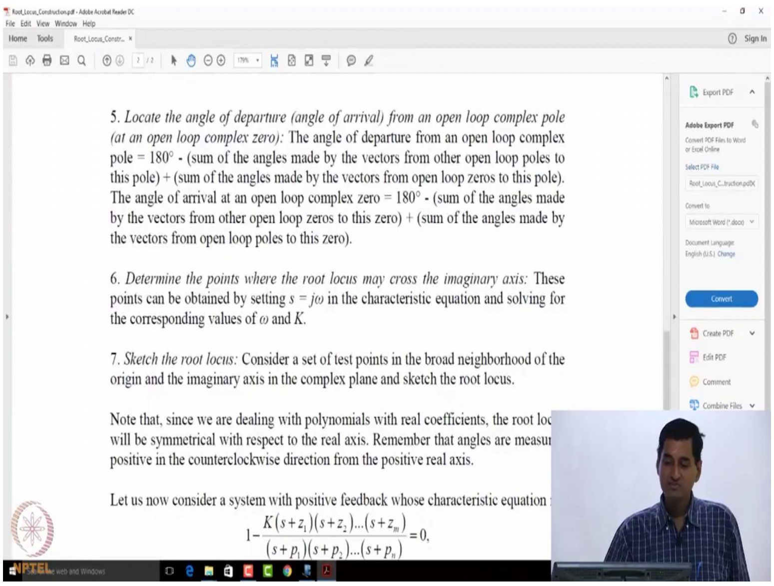

And the next step was to figure out the angle of departure from a complex pole or angle of arrival at a from a at a complex open loop zero right. So, we figure out, this is the formula. So, we did an example yesterday right, to essentially see, how we can calculate right. So, please note that, step 5 applies only when we have a what to say complex conjugate open loop pole pair, or complex conjugate open loop zero pair right. So, if you have in the example that we just started off with you know like, there are two open loop poles, both were real right. So, step 5 did not apply to that problem right.

And step 6 is what we are essentially going to look at today ok, and then we will complete the root locus construction right. So, what is step 6, step 6 is to determine the crossover points ok. Like what are these crossover points, crossover points are nothing but points where the root locus cuts the imaginary axis. So, imagine that you know like in the s-plane, the root locus essentially we want to figure out, where it cuts the imaginary axis. And why are these called crossover points, because I can have, for example, let us say two branches of root locus coming like this, and then they cross over from the right half plane into the left half plane right. So, please note that the imaginary axis divides the s-plane into RHP and LHP right.

So, a cross over point as the name indicates, you know like is a point at which the root locus transitions from one half plane to other, it can be either from right half plane to left half plane, or left half plane to right half plane either way. So, but you can immediately see that the imaginary axis, the boundary right between the two half planes, so that is why, you know like we are looking for where the root locus cuts the imaginary axis ok. Those points are what are called crossover points.

And how do we figure those out, you know like by substituting s equals j omega in the closed loop characteristic equation. Why, because if any branch of root locus cuts the imaginary axis, within which includes the origin by the way right. So, then what is going to happen, the real part is going to be 0 right, on the imaginary axis that is what it is right. So, if you consider a complex variable s of the form sigma plus j omega, if the complex variable is purely imaginary, that means, sigma is 0 right, so that is why we search for roots of the form s equals j omega, for some value of K right that is what we are going to look for, so that is why we substitute s equals j omega in the closed loop characteristic equation.

So, let us consider, what we what example that we are looking at. For the example that we are looking at G of s H of s was K divided by s plus 4 times s minus 1, I just expanded it as s squared plus 3 s minus 4 right. So, now, the closed loop characteristic equation is this right, 1 plus G of s H of s equal 0 that is something, which we already know all right. So, we just plug it in and simplify, we get the closed loop characteristic equation as s squared plus 3 s plus K minus 4 equal 0 right.

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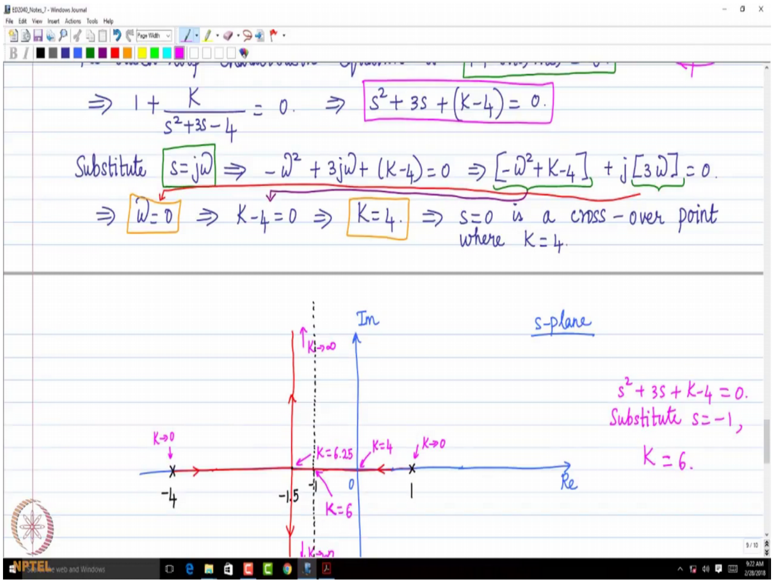

Now, we substitute s equals j omega into this particular equation right. So, this is the closed loop characteristic equation right, because please remember, the roots of this closed loop characteristic equation are the closed loop poles right. So, we are looking for closed loop poles, which are which are what to say imaginary.

So, we essentially basically substitute sigma the real part to be 0. So, the structure is of the forms, j omega ok. Omega can be 0, because we can also have the crossover point of the origin right. See for example, we can have a crossover happening from the positive real axis to the negative real axis, and vice versa, you can have crossover on the real axis also right.

So, we substitute s equals j omega, s squared becomes minus omega squared, then we get 3 j omega plus K minus 4. Then we collect the real part and the imaginary part. So, the real part is minus omega squared plus K minus 4, the imaginary part is 3 omega right. So, obviously, when both are equal to 0 alright so, when this complex function is equal to

0, the real part should be 0, the imaginary part should be 0 right. So, real imaginary part being 0 gives me?

Student: (Refer Time: 07:35).

From here, I can immediately figure out omega is 0 right that is the only potential that is one potential solution for omega right, a crossover point. But, once again we need to check the value of K ok, whether it is positive or negative right. So, what do we do, we check now process the real part. So, if you look at the real part, if I substitute omega equals 0 in the real part what happens, I get K minus 4 equals 0, or in other words K is equal to 4.

So, omega is equal to 0, and K equals 4 is a potential solution of the closed loop characteristic equation, when we are searching for a crossover point right. And this is a valid solution for this problem, because K is assumed to is taken to be positive right. So, you see that omega equals 0 means s equals 0 right that means, the origin right. So, and at that point, the value of K the gain K is positive right. So, the origin is a valid crossover point right, as far as this particular example is concerned, correct, so that is step 6 ok. Is it clear how to calculate what is meant by a crossover point and how do we calculate them.

And why are we interested in crossover points, because a crossover point indicates when I am transitioning from one half plane to another right, is it not, because that is very important for us as far as stability is concerned right. See I have to design my closed loop system such that I choose only those values of K, which would stabilize my closed loop system right.

So, a crossover point represents, when the root locus may transition from the right half plane to the left half plane, which is going coming from unstable to stable regions or vice versa right. So, I know which ranges of K to choose, which not to choose right, for closed loop stability, is it clear ok, so that is step 6 determining the crossover point.

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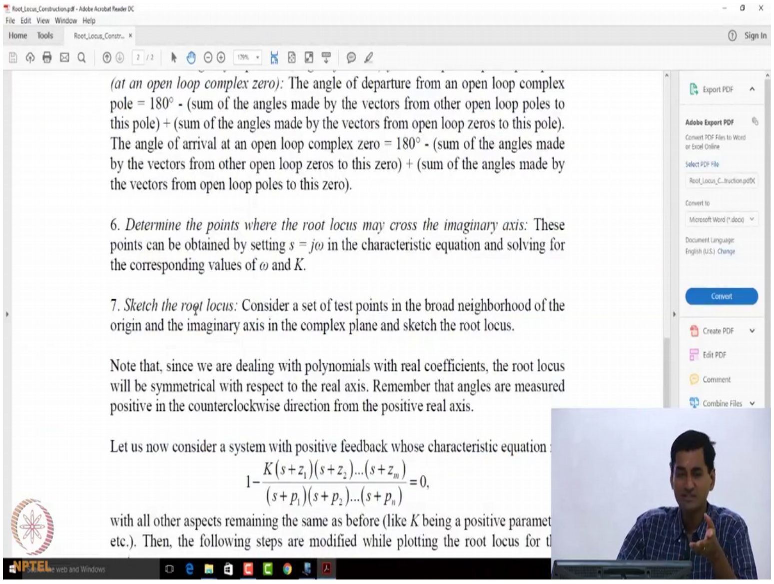

So, the final step is to sketch the root locus ok, like by enlarge what we need to do is that, like we need to essentially do steps 1 to 6. What I expect of you, and you draw the root locus by hand is to what I say methodically do steps 1 to 6, by the time you will have an idea, you know as you keep on constructing root loci you will have an idea as to how to construct the final root locus approximately.

The final step is to essentially consider a set of test points on the broad neighborhood of the origin and the imaginary axis in the complex plane and sketch the root locus, which is not very tractable or very easy to do, when we do by hand right. So, when you for example, if you use software like mat lab, you can pretty easily get the actual root locus. But here you know like we will just use steps 1 to 6 to essentially brought the root locus ok, is it clear.

And once again you know like please use the r locus command in mat lab to sketch the root locus for all the problems that we are doing right. Even in homework number 2, which I have given you, you know like you need to construct root locus for certain problems. Sketch by hand, and then just check your solutions ok, by plotting the root locus and mat lab right.

So, let us let us construct based on whatever we have learnt right. So, we already know that one branch of the root locus is going to start from minus 4 ok. So, we are always going to have you know like the root locus to start at an open loop pole as K tends to 0.

So, one branch of the root locus starts from minus 4, and another oops and another starts from plus 1 in this direction, and of course, there is a crossover at 0 ok. At the crossover point, see you need to mark all the important points ok. The crossover point the value of K is 4 right.

And where do these two points intersect minus 1.5 right, so that was the break-away point right. So, let us say you know like I have minus 1.5 to be somewhere here ok. So, what is going to happen is that they come here, and then they intersect each other.

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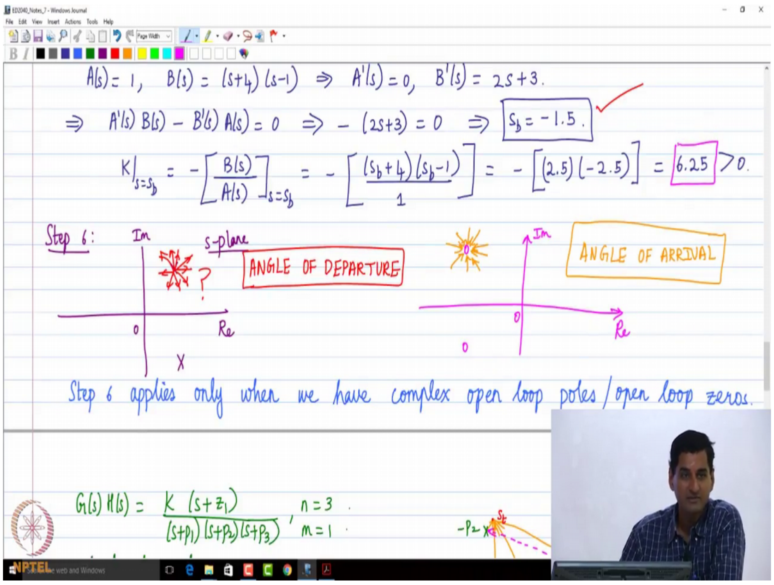

And what is the value of K at minus 1.5? If you go back to step 4 at the break-away point what was early of K that we calculated. So, it was 6.25 that is what we can see right. So, it was 6.25, so we need to essentially mark that ok. Now, they are going to break-away ok.

Now, the question is that like how does how does this evolve with time right. So, and we know that the asymptotes are going to also intersect at minus 1.5 for this problem ok. Please note I am repeating once again, in this particular problem it so happens that the point of intersection of the asymptotes on the real axis and the break-away point happened to be the same ok, it is not the case in general ok.

So, in this case, the asymptotes also intersect at minus 1.5. And what are we going to have? As angle of the centaurs plus 90 n minus 90 so, what we will have in this problem is that one branch will just take off vertically upwards ok, and another branch will just go vertically downwards ok. So, this is what happens as K tends to infinity ok. These is how you should mark the root locus and mark all the critical points as well right, so that is the complete root locus for this particular problem ok so, using all the steps. Have you used all the steps? Yes, right. We have marked all the critical aspects of the root locus.

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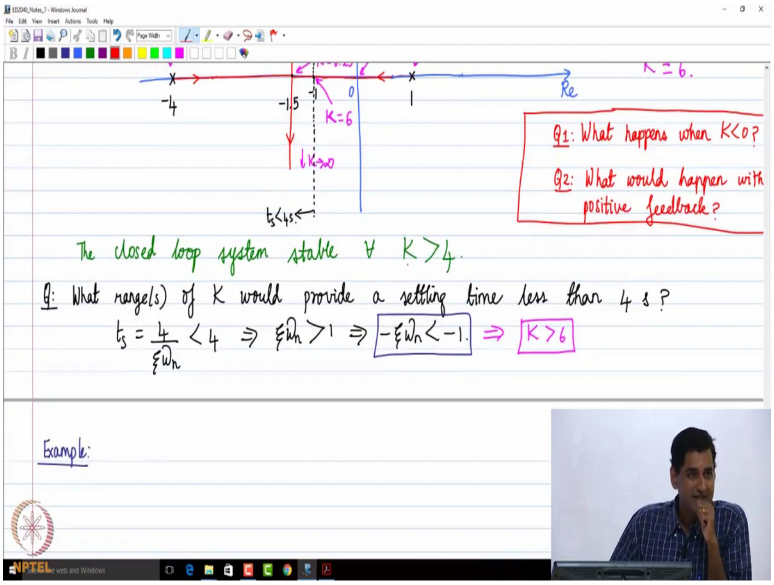

And immediately we can see from the root locus is that the closed loop system is stable for all K. When is the closed loop system stable?

Student: K greater than or equal to 4.

K greater than 4 right. Why, because you see that one branch of the root locus is always going to be in the left half complex plane ok. The 1 starting from minus 4 ok but, another branch which starts from plus 1 migrates into the left half plane only after K is greater than 4. So, you need both the all the closed loop poles to be in the left half complex plane right. So, the root locus gives you a very nice graphical representation to essentially figure these things out right, so that is the advantage with root locus right. You can get a general perspective ok.

So, let me ask you another question ok. So, what range of K would see closed loop stability is for k greater than 4 would provide a settling time, of course using the 2 percent criteria less than 4 seconds. So, from the root locus can we figure out you know like what ranges of K would give me a settling time of less than 4 seconds. So, what was the expression for the settling time? Now, t s was 4 divided by zeta omega n right, so that should be less than 4 or in other words zeta omega n should be greater than 1, which implies that minus zeta omega n should be less than minus 1 right.

So, let us say I draw a vertical line at minus 1. See we already discussed this right, when we last week you know when we want to get the how to translate performance specifications into regions of poles right. So, you see that any if all the closed loop poles lie to the left of this vertical line at minus 1 that will give me an assurance that the settling time is less than 4 seconds right.

Now, what value of K would get me that? Obviously, what I need to do? I need to find out what is the value of K at this point right. How do I find what is the value of K at that point now?

Student: (Refer Time: 17:08).

I look at the closed loop characteristic equation right, and substitute s equals minus 1 right, so that that is that is why you know we are doing all these steps right. Look at the closed loop characteristic equation right. What are the closed loop characteristic equation, s squared plus 3 s plus K minus 4 equals 0 right. Substitute s equals minus 1. Why am I doing it? Because that is the point after which beyond which you know like the both branches of the root locus go to the left of the minus 1 line right.

So, when I if substitute s equals minus 1, what do I get for K? This becomes plus 1, this is minus 3, minus 7, then I will get K to be what is the value of K, 6 right so, 6 ok. So, for any value of K, which is greater than 6 right. We will get a settling time less than 4 seconds ok, so that is the guarantee we can have. Of course, please note that we are using the expression for the step response of an under damped 2nd order system to come up with the settling time response. Please remember that ok.

So, this is essentially assuming that you know like you are going to have design an under damped 2nd order system by enlarge ok, we need to keep those things in mind right. So, when we do such analysis right. But, at least like it gives us a first get value beyond which we can go and tune our experimental system ok, so that is that is the benefit of this analysis ok. So, this is the final root locus plot ok, is it clear ok so, any questions on this? Right ok.

So, now what we are going to do is that like I am going to do an example ok. Like another example, and then we will quickly run through all these steps right. So, let us do another example for a root locus. Yes (Refer Time: 19:36).

Student: (Refer Time: 19:37) while you put a s equal to minus 1 then (Refer Time: 19:40).

Yeah ok. See at what point of the in the root locus thus both root locus branches you know lie to the left of this minus 1 vertical line.

Student: (Refer Time: 19:56).

That is why, because that is the condition that we obtained right ok.

Student: (Refer Time: 20:08).

We discussed this last week also right, so that is for settling time condition we converted into a constraint on the real part of the pole. See please remember what was minus zeta omega n, it was the real part of the poles of the under damped 2nd order system right ok. So, essentially we want if our dominant dynamics is going to be an under damped 2nd order system, we want the what to say the real paths to be to the left of this minus 1 line right. So, we I think we did it last week right so, sometimes 2 or 3 classes ago.

But, then like we need to take it a pinch of solved right. So, because see between K equals 6 and 6.25, the system is really over damped ok, it is not an under damped, please remember right. So, but then you know like we still use this analysis to get an approximate range that is what we are doing right. But, the root locus will tell you that ok. By enlarge we want to design an under damped system, so I would choose maybe a K beyond 6.25 in this example.

Student: How do you understand that (Refer Time: 21:12).

Over damped right; Why, because between 6 and 6.25 what can you say about the two branches of the root locus, both are on the real axis right. So, both closed loop poles are going to be real. So, what do you call as a call a 2nd order system, where both with distinct real poles? Over damped right; At K equals 6.25, it becomes critically damped. And then K beyond 6.25 becomes under damped that is what we can easily look from there.

See the so you now you can appreciate the value of root locus right. It gives you a graphical representation from which you can what to say get all these interpretations right. Yeah.

Student: Value of K to the left of the break-away point (Refer Time: 21:58).

Value to the left of the break-away point it goes from 0 to 6.25. So, you pick any value of s, you substitute it here, you will get the corresponding value of K right. See you are asking about this range right from minus 4 to minus 1.5, it scales the same way as 1 to 1 minus 1.5 right. So, please note that you know another interesting thing about the root locus is that. The root locus plot does not scale linearly with K ok. See for example, you know like as we construct more examples you will see that to travel what we think is a very large distance the change in K would be very small relatively smaller right.

And sometimes you know like we were we are going to do another example. Where you will see that in a very finite distance you know it would go from 0 to infinity ok, it depends on the problem right. So, it does not scale linearly you know like as you vary K right; Yeah fine any other questions?

Student: No.

So, I am I am going to leave you with two questions right. So, question 1, you think about these questions, and then like we will answer them in the next class right. So, what happens when K is negative? See in general, my K can go from minus infinity to plus infinity right. Do you agree? Right. See K is a design parameter you know like I can K is a real number, so I can have K going from minus infinity to plus infinity.

See think of K let us say as a proportional gain right, it is a real number ok. We did only for the part going from 0 to infinity. What about minus infinity to 0, how do you construct the root locus for that ok, what steps will change? Anyway I have given those changes in the document. Please read through them, think and then come back to the next class. We will discuss them once again right.

And question 2, what would happen with positive feedback? Ok, that is the second question we need to ask ourselves. So, we looked at negative feedback. But, let us say if we are doing positive feedback right in some example, some case right, how would the steps of the root locus change once again right? Please think about it.

Why would with positive feedback, why would the root locus steps change, because the closed loop characteristic equation is going to be now 1 minus G of s H of s equals 0 right. We did everything considering 1 plus G of s H of s equals 0. Then the magnitude condition will remain the same, but the angle condition will change right ok. So, you think about what changes would occur right, is it clear ok. So, please at think about both these questions, the answers have already been provided in the (Refer Time: 25:06).

I just want to read and think, and come back to the next class, is it clear?