Control Systems
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Lecture – 17
First Order Systems
Part – 1

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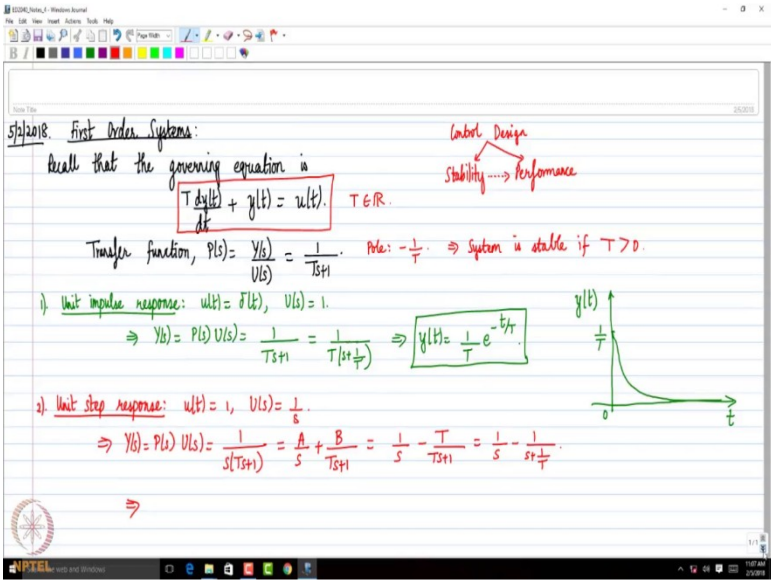

In this lecture we will discuss about performance of a first order system and obtain some quantifiable parameters that can be used to evaluate a systems response. The general governing equation of a first order system is given by

$$T\frac{dy}{dt}+y(t)=u(t) \quad ,$$

where T is a real number in general. This system has a pole at $\frac{-1}{T}$. We can immediately see that system is stable if $T>0$.

Let us look at the unit impulse response. That means

$$u(t)=\delta(t), U(s)=1.$$

$$Y(s)=P(s)U(s)=\frac{1}{Ts+1}=\frac{1}{T\left(s+\frac{1}{T}\right)}.$$

Taking inverse Laplace transform,

$$y(t)=\frac{1}{T}e^{\frac{-t}{T}}.$$

That is the unit impulse response. If we want to plot this we look at the following,

$$t\to\infty, y(t)\to 0 \quad \text{and} \quad t=0, y(t)=\frac{1}{T} \; .$$

Based on these, the plot of $y(t)$ can be drawn as shown.

Here at $t=0, y(t)\neq 0,$ but according to the definition of transfer function the initial condition should be 0. The reason for this is that we are giving an impulse input at $t=0$. Ideal impulse instantaneously goes to a very high magnitude at $t=0$ and comes back again. During this the output increases from 0 to $\frac{1}{T}$. That is how it should be interpreted.

So, the unit impulse response of a first order system increases from 0 to $\frac{1}{T}$ almost instantaneously and then decays exponentially with the exponent being $\frac{-t}{T}$, and then goes to 0 as $t\to\infty$.

Let us calculate the unit step response.

$$u(t)=1, U(s)=\frac{1}{s}.$$

$$Y(s)=P(s)U(s)=\frac{1}{s}\frac{1}{Ts+1}=\frac{A}{s}+\frac{B}{Ts+1}$$

Solving the partial fractions, we get $A=1$ and $B=-T$.

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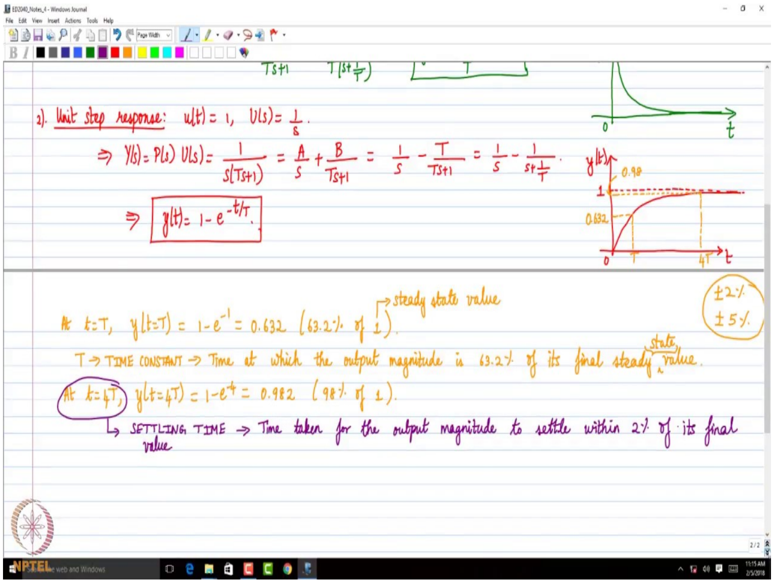

Taking inverse Laplace, we get

$$y(t) = 1 - e^{\frac{-t}{T}}.$$

As $t \to \infty, y(t) \to 1$ and $t = 0, y(t) = 0$.

$y(t)$ is going to increase from 0 and go to 1 as $t \to \infty$, based on this we can plot the response as shown.

We can observe that at $t = T$, $y(t) = 0.63\ \ 2$.

0.632 is 63.2% of 1, which is the steady state output. So, $t = T$ happens when the output reaches 63.2% of the steady state value. T is called as time constant and it is defined for a stable first order linear time invariant system as the time at which the output magnitude is 63.2 percent of its final steady state value. The time constant is a single most important parameter characterizing the first order system dynamics. The time constant quantifies how fast the system responds.

Now, if we substitute $t = 4T$, the value of the output is going to be 0.982. This is close to 98% of the final value. We draw a band around the final value which is usually taken as ± 2 or ± 5 . We consider ± 2 , that means the output is 98% of the final value. That happens to be four times the time constant and this parameter is called as the settling time.

Settling time answers the question; what is the time taken for the output magnitude to settle within 2% of it is final value? This is important to evaluate how fast a system is.

These parameters are defined based on the unit step response. If we want to evaluate time constant and settling time, we you have to use step as the input. We need not use unit step all the time. We can always scale the input and the output accordingly.

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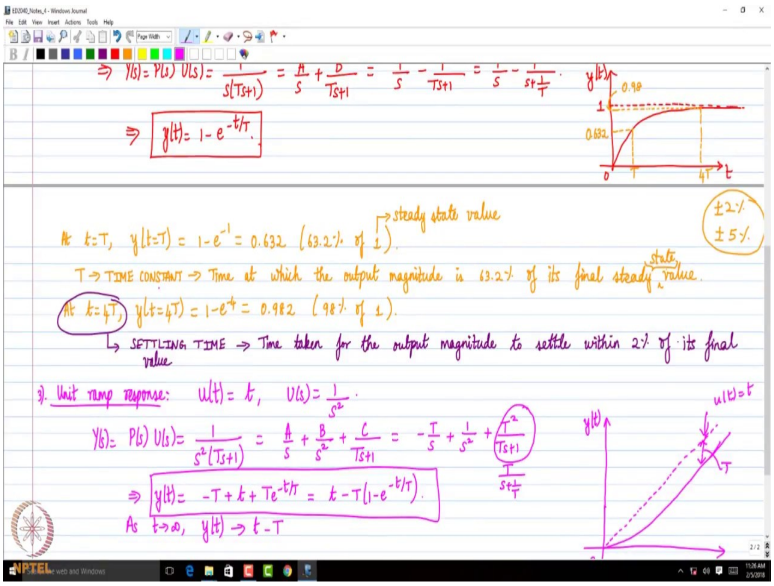

Let us look at the unit ramp response.

$$u(t)=t, U(s)=\frac{1}{s^2}.$$

$$Y(s)=P(s)U(s)=\frac{1}{s^2}\frac{1}{Ts+1}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{Ts+1}$$

Solving the partial fractions, we get $A=-1$, $B=1$ and $C=T^2$.

Taking inverse Laplace, we get

$$y(t)=-T+t+Te^{\frac{-t}{T}}=t-T\left(1-e^{\frac{-t}{T}}\right).$$

As $t\to\infty, y(t)\to t-T$ and $t=0, y(t)=0$. The plot of $y(t)$ is shown in the diagram. The gap between the function t which is your input and the output is going to be T

graphically. Please note that unit ramp is an unbounded signal. So BIBO stability is not applicable in this case.

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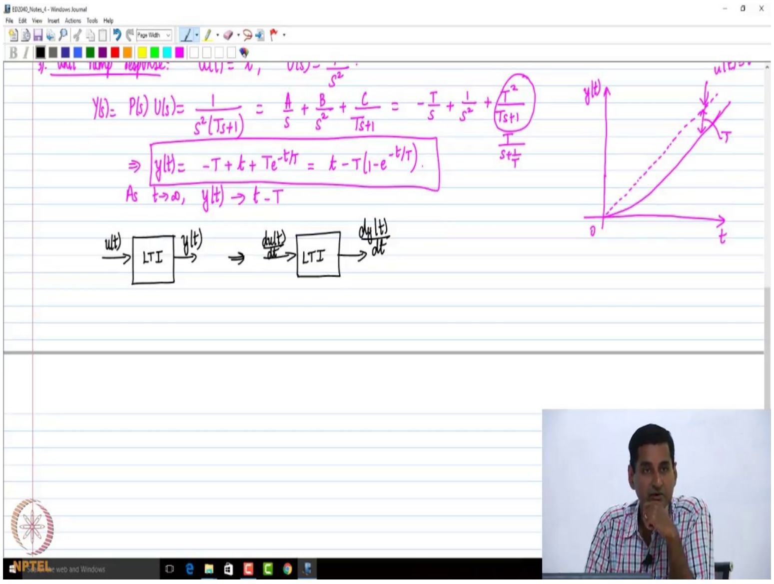

If we look at all the three responses, we can see that the step response is the time derivative of the ramp response. And the step input is the time derivative of the ramp input. This is a very nice feature of stable LTI systems. If we give $u(t)$ and we get $y(t)$, this implies that to the same system, if we give $\frac{du(t)}{dt}$ we get $\frac{dy(t)}{dt}$.