Control Systems
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Lecture – 16
Closed Loop System
Part – 2

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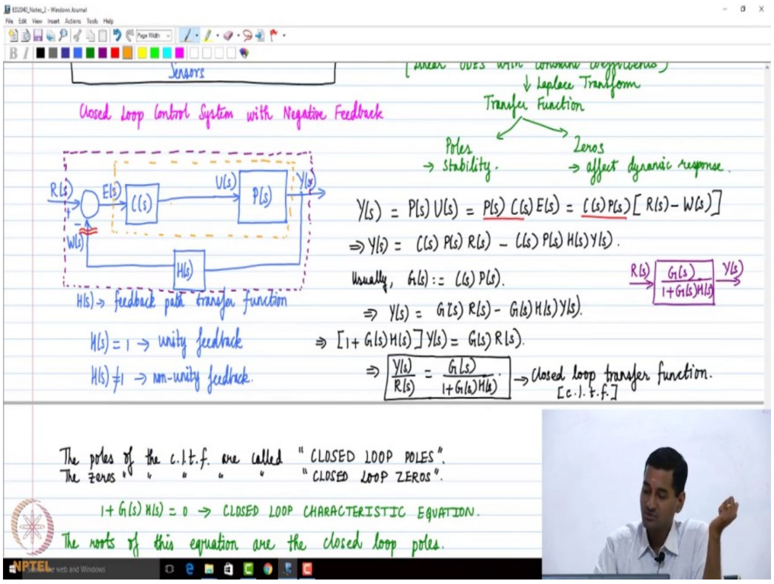

Now, it can be observed that the closed loop transfer function of a positive feedback system is,

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}$$

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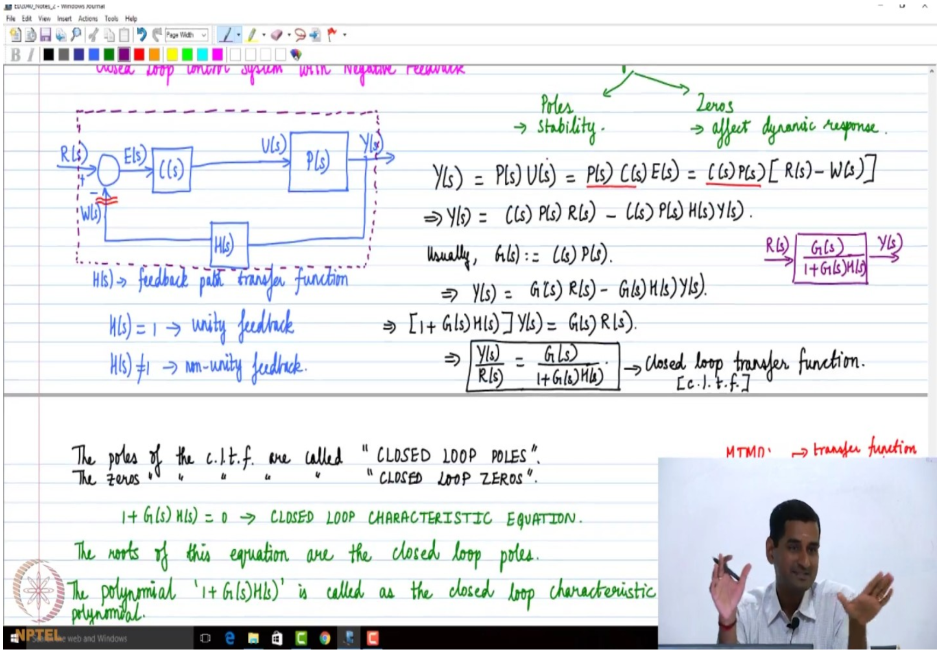

Now, some more terminology ok; so, suppose if I break open the loop at this point (near W(s)), ok. Suppose, let us say I have a closed loop and I break open the loop and this point whatever transfer function I get, which relates the signal at this point which is W(s) to what goes as the input to the system which is E(s) is what is called as the open loop transfer function, ok.

$$\frac{W(s)}{E(s)}=\frac{H(s)P(s)C(s)E(s)}{E(s)}=G(s)H(s)$$

So, this is what is called as the open loop transfer function, ok. Why is it called open loop transfer function? As I told look at the ratio of the signals that I am taking right, W and E, right. So, as I told you imagine that you cut open the loop here, right, you open the loop the signal that you are getting at this junction is going to be W and the input to the open loop is going to be E, right. So, that is why we are essentially taking W(s) divided by E(s) as the open loop transfer function that is a definition, and that is going to be G(s) times H(s).

The poles of the open loop transfer function are called as open loop poles right and the zeros of the open loop transfer function are called as open loop zeros, ok. So, that is once again some terminology we should remember, ok. So, if you encounter the adjective open loop we are talking about G(s)H(s), ok. So, I hope it is clear why it is open loop right. So, I am opening the loop, alright.

So, you may ask me the question, hey is it going to be something which is physically meaningful? Not necessarily. Ok, but you will see that this is going to be used, right. So, it is an indicator of what will happen without feedback that is what we have. Suppose, like let us say I put all these elements in the loop, but I do not close the loop, right which I will never do in practice by the way, right. So, see then what is the point of designing a closed loop feedback system, right? Suppose, if I do not close the loop what is the transfer function I am going to get, ok. So, that is what is called as open loop transfer function. Why is it important? What is the open loop transfer function? G(s)H(s), right. G(s)=C(s)P(s)H(s).

So, you see that the open loop transfer function is nothing, but the product of all the transfer functions that you have, right. So, you have three blocks, right. C(s), P(s) and H(s) you see that immediately the open loop transfer function is formed by the product of these three transfer functions, right, that is why you will see that it is useful, as we go along, ok.

When we go to advanced tools used in control design you will see the impact of this open loop transfer function. Immediately you see that the closed loop characteristic equation is one plus open loop transfer function, one can immediately observe that, right.

Closed loop characteristic equation = 1+ OLTF

In many references we will see that the same feedback diagram is modified in this way. The controller and plant are combined and written as G(s).

So, now as I told you we are still looking at P(s), right. So, we are still in the plant transfer function. So, we are once again analyzing the plant or the system, but as we move along we will extend the concepts to the entire system, ok. So, now, we are looking at whether a plant is stable or not by looking at the poles of the plant. So, when we design a closed loop feedback system we want the closed loop to be stable. So, we look at the closed loop poles we will extend it to the closed loop transfer function, ok. So, that is something which we are going to do.

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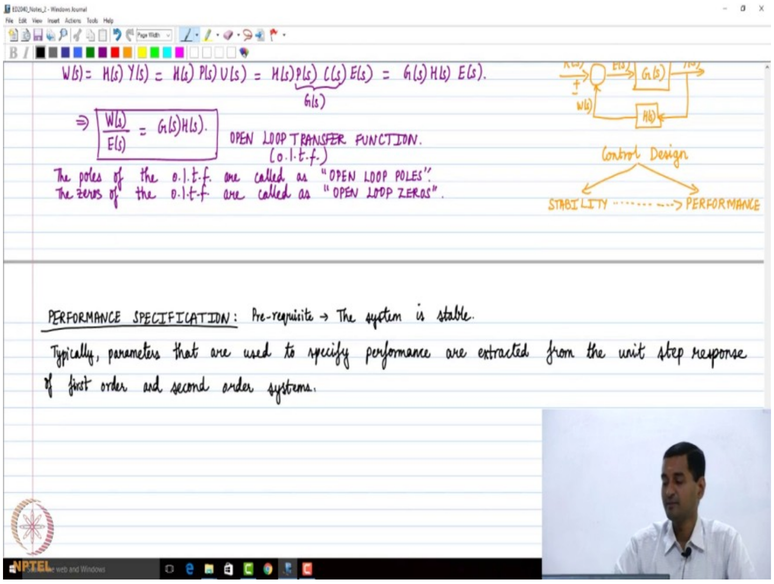

And, one important thing is that in any control design ok, we, as control designer should look at two important aspects, in this order. The first one is that of stability ok, the second aspect is that of performance, ok. Both are important, but, stability is more important than performance, right. See, if I do not have a stable system to begin with I cannot even talk about it is performance right first I should ensure that the system I am designing is stable, then we talk about performance, ok.

So, what I am going to do is that like I am going to draw a dashed arrow to indicate that when you design both are important for a control designer, but you first you ensure stability then we talk about performance, ok. So, now, what we are going to start discussing from now is that like how can I quantify performance, right. So, that is something which we are going to learn, right.

So, we are going to learn about performance specification, right. So, what we have done is that like we have looked at stability you know the notion of stability that we are going to follow in this particular course is that of bounded input bounded output stability. And we have seen that you know the criteria for ensuring bibo stability is that all transfer system transfer function poles should be in the left of complex plane ok. So, that is the criteria right for bibo stability, right.

let us say we want to develop a controller for regulating the temperature of air in this room, right. Now, let us say all 60 of us design stable controllers, ok. Now, I have 60 controllers in my hand how do I evaluate which is the best one, right? So, from a practical perspective I need to evaluate, right. So, then what can be some parameters which I can use for evaluation. Now, what do you think I can use as criteria for performance evaluation?

So, let us say I come into this room in summer, let us say that room temperature is let us say 38 degree Celsius, right. I wanted to go to 25, right. I look at each controller, right and then like I see I set, I set my reference as 25 degree Celsius I may evaluate how fast it will go to 25; that is one.

Then, how I would say accurately it can maintain it around 25 how fast it can settle around 25 and how close will it remain to 25, right. See, if I have an air conditioner let us say which goes from 38 to 25 in 10 minutes, but then it remains what to say between 23 and 27 you know like, I may not be very happy right. So, let us say it cuts off at 23 degrees Celsius and cuts in at 27 degree Celsius, right. So, I may not feel very happy right, because I am going to have a four degree Celsius variation right, but on the other hand let us say an air conditioner maintains the temperature between 24.5 and 25.5, I may be very happy, right. So, that is one performance specification, right.

So, then how fast it will settle, right. How fast it may overshoot the value that is what we have also looked at, right. So, there is I am given a reference value how fast or how far away will it go from the desired value, right that is another performance specifications right what are the steady state errors all right. So, the speed with which it will go to the final desired value, the band to which it will regulate around the final value right, how much it will overshoot the final value or the desired value all those are performance specifications that we would be interested in, right and that is what we are going to learn and quantify, right.

So, typically the prerequisite is that the system is stable, that is why I discussed stability first, right. So, because without stability there is no point in discussing performance, right. So, as a prerequisite you know like I want my system I can only do performance analysis of those systems that are stable to begin with, and typically parameters that are used to specify performance or quantify performance I should say are extracted from the step response of first order and second order systems, ok. So, we would see why, right.

So, typically the parameters that quantify what to say performance requirements are motivated and calculated, the formulae are extracted from the step response or the unit step response of what are called first order and second order systems, we would see why, as we go along ok. The motivation will become clear when we go to advanced control design you will see that we learnt about dominant poles right yesterday. So, by and large even if you have a tenth order system, ok, by and large you are going to have a dominant dynamics that corresponds to a first order system or a second order system by and large ok, there can always be exceptions, ok.

Typically you are going to have the dominant dynamics which can be captured by a first order or a second order system, ok. That is why we are motivated in using these systems for extracting performance parameters ok, we will revisit this point later right ok. So, consequently let us look at let us start today with first order systems.

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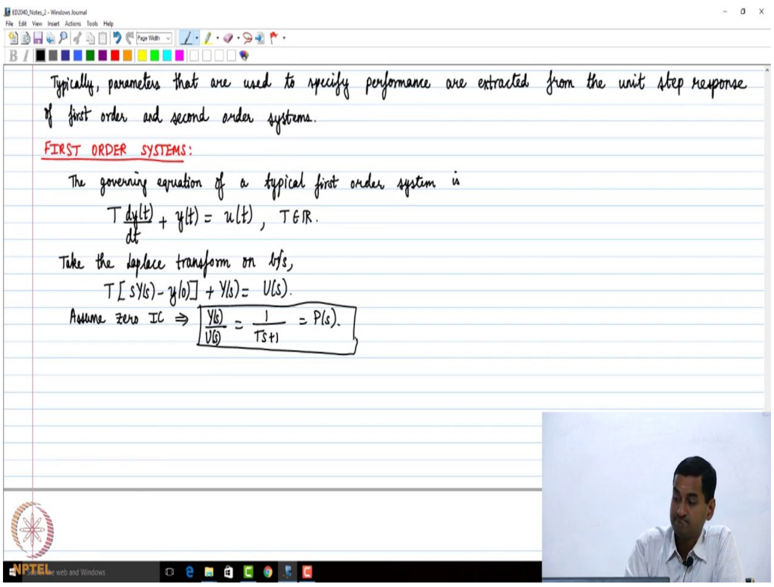

First Order Systems:

So, I am going to start with a typical first order system model and I am going to leave you with a set of questions, ok. So, what is a first order system the governing equation of a typical first order system? See, in fact, like it really depends on whom you talk to suppose let us say you are discussing this what to say this with a chemical engineer for example, right. They would say a model for a first order process, ok. They will always be looking at processes, right.

So, the plant for them is a process, ok. So, if you if you look at references that are written by chemical engineer they will say that oh it is a first order process. So, please do not get confused; it is the same thing, ok. So, we for us we are looking at systems, right.

The governing equation of a typical first order system is,

$$T\frac{dy(t)}{dt}+y(t)=u(t), T\in R$$

Take Laplace transform on both sides,

$$T(sY(s)-y(0))+Y(s)=U(s)$$

Assuming zero initial conditions,

$$\frac{Y(s)}{U(s)}=\frac{1}{Ts+1}=P(s)$$

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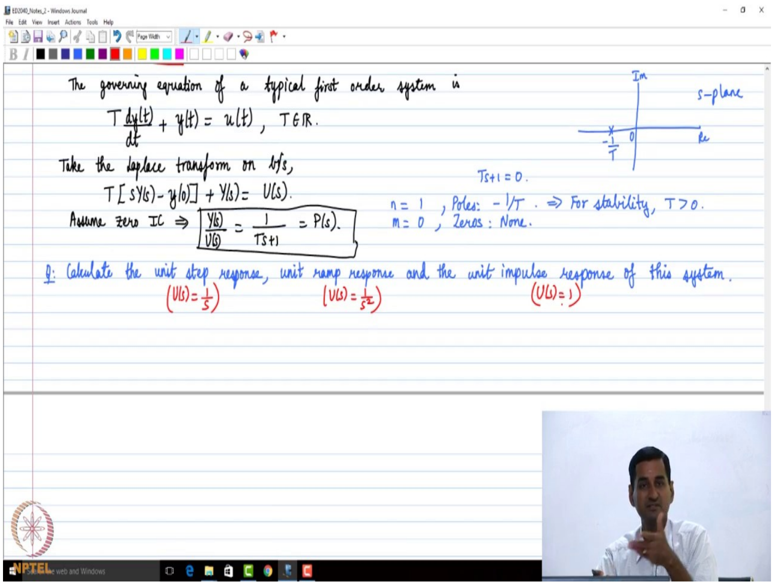

Here n=1; m=0.

So, we have a pole at -1/T, right. So, you need to solve TS + 1 = 0, As far as zeros are concerned there are none. So, this implies that for stability what should happen?

Student: T must be positive.

T must be positive, ok. So, you will see that stability requires that it should be positive, ok. So, we are going to have a pole at minus 1 over capital T. So, now, what I am going to leave you with are these questions, ok. So, you calculate the unit step response ok, unit ramp response and the unit impulse response of the system ok, that is a homework which I am going to leave you with. We would restart from the same point on in the next class, right. So, calculate the units unit step response means what do you use for U of s?

Student: (Refer Time: 23:20).

$U(s)=\frac{1}{s}$. Unit ramp means?

Student: 1 by S square.

$U(s)=\frac{1}{s^2}$

For unit impulse U(s)=1.

So, please use this U(s) pretty straightforward, ok. It should not take you more than 5 minutes to calculate all the three responses. Do please get expressions, make observations ok, what do you observe from them and then like you come back to the next class we will restart our discussions from here fine, ok.

Thank you.