Control Systems
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Lecture - 15
Closed Loop System
Part – 1

Let us briefly recap where we are, what we are doing and where we are headed.

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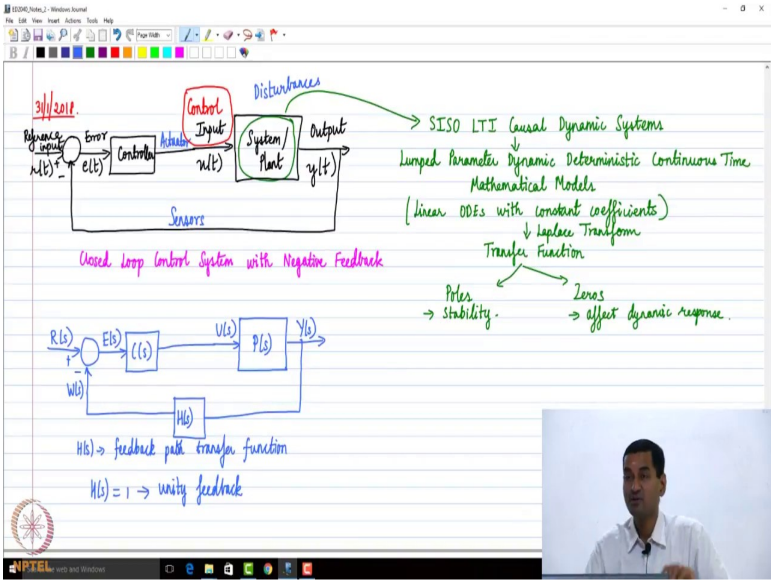

We consider a dynamic system or a plant to which we provide an input $u(t)$ and we obtain an output $y(t)$. We want to regulate the output of the system to a desired value. So, we give a reference input, ok. The reference input tells us what is the desired value of $y(t)$. Then we sense the output and give it as a feedback and compare it with what we want and then we get an error. We denote it by $e(t)$ and this error is passed through a controller which then calculates the control input. This is the overall objective of this course; that is we want to learn how to design this closed loop feedback system control system. This is also called as an output regulation problem.

Referring to the block diagram, right now we are in the system block. The class of systems that we are considering are SISO linear time invariant causal dynamic systems. We used deterministic continuous time mathematical models to study these systems. We represented these systems by linear ordinary differential equations with constant coefficients. We then applied Laplace transform to obtain the transfer function representation. We have looked at the poles and zeros of the transfer function and their implications on BIBO stability. In the coming lectures, we will look at the controller block and the potential choices for this controller block.

In the block diagram, we understood the presence of sensor dynamics, actuator dynamics and disturbances. This is called as a closed loop control system with negative feedback. We can draw a similar block diagram where we can analyze the blocks with the corresponding transfer functions. We use the following representation. Plant transfer function ($P(s)$), controller transfer function ($C(s)$), control input ($U(s)$), output ($Y(s)$), Laplace transform of the reference input ($R(s)$), feedback ($H(s)$), error ($E(s)$) and filter in the feedback path ($W(s)$).

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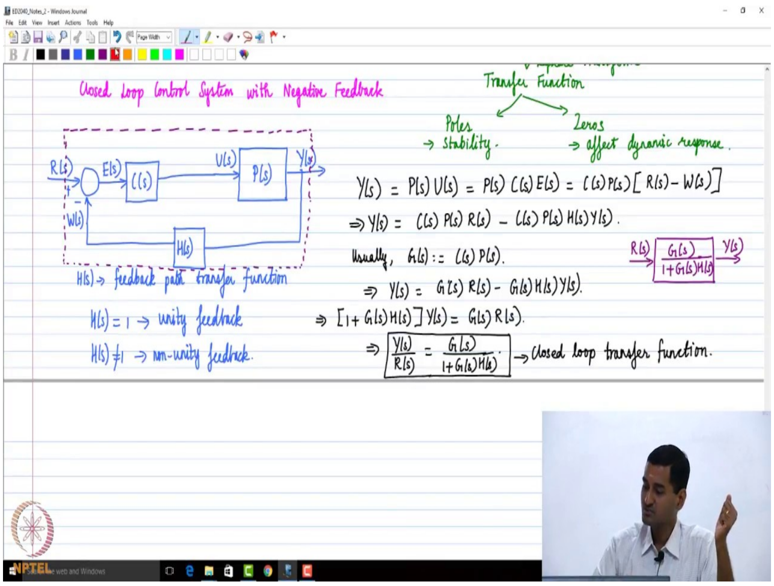

The advantage with going to the complex domain and drawing this block diagram is that the output signal at the plant block ($Y(s)$) right is going to be the product of the block transfer function and the input to the block.

$$Y(s)=P(s)U(s)$$

$$U(s)=C(s)E(s)$$

$$Y(s)=C(s)P(s)[R(s)-W(s)]=C(s)P(s)[R(s)-H(s)Y(s)]$$

Usually $G(s)=C(s)P(s)$ is the product of all the transfer functions is that come in the forward path.

$$Y(s)=G(s)R(s)-G(s)H(s)Y(s)$$

$$[1+G(s)H(s)]Y(s)=G(s)R(s)$$

$$\frac{Y(s)}{R(s)}=\frac{G(s)}{[1+G(s)H(s)]}$$

This is what is called as the closed loop transfer function because it represents all the components of the closed loop in a single block with $R(s)$ being the input and $Y(s)$ being the output of the block.

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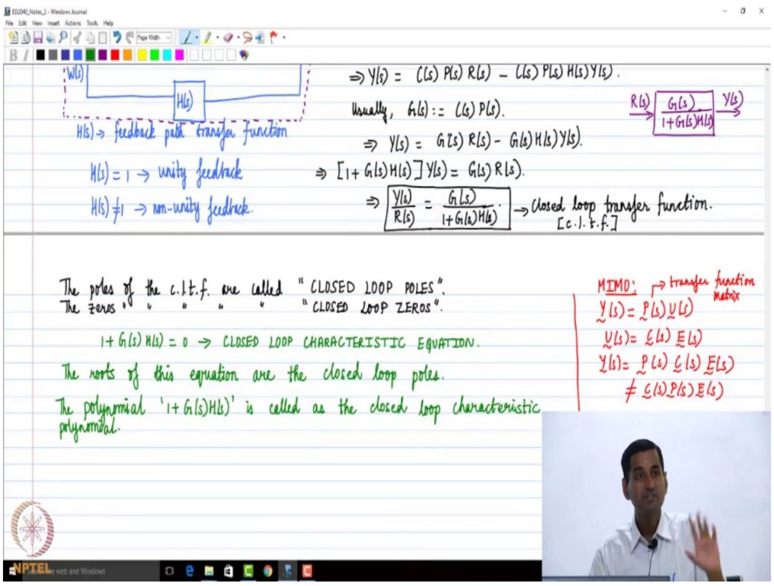

If we had a MIMO system, $Y(s)$ will be a vector and $U(s)$ will be a vector, then what we have is a transfer function matrix. In that case we need to be careful about the order of multiplication of two transfer functions. For example; multiplication of $C(s)P(s)$, we cannot write it as $P(s)C(s)$ because they are matrices.

We are going to abbreviate the closed loop transfer function as CLTF. The poles of CLTF are called closed loop poles and the zeroes of CLTF are called closed loop zeroes.

How do we calculate closed loop poles? Similar to the earlier discussions, poles are the roots of the denominator polynomial. Here denominator polynomial is $1+G(s)H(s)$. The equation $1+G(s)H(s)=0$ is called as the closed loop characteristic equation. We solve the closed loop characteristic equation to get the closed loop poles. We are interested in closed loop poles because when we design a closed loop system, first the closed loop system has to be stable, for which all the closed loop poles must lie in the left of complex plane.