Control Systems
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Lecture - 14
Effect of Zeros
Part – 2

So, I am going to do one more example, I want you to do this example and tell me what the unit step response of this system is?

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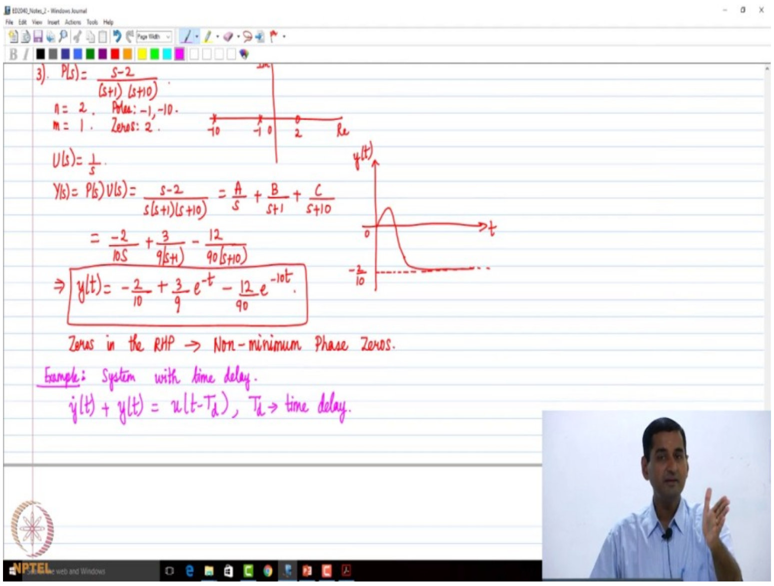

So, let us say, I take this system for example

$$P(s) = \frac{s-2}{(s+1)(s+10)}$$

Here n = 2, m = 1

Poles: -1 and -10.

Zero: 2.

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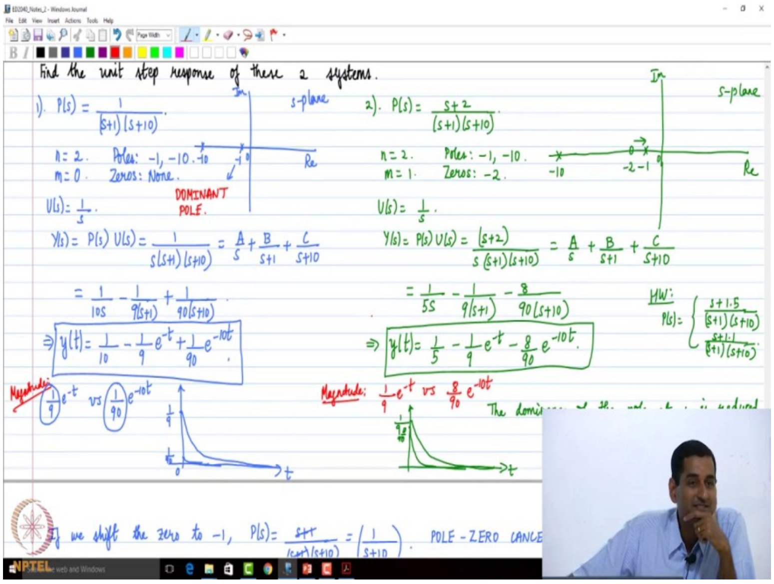

Previously, the zero was at -2, what we have done is that we have taken at to 2 ok, that is what we have done. Please calculate the unit step response and let me know what happens.

$$U(s)=\frac{1}{s}$$

$$Y(s)=P(s)U(s)=\frac{s-2}{s(s+1)(s+10)}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+10}$$

¿ $$\frac{-2}{10s}+\frac{3}{9(s+1)}-\frac{12}{90(s+10)}$$

Taking inverse Laplace transform,

$$y(t)=\frac{-2}{10}+\frac{3}{9}e^{-t}-\frac{12}{90}e^{-10t}$$

Once again, you could observe that the system is stable; poles are in the left of complex plane. Poles are the same, right see I have kept the poles to be the same in all these problems; why? Because, we want to investigate the effect of zeros; so, I am not even touching the poles, right. So we are working with the same poles, right.

So, we have shifted the zero to + 2 to the right of plane. So, the system is still BIBO stable;

As $t\to\infty$, $y(t)\to\frac{-2}{10}$.

So, the value can be negative also absolutely no problem, it needs to be a finite real number. And you can once again observe that the exponential terms have the poles as the exponents and the residues are also affected by the location of the zero. Now, we see that compared to example 2 the relative importance of e^{-t} has now increased because we have shifted the zero away from -1.

See -1 was the dominant pole when we had the zero at -2 the dominance was reduced. Now we have shifted the, what to say, pole to away from the dominant pole at -1, you can see that the relative magnitude has once again increased; the dominance has once again increased; that means that the magnitude of the e^{-t} term is $\frac{3}{9}e^{-t}$. So, you could immediately observe that the $\frac{3}{9}e^{-t}$ term will dominate ok, fine. So, that is the first observation ok. So, can you observe something else?

So, what is y(0)? Of course, I mean I can always double check see whenever we are using this analysis, we are assuming initial condition to be zero. So, once you complete the problem using this transfer function based approach, it is always a good idea to substitute t = 0 and ensure that y(0) is zero right because that is an implicit assumption that we have made.

So, you see that I start from y(0)=0, right and let us say, this is my final value ok. So, this I leave it to you, you can go to MATLAB and plot this function, let us say this is minus 2 by 10. So, what is going to happen is that the output function will go something like this and settle down to minus 2 ok, that is going to be the impact of the zero at 2.

So, you will see that in all those cases the initial value zero and you will see that the output would monotonically increase to the change to the final value; what do I mean by that? It is not going to change signs ok. So, for example, the final value is 1/10 and 1/5; the output function will always be non negative ok, it will go from zero to the final value smoothly without changing its sign. But in this particular problem, if you plot the output function which is enclosed in this red box, you will see that it will be at zero, it will start at zero at time t equals zero. Final value is going to be negative in this particular example it will so happen that initially, it will go, the values will be positive, then it will change sign and then it will come to the final value.

If I had written 2 - s it will become negative and then it will go to positive, then the effect is going to be reversed. Either way, you know like the conclusion that we can draw is that if you have a zero in the right half plane, your step response is going to have such a characteristic; where the response will start off in one direction, then reverse its direction and go and settle down in that way. So, let me give an example: let us say, we take a ceiling fan right; suppose if the ceiling fan had a zero in the right of plane, what will happen? Let us say I switch on that fan, what will happen? It may start rotating in, let us say clockwise direction after sometime it may stop and then start rotating in the counterclockwise direction. Picturize that; that is this case.

Suppose, if I had a door which has a zero in the right of plane, what will happen is that let us say if I push the door I expect it to go out. Imagine a door if it comes in and then goes out, right; Doors are not like that, but then like you can imagine right. So, that is going to be effect of this zero in the right of plane. And zeroes in the right of plane are called as **non minimum phase zeros** ok, we will figure out why it is called non minimum phase when we do frequency response ok, we will see what is the meaning of the adjective non minimum phase when we come to frequency response ok.

A **minimum phase system** is one which has all poles and zeros in the left of complex plane ok. So, that is the definition of minimum phase and non minimum phase is it clear ok?

Student: Also the final value of y of t seems to be the constant in the numerator divided by the.

Good observation ok. What his observation was that not only this problem; you take problem number 1, you substitute s equals zero in the transfer function what do you get?

Student: 1 by 10.

1 by 10. Second problem substitute s equals zero, 2 by 10 and what are 1 by 10, 2 by 10 minus 2 by 10 in these problems? It is this steady state output of a stable system when subjected to unit step input ok, but then we will generalize it using the final value theorem. Do not always substitute s = 0 to find the steady state value for any input, it does not work that way ok, it only works for this unit step input and that too for stable systems ok. So, please remember that right ok.

So, anyway we will come to that that is a good observation, but we will come to that ok. Now the question is when do we get a non minimum phase system in practice? So, I have to give you an example, right. I can get non minimum phase zeroes right for example, when we have a system with time delay, ok.

So, let us state let me take a simple example let us say I take

$$\dot{y}(t)+y(t)=u(t-T_d)\ T_d \rightarrow time\ delay$$

So, that what it means is that I give an input now that is going to affect the output only after an interval time interval of T_d. So, can we encounter time delay in practice, yes, we do right. Suppose let us say you know I want to push this desk, the force which I am applying is my input; displacement of the desk is the output. Will the desk start moving instantaneously?

Student: No.

No, right why not?

Student: Inaudible.

Exactly, you have to overcome static friction at the point the region of contact between the desk legs and the surface right. So, even if you start counting time from the moment I start applying a force on the desk right; the displacement happens only some time afterwards alright. So, that is a delay; isn’t it?

So, friction for example, right incorporates a delay in a system ok. So, we will see that many cases you know like you may have delays; see for example let us say I may give a voltage input to a motor. The motor shaft may not start spinning immediately, right, it may take a small interval of time before it starts spinning, right. So, question is that like is that delay time significantly high when you compare it to the time scale of interest that is a question we need to ask.

But assuming that is the case then what do we do right. So, I have to factorize timely. Let us say; now I want to find the transfer function right you take Laplace transform on both sides.

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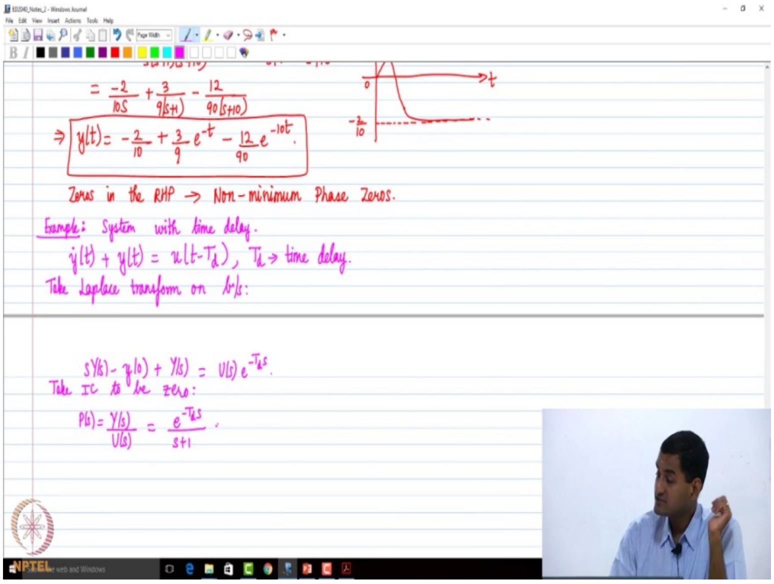

Taking laplace transform,

$$sY(s) - y(0) + Y(s) = U(s)\,e^{-T_d s}$$

Taking initial conditions to be zero,

$$P(s) = \frac{Y(s)}{U(s)} = \frac{e^{-T_d s}}{s+1}$$

Now, what are the values of n and m; what is the value of n?

Student: 1.

1. m=?

I have $e^{-T_d s}$, right, how can I, can I expand $e^{-T_d s}$ as a polynomial expansion? Yes, what will be the order of that polynomial?

Student: Inaudible.

Infinite, right; so, I can just go to any order I want, right. So, I have a problem now because the system has become infinite dimensional in a sense right. So, because we can apply the tools that we are going to learn in this course only for proper transfer functions; that means, that we need to have the order of the numerator polynomial to be less than or equal to the order of the denominator polynomial ok. That is that is when we can apply the tools that we are going to learn in this particular course ok.

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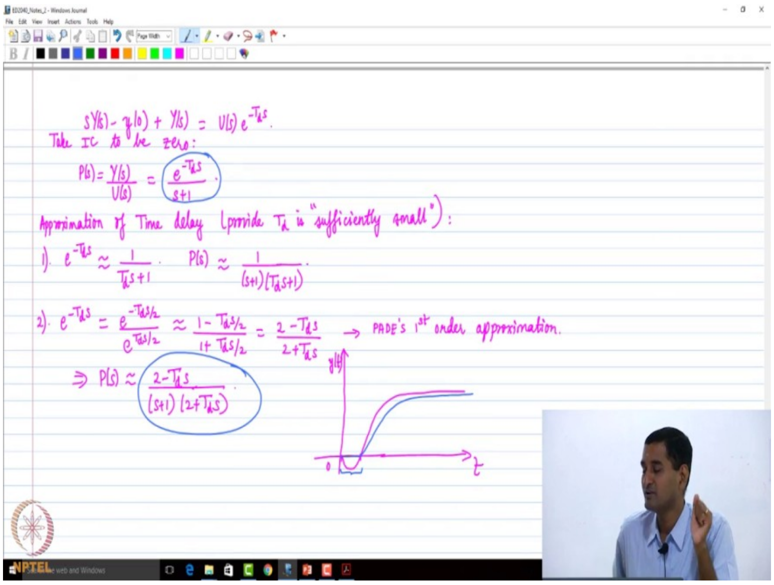

So, given this limitation we can do approximations of time delay, provided T_d is small.

<u>Approximations of Time Delay:</u>

1) $e^{-T_d s} = \frac{1}{T_d s+1}$

2) $e^{-T_d s} = \frac{e^{-T_d \frac{s}{2}}}{e^{T_d \frac{s}{2}}} = \frac{2-T_d s}{2+T_d s} \rightarrow Pade's\ 1st\ order\ approximation$

Now we have introduced a non minimum phase zero through the second approximation; you look at the numerator right T_d is anyway positive what is the zero of the transfer function? Approximate transfer function $2-T_d s$ right. So, that is it going to be in the right of plane.

If you plot the step response what is going to happen is that, the output will go the other way and then it will go and settle down assuming that we are stabilizing the system carefully right. So, this is indicator of the time delay. In reality what is going to happen is that the real system output may be something like this ok. So, let me show it in blue ok. So, the real system let us say there may be a time delay and then like it may go and go like this ok. So, in this interval there is no output at all, but then the approximation essentially tells us that the output may go in the opposite direction and go in the correct way ok.

So, the pink curves only an approximation of reality and why are we doing this approximate, then only we can design right ah. See I cannot design with this guy the tools that we are going to use in this learn in this course cannot be used for this transfer function, but I can use it for this transfer function ok. So, that is why we approximate, but the impact is that we introduce a non minimum phase zero which is going to behave like this; the dip in this curve is a fictitious dip, it does not happen in practice right because during that period the actual output is going to remain zero due to time delay.

So, what it says is that ok, look if I if a model time delay for this desk the actual system responsively that if I give a step force it is going to remain zero for some time and then like it is going to move alright. So, that is what is going to happen, but what I am predicting through this approximation is that if I apply a step force the desk is going to come slightly towards me and then go backwards; in reality, it is not going to happen, but that is the approximation we are going to have ok.

And why are we doing it? Because we the tools that we learn will be can be used only when the transfer function is proper. So, we are using this approximate transfer function ok. So, I hope it is clear when we can encounter non minimum phase zeros right when we have to approximate incorporate time delays and do the design process; we have to do this right, we introduce a non minimum phase zero ok.

So, what we have done in today's class is essentially look at the influence of zeros, right, that is what we have done today. So, we have looked at impact of poles yesterday. So, starting from tomorrow's class, we are going to look at as I told you stability and performance, right, we have looked at stability and tomorrow's class onwards, we look at performance specifications that will complete the system block.