Control Systems
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Lecture - 13
Effect of Zeros
Part – 1

In the previous lecture we looked at the effect of system poles on the response. We identified the fact that the location of the poles influences the stability of the system. And for BIBO stability all poles of the system transfer function should lie in left half complex plane. In other words, they should have negative real parts.

In this lecture we are going to do a few examples to convey the impact of zeros on the system response. To recall, the transfer function the ratio of the Laplace of the output to the Laplace of the input with all initial conditions being 0. And the transfer function is going to be a ratio of two polynomials $\left(\frac{N(s)}{D(s)}\right)$. The roots of $N(s)$ are called zeros of the transfer function. The roots of $D(s)$ are called poles of the transfer function. We have seen that are poles are connected to system stability. Now let us look at zeros.

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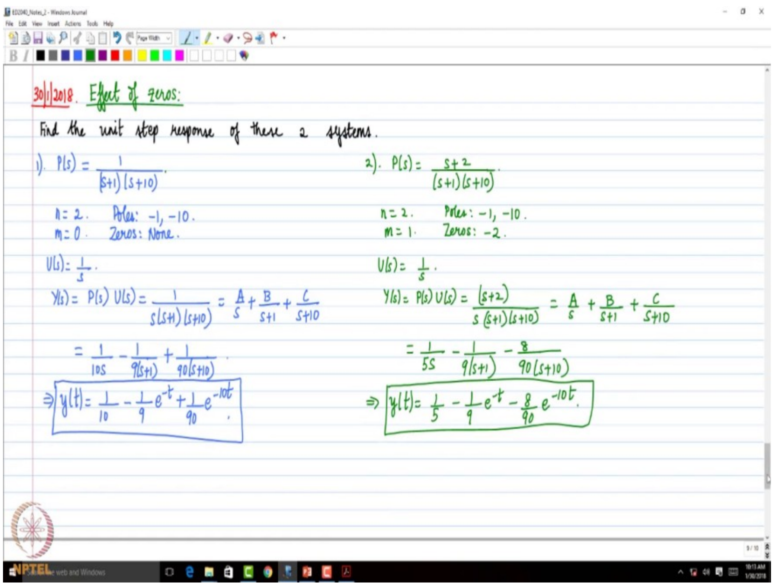

Let us consider two systems: the first plant transfer function is

$$P(s)=\frac{1}{(s+1)(s+10)}$$

The second system is

$$P(s)=\frac{S+2}{(s+1)(s+10)}$$

The order of these systems is 2 because the order of the denominator polynomial is always going to be the order of the system for the class of systems that we study.

For the first system, $n=2$ and $m=0$.

We want to figure out the unit step response.

$$U(s)=\frac{1}{s}$$

$$Y(s)=P(s)U(s)=\frac{1}{(s+1)(s+10)}\frac{1}{s}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+10}.$$

Evaluating the partial fractions, we get $A=\frac{1}{10}, B=\frac{-1}{9}, C=\frac{1}{90}.$

If we take the inverse Laplace transform, we get

$$y(t)=\frac{1}{10}-\frac{1}{9}e^{-t}+\frac{1}{90}e^{-10t}$$

We can observe that the two poles are in the left of complex plane. So the system is BIBO stable. As $t\rightarrow\infty, y(t)\rightarrow\frac{1}{10}.$ The output remains bounded for all time. And we can observe that the output function has exponential terms which correspond to the real part of the poles as we have discussed.

For the second system, $n=2$ and $m=1$.

The poles are at -1 and -10 there is a zero at -2. The difference between system 1 and system 2 is, we have introduced a zero at -2. Let us now calculate the unit step response.

$$U(s)=\frac{1}{s}$$

$$Y(s)=P(s)U(s)=\frac{S+2}{(s+1)(s+10)}\frac{1}{s}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+10}.$$

Evaluating the partial fractions, we get $A=\frac{1}{5}, B=\frac{-1}{9}, C=\frac{-8}{90}$.

If we take the inverse Laplace transform, we get

$$y(t)=\frac{1}{5}-\frac{1}{9}e^{-t}-\frac{8}{90}e^{-10t}.$$

Even in this solution we can see that as $t\to\infty, y(t)\to\frac{1}{5}$. So, it is bounded, the system is BIBO stable because the poles are in the left half plane.

When compared to the first system, coefficients of exponent terms have changed due to the zero. Let us see what the relative impact due to this is.

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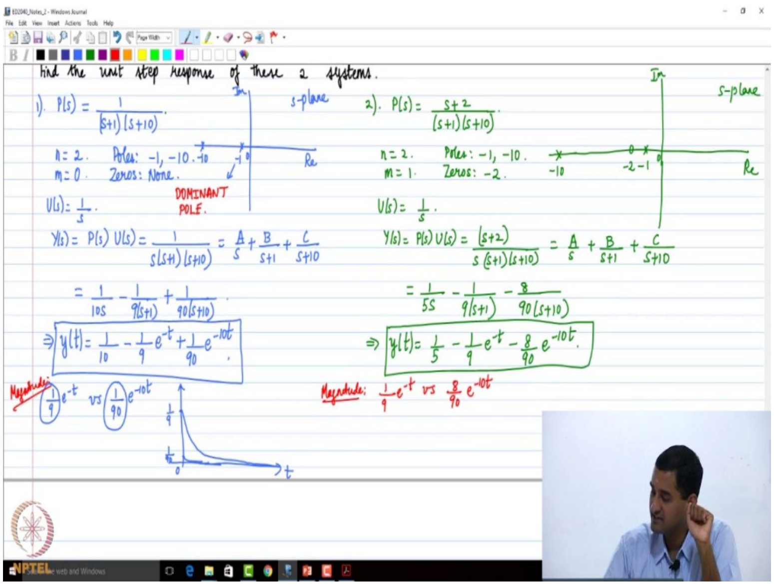

Let us draw the S plane, the poles are at -1 and -10 for both systems. In this case of system 2, we have a zero at -2. If we look at the forced response in problem number 1, e^{-t} will dominate the output term for a longer time because as the exponent becomes more negative the exponential function is going to decay to 0 much faster. If we consider the magnitude of the first term, $\frac{1}{9}e^{-t}$ versus $\frac{1}{90}e^{-10t}$, $\frac{1}{9}e^{-t}$ is going to dominate for a longer time.

Another point is that the residues multiplying the two exponential terms $\left(\frac{\frac{1}{9}\wedge 1}{90}\right)$. So, not only does e^{-t} dominate e^{-10t} but also the residues are such that $\frac{1}{9}e^{-t}$ will certainly dominate $\frac{1}{90}e^{-10t}$. A pictorial representation is given in the slide. For this reason, the poles that are closest to the imaginary axis are called dominant poles since they dominate the output for longer time. Here -1 is the dominant pole.

Let us now consider system 2; here we introduced a zero which is closer to the pole at -1. As we can see, it affects the residues. We can see from the pictorial representation that the dominance of the -1 pole has reduced in comparison with system 1.

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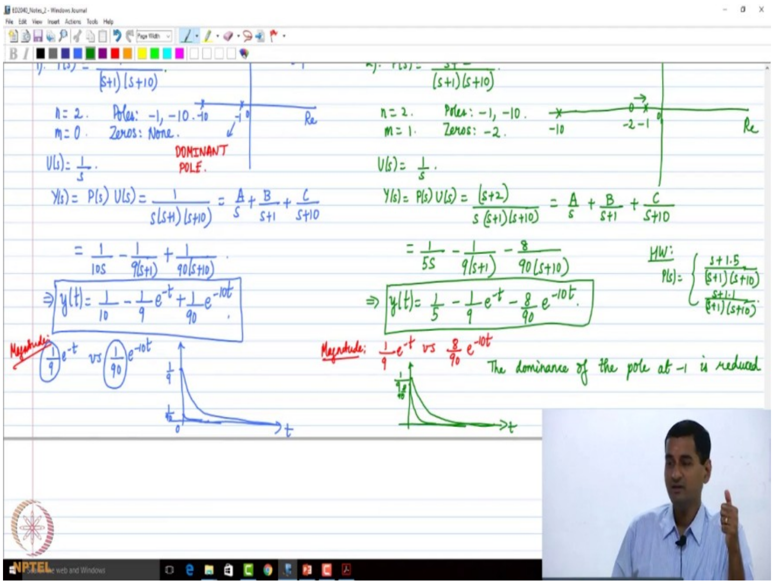

the consequence of the introduction of the zero at -2 which is closer to -1 is that the dominance of the pole at -1 is reduced. As we keep on moving the zero closer and closer to -1, the dominance of the pole at -1 will keep on reducing further and further.

We can see that the zero is going to affect the dynamic response of the system but it is not going to have a direct impact on the stability. Because stability is dependent on the location of the poles. Zeros will affect the coefficients of the response function.

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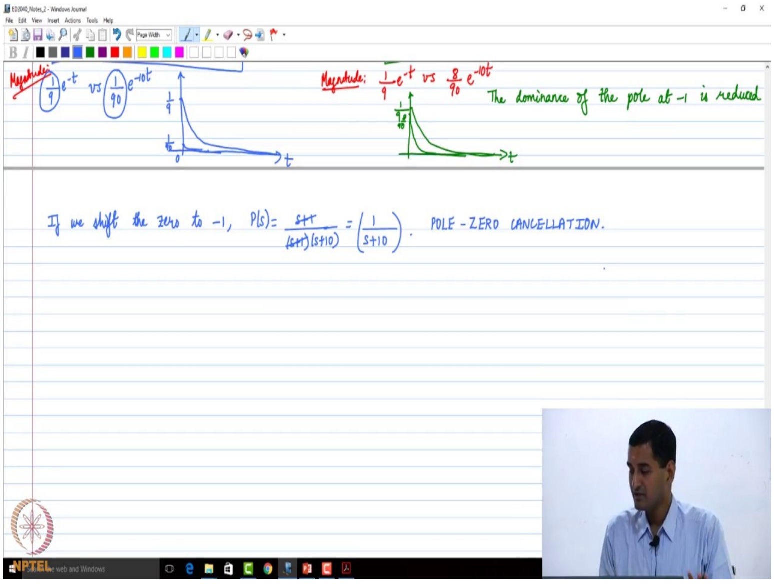

If we shift the zero to -1, we are going to have,

$$P(s) = \frac{S+1}{(s+1)(s+10)} = \frac{1}{(s+10)}$$

The zero is going to cancel the effect of the pole completely. What was a second order system to begin with would start acting like a first order system. This effect is what is called as pole-zero cancellation. Typically do not do this, we will see why.

We will see that, doing control designs, we can introduce system zeros. For example, if we do a proportional derivative control, we can introduce a system zero. During the design process then we need to be in a position to how to handle those zeroes. We shall learn this later.