Discrete Mathematics
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Lecture -16
Relations

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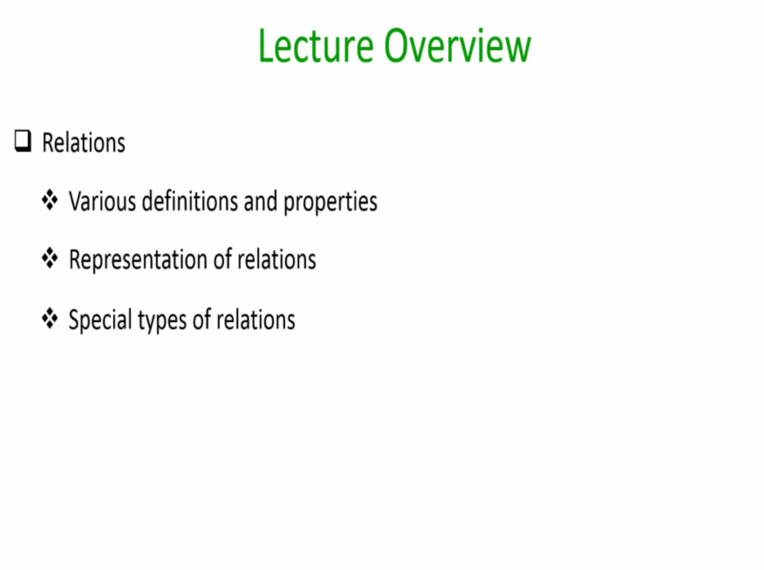

Hello everyone, welcome to this lecture on relations. Just to recap, in the last lecture we introduced the definition of sets and various set theoretic operations, we also saw various set theoretic identities. The plan for this lecture is as follows. In this lecture we will introduce what we call as a relations, we will see their various properties, we will discuss how to represent relations and we will see some special types of relations.

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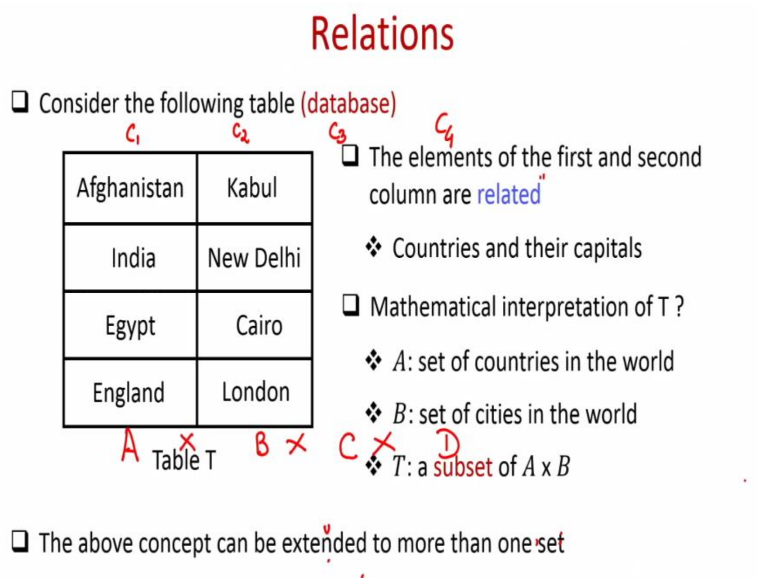

So, what is a relation? So, let me begin with this example, you consider this table and people who are familiar with databases, they very well know what is a table. Table basically consist of several columns and each column have some entries. So, I call this table T which has two columns, column number 1 and column number 2. In column number 1, you have some countries and in column number 2, you have some cities listed.

Now if your general knowledge is good then it turns out that the elements you can spot here that elements of the first column and the second column they are related by some relationship. And the relationship here is that, in the first column I have listed some of the countries and in the second column I have listed the capital of the corresponding countries. Of course, I can add multiple entries in this table and I am not doing that.

Now how do we mathematically interpret this table? Is there any mathematical interpretation or mathematical abstraction by which you can define this table? Well the way I can mathematically interpret this table T is as follows. I can imagine that I have a set A which is defined to be the set of all countries in the world. Well I do not know right now how many exact numbers of countries in the world definitely it is more than 200.

So, A has more than 200 elements. Whereas B is another set, which is defined to be the set of all cities in the world. Again this is a well defined set because we know the list of all cities in this world, so both these sets are well defined. Now if I take the Cartesian product of A and B, what will I obtain? The Cartesian product of A and B will be a set of the form (a, b), where a will be some country.

Namely, it will be belonging to A and b will be some city. As of now when I take A x B, there is no relationship between the elements a,b, I am just picking some country and some city, country, city. I have listed down all possible pairs of the form country, city and this will be an enormously large set. Now if I take a subset of that A x B, a special subset of that A x B and call it T and what is that special subset?

Namely I take a subset where the 'a' component is a country and 'b' component will be the capital. And if I take only components or pairs of the form (a,b) of this form, where a is the country and b is the capital and list down all such (a, b) pairs I obtain this table T. So, you can imagine that this table T here is nothing but a special subset of A x B. So, to demonstrate, my A x B could also consist of elements of the form Afghanistan, New Delhi, it will also have elements of the form India, Cairo and so on. I am not taking those elements, I am taking only those elements (Afghanistan, Kabul) from A x B, I am taking only the element (India, New Delhi) and so on. So, I am taking a subset of A x B and only those subsets which have a special relationship among the, a component and the b component.

So, that is a loose definition of a relation. A relation here is basically a subset of A x B, if I am considering two sets A and B and of course whatever I have discussed here can be extended where I have multiple sets. What do I mean by that? In this example I had only two columns, C_1 C_2. And C_1 was having entries from set A and C_2 was having some entries from B. What if I have a database consisting of 3 columns?

Say there is a third column as well, where the third column denotes population. So, those entries will be coming from a set C, what if I have a fourth column which denotes another feature of the table, say the climate or the temperature of the respective countries. So, those entries will be coming from another set D and so on. And the table with some specific entries will be considered as a subset of the Cartesian product of all the big sets A, B, C, D from which the elements in your column C_1, C_2, C_3, C_4 are occurring here.

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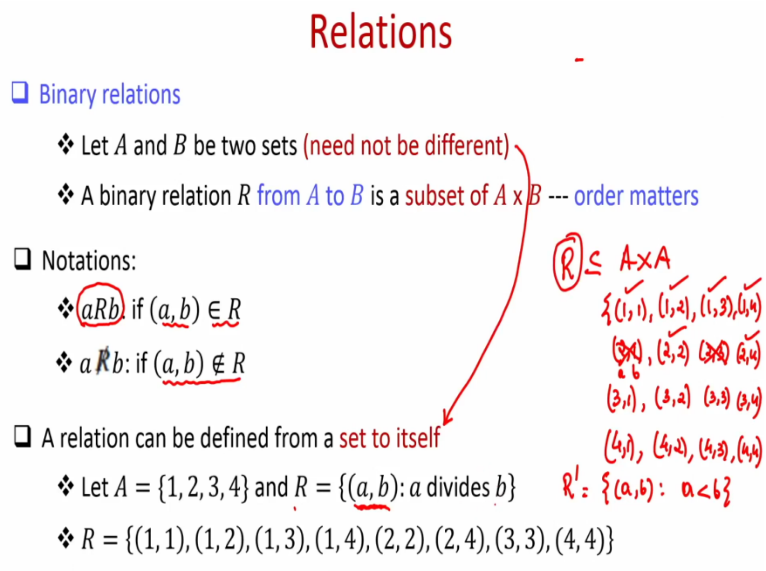

So, that is how we are going to define a relation, so we will focus in mostly on binary relations and by binary relations I mean, we will be working with two sets A and B, but whatever we are discussing here can be generalized for extended for any number, it can be generalized for n-ary relations which are defined over n sets. But for this course and for most important cases, we will be focusing on binary relations.

So, how do we define a binary relation? So, we are given two sets here, call them A and B and they need not be different, I stress here, they can be the same, definition does not say that they have to be different sets, because we are defining a relation in an abstract fashion. Then a binary relation from A to B is a subset of A x B. So, I have highlighted the term from A to B by a different color because the order of the relation matters.

So, if you are defining a relation from A to B then the relation should be a Cartesian product of A x B. Whereas if your relation is from B to A, then that relation should be a Cartesian product of B x A or it should be Cartesian product of B and A, it should be a subset of B x A. So, the order matters here a lot. Now when I say a subset my relation R could be empty as well, that means my table could have zero entries, that is also possible, that is also a valid table.

So, it is not necessary that a relation always should have at least one element of the form (a,b) it could be empty as well. So, we use some notations here when dealing with relations. So if the element (a, b) belongs to the relation R then I will be often writing this expression aRb, I will be saying a followed by capital R followed by b, to denote that a is related to b whereas if the element (a,b) does not belong to the relation R, then I will strike off R in this expression. So, as I said here when I am defining the binary relation my sets A and B could be the same, they could be different. So, I can define a relation from the set to itself so let me demonstrate this. So, imagine I have a set A consisting of the elements {1, 2, 3, 4} and I define a relation R consisting of all elements of the form (a, b) we are both a and b are from the set A such that a divides b.

So, basically I am defining a relation over A x A. So A x A here will have many elements here, it will have (1, 1) it will have (1, 2), (1, 3), (1, 4). It will have the elements (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4) and (4, 1), (4, 2), (4, 3), (4, 4). So, I will be taking only those components, (a, b) from this collection A x A where the first component a divides the second component b.

So, it turns out that I will be taking only (1, 1) because one divides one. I will be taking (1, 2), I will be taking (1, 3), I will be taking (1, 4). But I would not be taking (2, 1) in the relation R, because for (2, 1), a is 2, b is 1 and 2 does not divide 1. I will be taking (2, 2) but I would not be taking (2, 3) because 2 does not divide 3, I will be taking (2, 4) and so on. So, the elements of the relation are listed down here.

I could define another relation, I could define a relation R’ collect consisting of all (a, b) such that a < b. That is another relation and that will be consisting of other pairs, it might be consisting of other pairs different from the pairs which you have listed down in R. So, it depends upon the property which you want to be satisfied by the elements of the relation that defines that tuples or the pairs which will be present in that relation.

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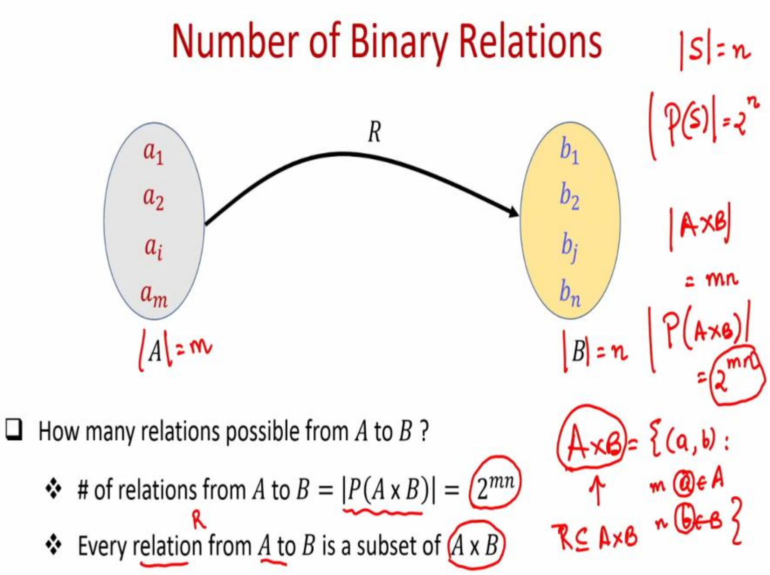

So, now an interesting question is that if you have a binary relation defined from the set A to B how many such binary relations can you define? Can I define any number of binary relations or is there an upper bound on the maximum number of binary relations that I can define? Namely how many tables I can form with two columns, where the first column can take entries from the set A and a second column can take entries from the set B.

Well it turns out that I can form 2^{mn} number of binary relations provided A has m number of elements and B has n number of elements. And this simply comes from the observation that you take any relation R, it is nothing but subset of A x B. So, you have this bigger set A x B consisting of all elements of the form (a, b) where a is from A and b is from B, you pick any subset of this A x B that gives you a relation.

So, the number of binary relations is nothing but how many different subsets of A x B you can form. Namely the cardinality of the number of relations will be the same as the cardinality of the power set of A x B and what will be the power set of A x B, so you recall the theorem that we proved in the last lecture. The last lecture we proved that if the cardinality of a set S is n, then the cardinality of the power set is 2^n.

There will be 2^n subsets of a set consisting of n elements. Now what will be the cardinality of A x B? Well the cardinality of A x B will be m times n. Why? Because it is the collection of all (a,

b) pairs, where a is from A. So, how many different values of a you can have m and how many different values of b you can have, n. So, you have m times n number of (a,b) pairs, which will be present in A x B.

So, your cardinality of A x B is mn and that means the number of subsets of A x B that you can form is 2^{mn}. That means these 2^{mn} is the maximum number of tables that you can form with 2 columns, where the first column takes entries from the set A and the second column takes the entries from the set B.

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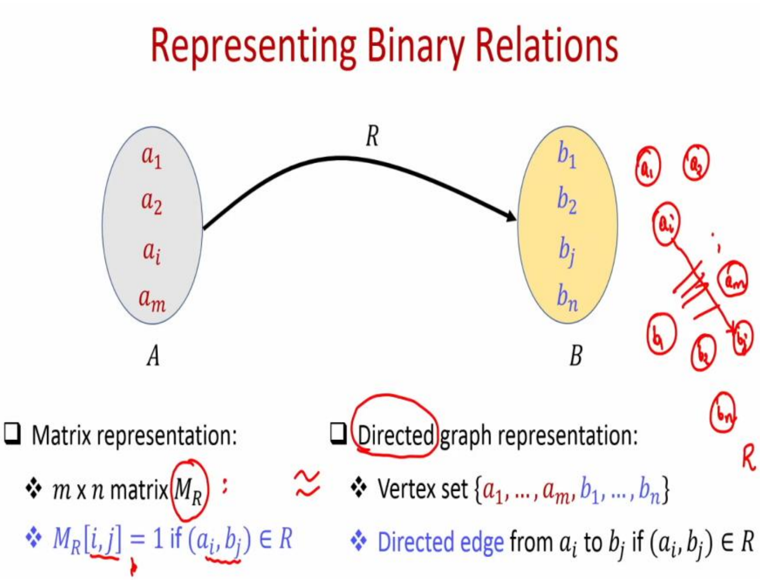

Now the next question is how do we represent binary relations? So, there are some well known methods for representing binary relations, the first method is the matrix representation. So, since we are dealing with binary relations the matrix representation here will be an m x n matrix, m because there are m possible elements from the set A and n columns because I have n possible elements from the set B.

And I will be denoting this matrix by M_R, where R is the relation. And what will be the entries of this matrix. So, it will be a Boolean matrix where the entry number i^{th} row and j^{th} column will be 1 provided the element (a_i, b_j) is present in the relation R. So, what I am saying here is that imagine you have constructed a table or a relation R, so it will be either consisting of 0 number of rows or some number of rows and each row will have 2 columns.

So, if you have a row where an element a_i is present in the first column and b_j is present in the second column, that means you have (a_i, b_j) present in the relation R, then in the matrix representation what you will do is you will go to the i^{th} row, j^{th} column and you will put an entry 1. Whereas if the entry number (a_i, b_j) is not there in your database or in the relation R if it is not present, then the i^{th} row and j^{th} column entry will become 0.

That means you want to denote there that element a_i is not related to the element b_j as per your relation R. So, that is the matrix representation, so if I want to now look back into the question that how many binary relations I can form, well it turns out to the answer for how many different or distinct Boolean matrices of dimension m x n you can construct because each Boolean matrix will correspond to one binary relation.

You cannot have two different relations or two different tables represented by the same Boolean matrix that is not going to happen. So, this is the matrix representation for representing a relation, we have another representation which we call as the directed graph representation. So, what do we do in this representation, we draw a graph and by graph I mean a collection of vertices and edges.

The vertices will be the nodes a_1, a_2, a_m, and b_1, b_2, b_n and it will be a directed graph that means the edges here will have a direction associated, how the edges are added in this graph? So, if this graph represents the relation R, and if in the relation R the element a_i is related to the element b_j, that means in the database of R, you have a row with first column being a_i and the second column being b_j, that means a_i is related to b_j.

Then what you will do is you will take the node a_i here and b_j here and you will add a directed edge from a_i to b_j, so the edge here denotes that a_i is related to b_j. The direction here matters, if the edge from a_i to b_j is present that does not mean that the edge from b_j to a_i is also present. That depends whether b_j is related to a_i or not, you might have a relation where only a_i is related to b_j.

But b_j is not related to a_i in which case the reverse direction edge may not be present. And now you can see here that the matrix representation and the directed graph representation they are equivalent to each other. If you have the entry a_i, b_j in the matrix representation 1, then you will have the edge from a_i to b_j, whereas if in the i^{th} row and j^{th} column of the matrix, you have the entry 0, then this edge from a_i to b_j will not be present.

You might be wondering why we have two different representations. We will be seeing that depending upon how we want to prove or whether what kind of properties we want to prove regarding the relations, the representation matters a lot. There might be cases where if we use the matrix representation then the arguments become very simple, whereas there might be cases where we want to deal with the directed graph representation.

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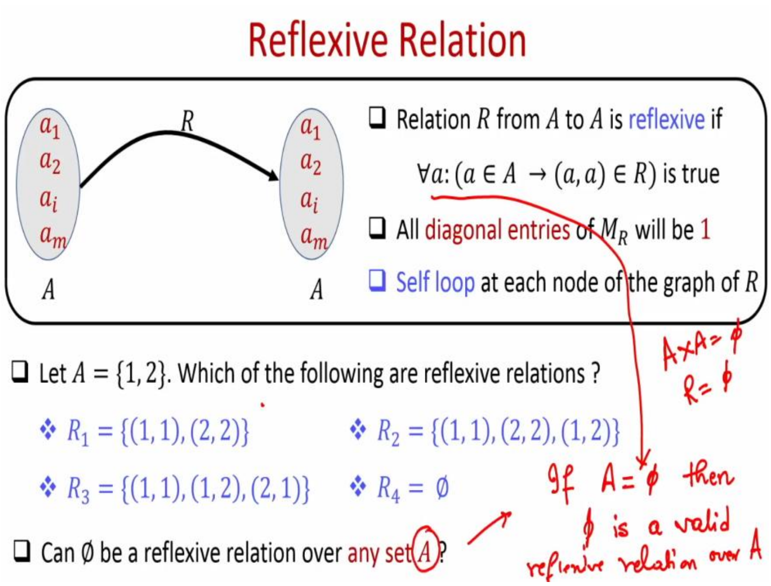

Now, we would define some special types of relations. So, the first special type of relation is a reflexive relation, it is a relation defined from the set to itself. And when do we say that the relation is reflexive, as the term reflects here signifies, the relation will be called a reflexive if every element from the set A is related to itself as per the relation. That means you take any element from the set A, the element A should be related to itself as per the relation.

If this is true for every element a, from the set A then I say that my relation is reflexive. Even if there is one a, one element a, which is not related to itself in the relation, then it will not be considered as a reflexive relation. So, now if I want to interpret the matrix representation of a reflexive relation, it is easy to see that its relation R is reflexive. Then the matrix M_R will be n x n matrix.

Because the relation is defined from the set to itself and if it is reflexive then all the diagonal entries will be 1. Because I will be having (a_1, a_1) present in the relation, I will be having (a_2, a_2) present in the relation and in the same way I will be having (a_n, a_n) also present in the relation. So, since (a_1, a_1) is present that means the entry (1, 1) will be 1, (a_2, a_2) is present that means the entry (2, 2) in the matrix will be 1.

Since the a_n is related to a_n that means the entry number (n,n) in the matrix will be 1 which is equivalent to saying that all the diagonal entries will be 1. There might be additional entries in the relation apart from these reflexive entries that is also fine. The definition of the reflexive relation says that you only want the guarantee that every element from the set A should be related to itself as per the relation.

There might be other elements which are related to the elements of A as per the relation R, I do not care about those elements. If I focus on the graph representation of reflexive relation, then it will be a special type of graph where I will be having a self-loop at each node of the graph. Because a_1 is related to itself, that means I will be having a loop or a directed edge from a_1 to a_1. Since a_2 is related to itself, I will be having an edge from a_2 to itself and so on.

So, now let us see some examples of reflexive relations. So, I have a set here A consisting of 2 elements {1, 2} and I have given you many relations over this set. We have to find out which of these relations are reflexive relations. So, let us start with the relation R_1. It is reflexive because indeed the element (1, 1) is present in the relation and the element (2, 2) is present in the relation. So, this satisfies the definition of reflexive relation.

My relation R_2, it also satisfies the definition of reflexive relation because (1, 1) is present (2, 2) is present and I have (1, 2) is also present here, but that is fine because even the element (1, 2) does not violate the truth of this universal quantification for the relation R_2. What is this universal quantification? It says that if 1 is present in the set A then (1, 1) should be in your relation.

If 2 is present in your set A then (2, 2) should be present in your relation. That is all, it does not say whether anything additional is present or not. Now I come to the relation R_3, my relation R_3 is not a reflexive relation, because (2, 2) is not present here. That means if I take this definition of reflexive relation here, this universal quantification is not true, because indeed I have an element 2 in my set A, and 2 is present in A, but (2, 2) is not present in R_3, this is not happening. That means this universal quantification is false and that is why R_3 is not an example of reflexive relation and what about R_4? Again, R_4 is not a reflexive relation over A. Because I have 1 present in A, but (1, 1) is not present in R_4. If I consider this implication, this implication is false because 1 is present in the set A, but (1, 1) is not present in R_4.

Because R_4 is empty in the same way 2 is present in A, but (2, 2) is not present in R_4, that means this implication is also false. And since both the implications are false here this universal quantification for this relation R_4 is not true. That is why relation R_4 is not an example of reflexive relation. Now here is an interesting question for you: can it happen that the set ϕ or the relation ϕ is a reflexive relation over a set A.

So, remember ϕ is also a relation because ϕ is a subset of Cartesian product of any A x A. So, the question here is: is it possible that ϕ is a valid reflexive relation over some set A? Might look no, but the answer is yes. If A is empty by itself then ϕ is a valid reflexive relation over A. This is because if A is ϕ, then A x A will also be ϕ. And the universal quantification which is there in the definition of reflexive relation will be true for the relation R equal to ϕ.

So, you take R equal to ϕ, so remember ϕ is always a valid subset of A x A and for the relation R equal to ϕ this universal quantification is true for a being ϕ, because the definition says for every a belonging to ϕ. But no element belongs to ϕ, so vacuously this implication will be true for the relation R equal to ϕ. So, we can have ϕ as a valid reflexive relation provided the set over which the relation is defined is an empty set. But if A over which the relation is defined is non empty, then ϕ can never be a valid reflexive relation.

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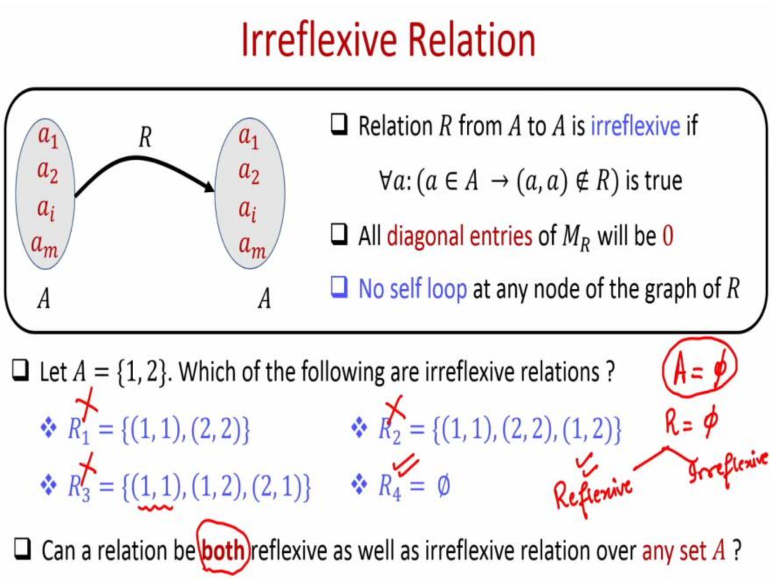

Now let us define another special relation defined from the set to itself which is called the irreflexive relation. And the requirement here is that you need that no element should be related to itself in the relation that means you take any element a from the set A, so this universal quantification over the domain is the set A. You take every element a from the domain or the set A, (a,a) should not be present in the relation

Or the element should not be related to itself. So, it is easy to see that if your relation R is irreflexive, then none of the diagonal entries should be 1 in the relation. So, the matrix for your irreflexive relation will be an n x n matrix. Because the relation is defined from the set A to itself and (a_1, a_1) is not there in the relation, that means the entry number (1, 1) in the matrix will be 0. Similarly (a_2, a_2) is not there in your relation.

That means the entry number (2, 2) in your matrix will be 0 and so on, that means the diagonal entry will be just consisting of 0's or equivalently in terms of the graph representation no self loops will be present, because a_1 will not have any directed edge to itself, a_2 will not have any directed edge to itself and so on. So, again, let me demonstrate irreflexive relations here, so my set A is {1, 2} and I have taken the same 4 relations here.

It turns out that relation R_1 is not irreflexive because you have both (1, 1) and (2, 2) present. Similarly R_2 is not irreflexive, R_3 is also not irreflexive because you have (1, 1) present here, whereas R_4 is a valid irreflexive relation because no element of the form (a, a) is present in R_4. Now it might look that any relation which is reflexive cannot be irreflexive or vice versa but or equivalently can we say that is it possible that I have a relation which is both reflexive as well as irreflexive defined over the same set A.

Well the answer is yes because if you consider the set A equal to the empty set, and if you take the relation R, which is also the empty relation. That is the only relation possible over an empty set A then this relation R is both reflexive as well as irreflexive. It is reflexive because at the first place there is no element present in your set A and hence there is no chance of existence of any (a, a), present in the relation R equal to ϕ.

And due to the same reason since no element is present in the set A you do not need any (a,a) to be present in R. So, the relation R equal to ϕ satisfies the implication, this universal implication given in the definition of reflexive relation as well as irreflexive relation vacuously. So, we can have a relation defined over a set which can be simultaneously reflexive and irreflexive and that can happen in the special case when the set is an empty set.

If A is non empty, then definitely you cannot have a relation which is both reflexive as well as irreflexive.

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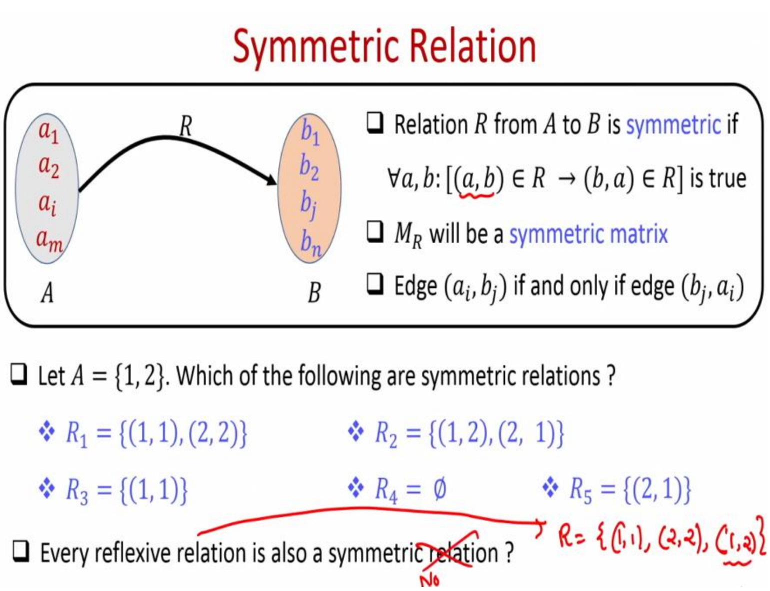

Now let us define symmetric relations, so this relation can be defined from a set A to B where B is might be different from A. So, the relation is from A to B and we say it is symmetric, so as the name suggests symmetric we want here the following to hold, whenever a is related to b as per the relation R, we need that b also should be related to a and that is why the term symmetric here and of course this universal quantification is the domain of a is A and domain of b is B.

I stress here this does not mean that you need every element of the form (a, b) and (b, a) to be present in the relation R, this is an implication. The implication here says that if (a, b) is present in R, then only you need (b, a) to be present in R. If (a, b) is not present at the first place in the relation, then I do not care whether (b, a) is there or not. I do not need (b, a) to be present, so the implication puts the restriction that this condition should be there should be true only if (a, b) is there in the relation.

So, it is easy to see that the matrix for a symmetric relation will always be a symmetric matrix, because if you have a_i R b_j, that means the i, j^{th} entrry will be 1 and since my relation is symmetric, that means I will also have (b_j, a_i) to be present. That means if I take the transpose of M_R, then in the j^{th} row and i^{th} column, the entry will be 1. Equivalently in terms of directed graph representation, if I have a directed edge from the node a_i to b_j and since my relation is symmetric, the edge from b_j to a_i will also be present. So, again let us do this example, I have set A = {1, 2} and I am defining various binary relations from A to A itself. That means in this case my A is equal to B here. Now which of the following relations are symmetric. So, it is easy to see that the first relation is a symmetric relation because this condition is true here.

I can say that since (1, 1) is present in the relation, I also have (1, 1) which can be interpreted as (b, a), also present in the relation. Due to the same reason since (2, 2) is present in the relation which can be interpreted as a being 2 and b being 2, I also have (b, a), present in the relation. Similarly the relation R_2 is a symmetric relation, the relation R_3 is also a symmetric relation because I have (1, 1) present in the relation.

And for symmetric relation (1, 1) also should be present in the relation, which is the case. Turns out that ϕ, is also a symmetric relation here. It satisfies the requirement of symmetric relation because at the first place there is no (a, b) present in my R_4. That means vacuously this implication that this universally quantified statement is true for R_4. And that is why R_4 is also a valid symmetric relation.

But R_5 is not a valid symmetric relation because I have (2, 1) present in my relation but (1, 2) is not present in the relation. So, here is a question for you, can I say that every reflexive relation is also a symmetric relation? So, remember reflexive relation means every element of the form (a, a) will represent in R. And apart from that I might have something additional also present in the relation.

So, if you are given a relation which is reflexive can I say that definitely it is also a symmetric relation and the answer is no. Take the example where A is equal to {1, 2} and let me define a relation R consisting of (1, 1), (2, 2) and say the element (1, 2). This relation is a reflexive relation, but this is not a symmetric relation. But this is not symmetric because you have (1, 2) present in the relation, but you do not have (2, 1) present in the relation.

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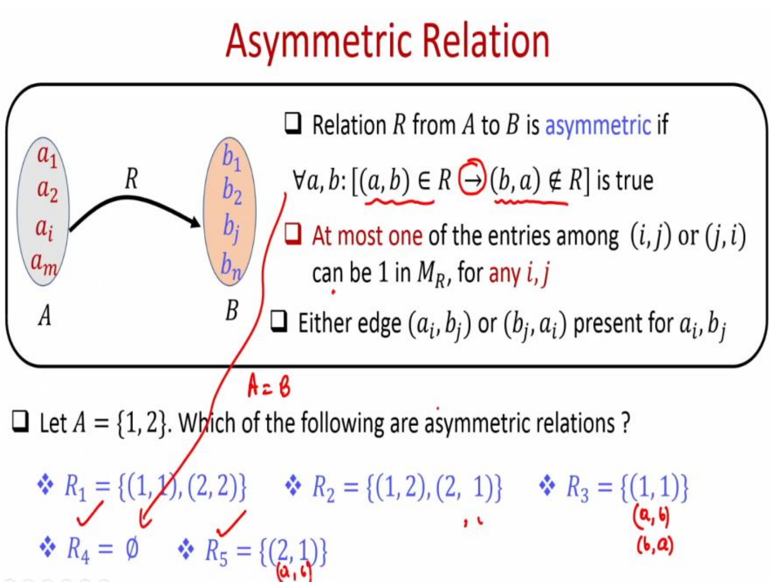

Now the next special relation is the asymmetric relation and the condition here is, if you have a related to b in the relation, then you demand that b should not be related to a. And again this is an implication that means this should hold only if (a, b) is present in the relation at the first place, if (a, b) is not present in the relation, vacuously this statement will be true. So, in terms of matrix notation the property of matrix for an asymmetric relation will be as follows.

You take any i, j^{th} entry, i^{th} row and j^{th} column, you can have at most one of the entries i, j or j, i being 1 in the matrix. You cannot have both entry number i, j 1 as well as j, i also 1. Because that will mean that you have (a_i, b_j) present in R, and (b_j, a_i) also present in the R, which goes against the definition of asymmetric relation. This automatically means that the diagonal entries will be 0.

Because if you have (a, a) present in the R, then that violates the universal quantification here, that serves as a counter example because you have (a, a) present in the R and this (a, a) can be treated as again (a, a) with a and b so here a only is playing the role of both a as well as b. So, you have (a, b) as well as (b, a) both present in this relation R and that serves as a counter example for this universal quantification and hence your relation will not be asymmetric.

So, none of the diagonal entries will be 1. In terms of graph representation, if you take any pair of nodes (a_i, b_j) then either you can have at most one edge, that means you can have either the edge from a_i to b_j or from b_j to a_i or no edge between a_i or b_j. So, this is a wrong statement here, so either edge a_i to b_j or no edge, that is also fine. Because if at the first place there is no relationship between a_i and b_j then that vacuously satisfies this universal quantification.

So, again, here I am taking A and B to be the same sets and I have given you some relations. So, let us see which of these relations are asymmetric. The first relation is not asymmetric because you have (a, b) as well as (b, a), only (1, 1) being present in this relation which serves as both (a,b) as well as (b, a). Due to the same reason R_2 is also not an asymmetric relation because you have both (a, b) as well as (b, a) present here.

Some a and b is there for which this universal quantification is not true. Your relation R_3 is also not asymmetric because you have (a, b) here written as well as (b, a) also present. Whereas the relation R_4 is an asymmetric relation over the set A, because at the first place there is no (a, b) present in this relation R_4, so R_4 vacuously satisfies this universal quantification and R_5 is also an asymmetric relation because you have only (a, b) present in this relation but no (b, a).

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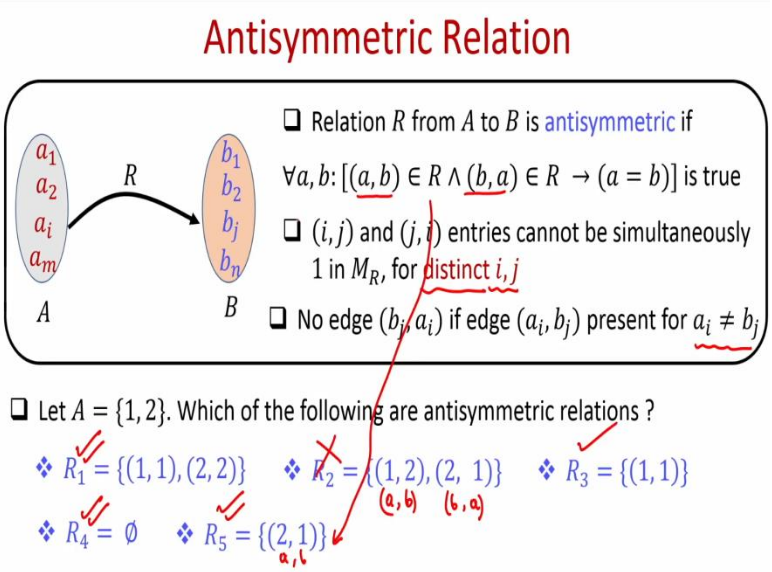

The next special relation is antisymmetric relation and the requirement here is the following. You want that if both (a, b) and (b, a) are present in your relation, that means if you have a case where an element a is related to b and b is also related to a, then that is possible only if a is equal to b. Contra-positively if a is not equal to b, then you can have either (a, b) present in the relation or (b, a) present in the relation or none of them being present in the relation.

That means for distinct elements, you cannot have simultaneously a R b as well as b R a. That is what is the interpretation of this condition. So, in terms of matrix properties if you focus on i^{th} row and j^{th} column where i and j are distinct, then only one of those entries can be 1. Of course both of them can be 0, that is also fine, because that means that neither a R b or nor b R a.

The condition demands that if at all a and b and b and a are both present in the relation, then that is possible only when a and b are same, if they are different and you cannot have both (a, b) as well as (b, a) present in your relation. In terms of graph theoretic properties if you have 2 distinct nodes a_i and b_j, then you cannot have an edge simultaneously from a_i to b_j as well as from b_j to a_i, that is not allowed.

Well, it is fine if you have no edge between these two nodes, that satisfies, that does not violate this universal quantification. So, here are some examples, relation R_1 is an antisymmetric relation because you have (a, b) present here namely (1, 1) and you also have (b, a), present here namely (1, 1), but the implication should be that 1 equal to 1 which is true, same holds for the element (2, 2).

So, this is an example of an antisymmetric relation. But R_2 is not an example of antisymmetric relation because you have a case here namely you have distinct (a, b) such that both (a, b) as well as (b, a) are present in your relation. R_3 is an example of an antisymmetric relation and R_4 is also an example of an antisymmetric relation because it satisfies this universal quantification vacuously.

R_5 is also an example of universal quantification, because you have (a, b) present here, but the (b, a) is not present in the relation R_5, that means the premise of this implication is vacuously true for R_5 and that is why this R_5 is not violating this universal quantification.

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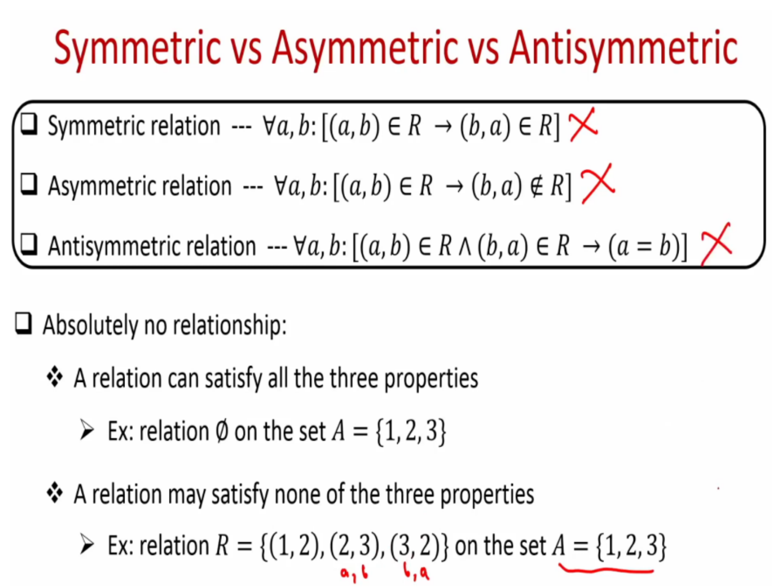

So, we have symmetric relation, asymmetric relation and antisymmetric relation. These are the definitions here and people often wonder that there is some relationship among these three different notions here, this some people think that something which is not symmetric will be asymmetric and similarly they try to conclude some relationship between the symmetric property, asymmetric property and antisymmetric property. But it turns out there is absolutely no relationship. You might have a possibility where you have a relation which satisfies all the three properties, namely if I take the set A and I take the relation R to be empty, then the relation R, which is an empty set here is symmetric as well as asymmetric as well as antisymmetric, it satisfies all these 3 universal quantifications.

Whereas there might be a possibility that a relation satisfies none of these 3 properties. So, if I take the set A to be {1, 2, 3}, and I take this relation R then it is not so, let us see which of these three properties it satisfies, so it is not symmetric because you have (a, b) here, but no (b, a). Only 1 is related to 2, but 2 is not related to 1, so it is not symmetric. It is not an asymmetric relation because you have (a, b) here and simultaneously (b, a) also, then this (2, 3) is there as well as (3, 2) is there.

So, it violates the universal quantification for asymmetric relations and it is not an asymmetric relation because you have (a, b) as well as (b, a) both being present here, even though your a and b are different. Then 2 is not equal to 3 but still you have (2, 3) as well as (3, 2) present in the relation. So, there is no absolute relationship among the notion of symmetric, asymmetric and antisymmetric relations.

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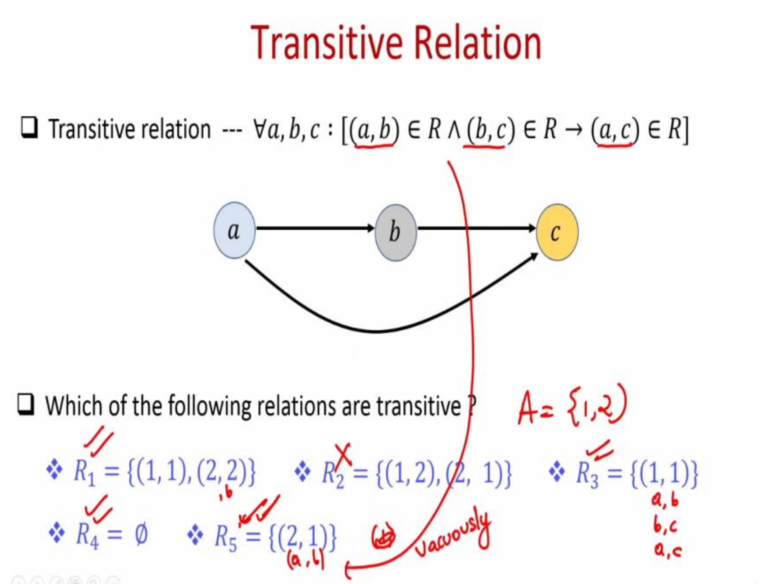

Now let us see the last important relation here, which is the transitive relation. And what do we mean by a transitive relation here, so a relation R is called a transitive relation if the following universal quantification is true. We want that if at all a R b and b R c in your relation, then a also should be related to c. In terms of graph theoretic properties, if you have an edge from a to b in the graph of your relation R.

And if you have; a directed edge from the node b to the node c in the graph of your relation R. Then we need that there should be an edge from a to c as well. And this should hold for every a, b, c, where the domain of a, b, c are from the sets over which the relation is defined. So, let us take this example, so consider the first relation it is a transitive relation, of course, so here everything is defined over a set say {1, 2} and a relation R_1 is transitive.

Because you have; (1, 1) present which can be also considered as (a, b) as well as (b, c) as well as (a, c).
So, again the same is true for (2, 2). But your relation R_2 is not transitive because you have a case here where you have (a, b) present, you also have (b, c) present but no (a, c) is present here. Namely (1, 1) is not present in your relation. Your relation R_3 is also a transitive relation because you have (a, b) present, (b, c) present and you also have corresponding, (a, c) present.

Your R_4 is a transitive relation because it vacuously satisfies this implication because at the first place there is no (a, b) and (b, c) present in your R_4. And your relation R_5 also satisfies vacuously this universal quantification, because you have (a, b) present but there is no (b, c) present that means there is nothing of the form (1, 2) here or (1, 1) here. That means vacuously this condition is true for R_5.

And that is why R_5 is also a transitive relation. So, that brings me to the end of this lecture. Just to summarize in this lecture we introduced binary relations and some special types of binary relations. We also discussed the 2 representations that we follow to represent any binary relation.