Numerical Optimization
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Lecture - 6
Convex Sets

Hello and welcome to this series of lectures on numerical optimization. In the next few lectures, we will study about convex sets and convex functions. Remember that this convex sets and convex functions play a very important role in the optimization theory, and it is very important to understand this theory before we actually move on to the other problems. So, let us start looking at convex sets first and then the convex functions. Convex functions are very nice functions, which do not have problems of local minima as we will see later. Now, these convex functions are typically defined on convex sets. So, we first study convex sets before moving on to convex functions.

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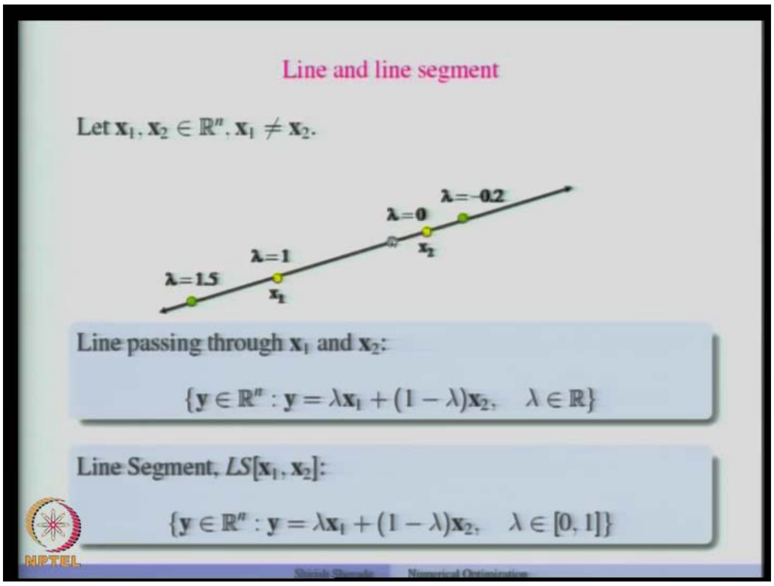

Now, before we get on to the actual theory of convex sets, so let me define some notations. So, on the display, you will see a line drawn passing through the points x 1 and x 2, where x 1 is not equal to x 2. Now, associated with each of the points on the line is a parameter lambda. So, let us assume that lambda is one at the point x 1 lambda is 0 at the point x 2 and beyond x 2 the value of lambda decreases. So, here you will see that for this point lambda is equal to minus 0.2 and it decreases further as you move away from x 2 in this direction. Now, if you move away from x 1 in this direction, the value of lambda increases.

So, any point on the line will have some real lambda associated with that point. So, the line passing through x 1 and x 2 can be represented by a set y, so that y can be written as lambda x 1 plus 1 minus lambda x 2, where lambda belongs to the set of real numbers. So, if we plug in lambda equal to 1, what we get is y is equal to x 1 as this quantity vanishes. If you plug in lambda equal to 0, we will get this quantity vanishes and we get y equal to x 2. So, a line is parameterized by a parameter called lambda which is a real number and if we are given two points through which the line passes, one can appropriately set lambdas at those points and find the equation of a line which passes through these points. Now, if you restrict our self only to the line segment x 1 and x 2, then we will denote it less x 1, x 2 and that can be written as y is equal to lambda x 1 plus 1 minus lambda x 2.

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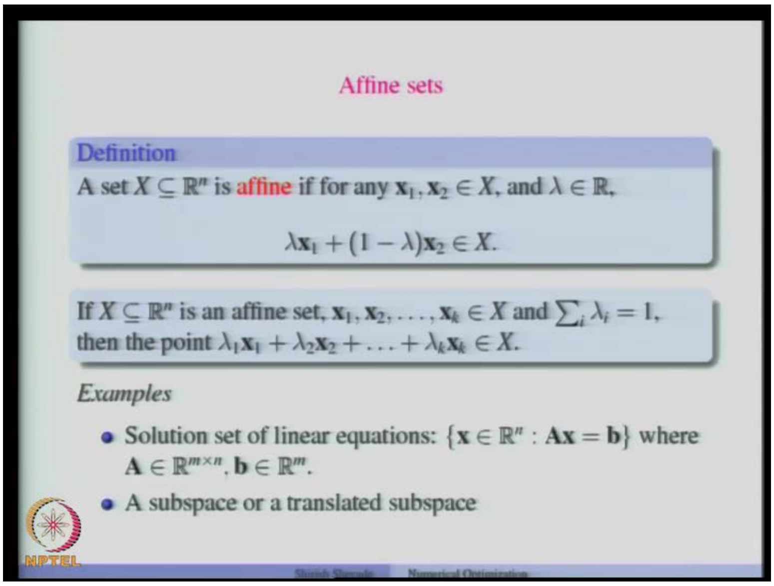

Now, remember that lambda takes the value in the close interval 0 to 1, so unlike the line where lambda can take any value from the set of real numbers here in when we talk about the line segment lambda takes the value in the range 0 to 1. So, this is the n segment x 1 x 2 that we are talking about. Now, let us look at what are called affine sets, a set x which is subset of r n is said to be affine. If you take any two points in the set x 1 and x 2 and take any lambda which is a real number, then lambda x 1 plus 1 minus lambda x 2 should belong to x.

So, what this essentially means is that if you take any two points from the set x and take a line passing through those two points, then that line lies in the set x. So, such a set is called an affine set. Now, this definition can be generalized to this t of k points. So, earlier we took only two points, x 1 and x 2. Suppose, we take k points x 1 x 2 up to x k which are part of the set x, and if we choose lambda which are lambda i h, so there will be a lambda associated with each of these points, but if we choose those lambdas such that sigma lambda equal to 1, then if x is an affine set, then sigma lambda x i is going from 1 to k belongs to the set x.

So, this is just an extension of this. So, remember that when we have an affine set, we take any points, any set of points from that set when sigma lambda x i is 1 if we ensure that sigma lambda is 1 for the affine set. Now, here are some examples of affine set. So, if we consider a solution set of linear equations, where a is m affine matrix and b is the m dimensional vector, then the set of all x is that x equal to b form the solution set of this linear equations. Now, one can easily show that this set is an affine set. For example, if we take two solutions, x 1 and x 2 if this system of linear equations, then a x 1 equal to b and a x 2 equal to b.

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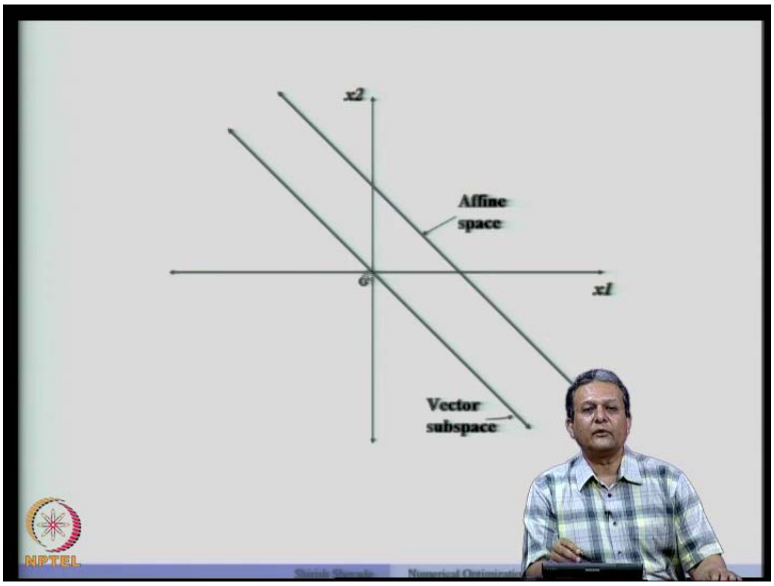

So, if we take lambda, so lambda into x 1 plus 1 minus lambda into x 2, one can show that also satisfies. So, if we take axis that x equal to lambda x 1 plus 1 minus lambda x 2, that also satisfies the system a x equal to b. So, any solution set of linear equations of this type is an affine set. Another example of an affine set is a subspace or a translated subspace. For example, this is the vector subspace because you will see that it is in to dimensional space. This is the vector subspace because it passes through the origin. Now, if I translate it that becomes an affine space. Now, both are affine sets because if you take any points, any two points in this set, then the line passing through those points belongs to the same set. So, these two are, both are affine sets.

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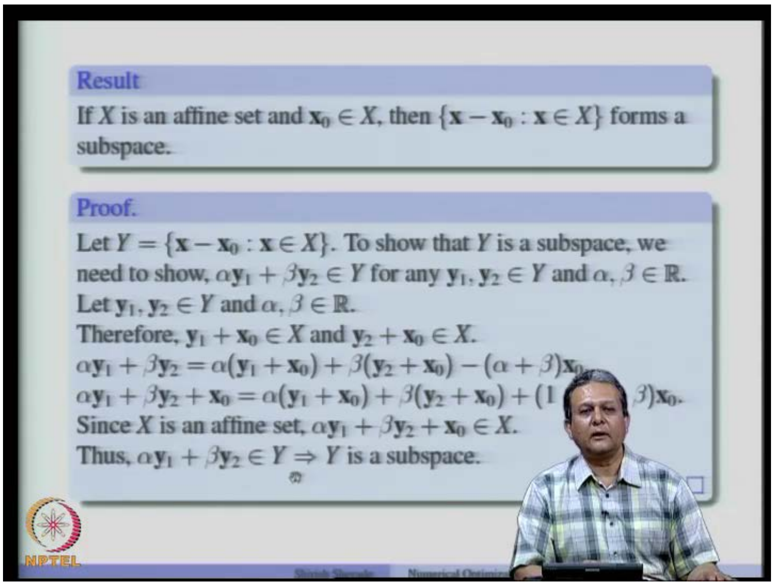

In fact, one can show that if we have an affine space and if you translate it to the origin, then it becomes a vector space. So, here is a result which says that if (( )) affine at x and let us take a point x 0 from that set x, then the set of all points x minus x 0, where x belongs to the set x, it forms a subspace and it is very easy to do that. So, let us look at the proof of this result. So, let us consider the set y. So, we are given x naught which will belong to the set x. So, let us consider the set y, such that which is the collection of x minus x naught where x belongs to x.

Now, we have to show that y is a subspace. That means what we have to show is that by the definition of subspace if you take any two vector, say y 1 and y 2 in the set y, then linear combination of those two vectors should also belong to the subspace y. So, that means that if we choose alpha and beta belong to r and alpha y 1 and beta y 2 should belong to y, so this is what we want to show that y is a subspace. Now, so let us consider y 1 and y 2 which belong to the set y and let us consider two scalars which are real numbers, alpha and beta. Now, since y 1 belongs to y and x naught belongs to x, so y 1 plus x naught clearly belongs to x and similarly, y 2 plus x naught belongs to x.

Now, let us see what happens to alpha y 1 plus beta y 2. So, alpha y 1 plus beta y 2 can be written as alpha into y 1 plus x naught plus beta into y 2 plus x naught minus alpha plus beta into x naught. Now, you will see that y 1 plus x naught belongs to xy 2 plus x naught, also belongs to x and x naught obviously belongs to x that is given to us. So, all these three vectors belong to the set x. Now, this affine combination of these vectors. Now, let us sum the co-efficients which are here. So, alpha plus beta minus alpha minus beta. So, that is 0, so that this is not an affine combination of the vectors in x. So, what we do is that to make it an affine combination, let us add x 1 x 0.

So, once you add x 0, so what do we get? So, these quantities remain the same and the one x 0 gets added to the third quantity. Now, you will see that y 1 plus x naught y 2 plus x naught and x naught belong to x and this is affine combination of the three vectors in the set x because alpha plus beta plus 1 minus alpha minus beta is 1. So, clearly alpha y 1 plus beta y 2 plus x naught belongs to the set x because x is an affine set and we are considering an affine combination of three vectors in that set.

So, clearly this left hand side quantity belongs to the set x, which means that alpha y 1 plus beta y 2 should belong to the set y. So, since x is an affine set, the left side quantity belongs to x and therefore, alpha y 1 plus beta y 2 belongs to y which implies that y is a subspace. So, if we have any affine set and if you translate it to the origin, then it becomes, so clearly you will see that since x 0 belongs to x origin, also belongs to the set x minus x 0. So, clearly an origin is the part of the set y and by this proof, it is a subspace.

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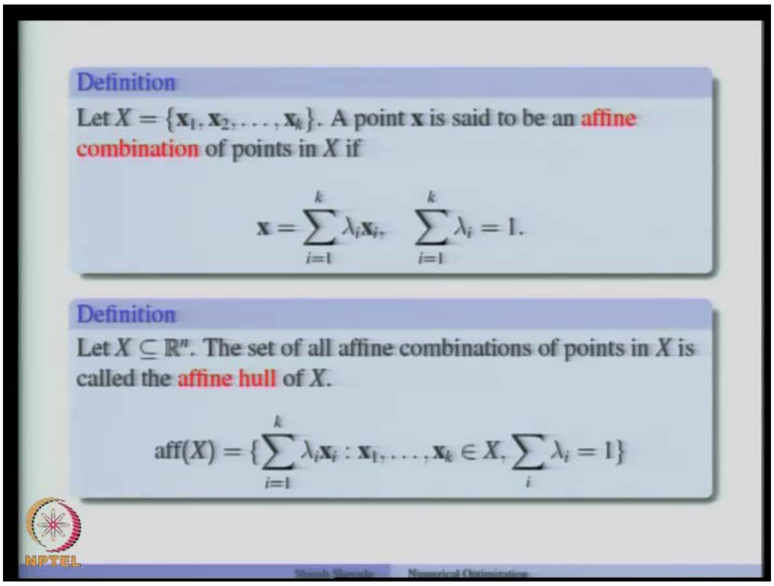

Now, let us look at some more definitions. Suppose, we are given a set of points x 1 to x k. Now, a point x is said to be an affine combination of points in x. If you can write x as a linear combination of x i's and the linear combination is such that sigma lambda is 1. So, the sum of the coefficients in the linear combination should be 1. Then we call x to be an affine combination of the points x i's, i going from 1 to k. Now, one can talk about this affine hull. So, affine hull is the set of all affine combinations of the points in x and it can be written as a set of all affine combinations of x i's in x.

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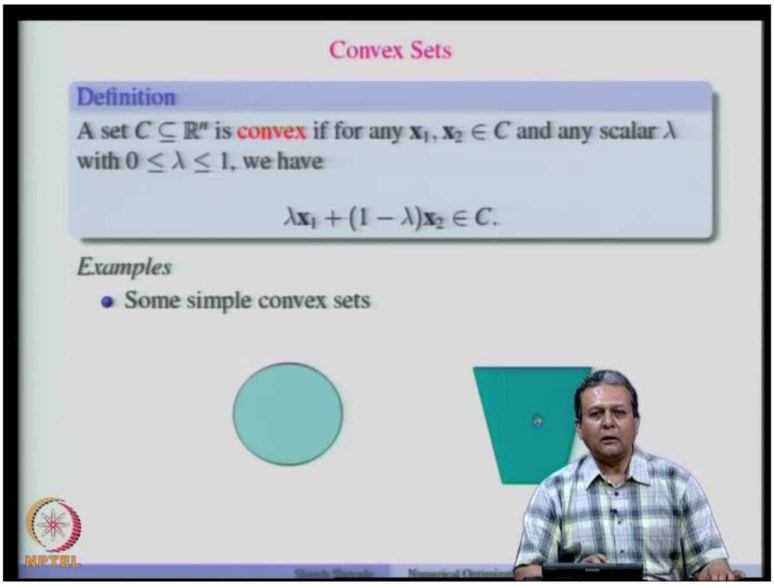

So, remember that sigma lambda is going to be 1. So, you take any k points in the set x, take an affine combination of them and collect all such affine combinations. We will get affine hull of the set x. Now, we will move on to the definition of convex sets and as I said that this convex sets and the convex functions are very important topics in the theory of optimization, and some of these ideas are used to derive optimality conditions for general linear and non-linear programic problems. So, let us see the definition of a convex set. Now, a set which are subset of r n is said to be convex. If you take any two points x 1 and x 2 has belonging to that set and a scalar lambda, such that 0 is less than 1 or equal to 1. So, lambda lies in the close interval 0 to 1.

Then, we have lambda x 1 plus 1 minus lambda x 2 which belongs to c. Now, what does this definition mean? So, this definition means that if we take any two points, x 1 and x 2 from the set c and take a line segment joining x 1 and x 2, so lambda x 1 plus 1 minus lambda x 2, where lambda is in the close interval 0 to 1 denotes a line segment joining x 1 and x 2. So, if you take any two points, x 1, x 2 belong to c, take a line segment joining those two points that the n segments should entirely lie within the set c. So, let us look at some examples.

So, here are a couple of examples of convex sets. Now, you can think of convex set has the following that along the boundary. Suppose, if you put a wall along the boundary of a given set, if you put a wall and then consider any two points in that set, then any two points, every point should see any other point of the set directly through a straight line path. So, if you take any two points here and if we take a line segment joining those two points that lies entirely within that set. Similarly, one can say that this is a convex set.

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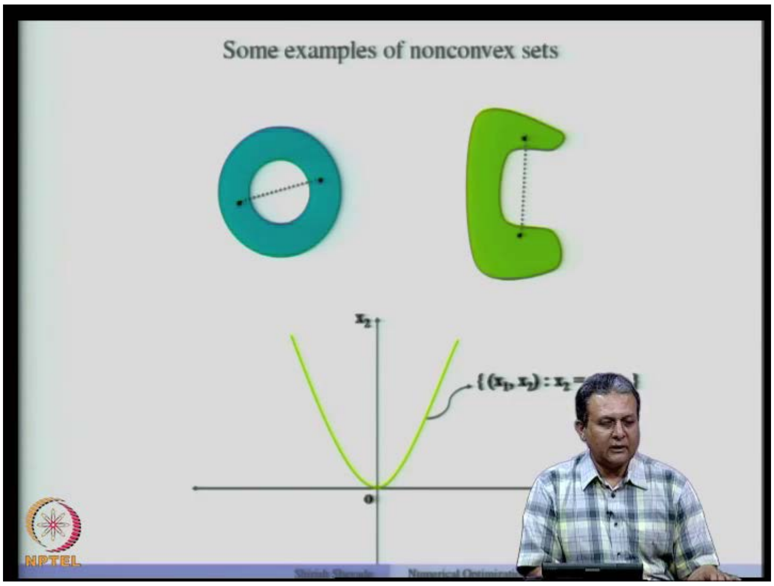

Now, let us see some examples of non-convex sets. So, on the left side, you will see that if I take this point and this point, so the line segment joining these two points does not entirely lie within the set. Although, some part of it lies, but not the entire line segments should lie within the set. So, if we again go by the earlier geometric interpretations, suppose if you put a wall around the boundary, then this point will not able to see this point because of the wall.

Similarly, this is not a convex set because the line segment joining these two points is not entirely within the set. So, these are some examples of non-convex sets. So, here is one more example. So, let us consider the set of points in two-dimensional space, x 1 x 2, where x 2 is nothing but x 1 square. So, this is the set of points which is shown by the green curve here. So, this set is not convex because if I take a point here and take a point here, the line segment joining them does not part of the set. So, these are some examples of non-convex sets.

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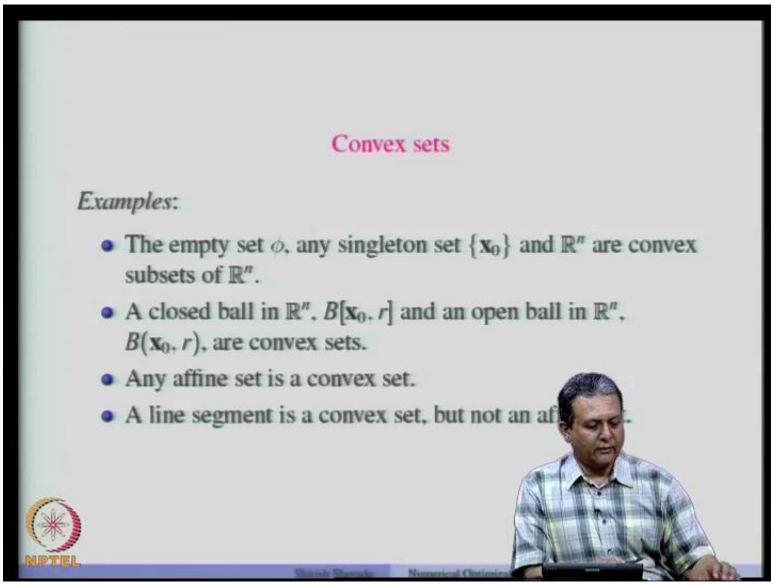

So, for convexity what we need is, for convexity of a set, what we need is that the line segment joining any two points of that set should lie entirely within that set. Now, let us look at some more examples of convex sets. Now, by convention, the empty set is a convex set. Then any singleton is also convex set because since we have only one point in the set, the set is obviously a convex set because there is no notion of a line segment in that set. Then the set of r n, the n dimensional set of a real numbers, these are all convex sets.

Now, if you take a close ball in r n. The ball with the center x naught and radius r where r is greater than 0 or if you take an open ball in r n which are the center x naught and the radius r which is again greater than 0, then they are all convex subsets of r n. Now, any affine set is a convex set because an affine set by definition is a set where if you take any two points, the line passing through the two points always lies in that set. So, if a line passes through the set, the line segment joining any two points also should lie in that set. So, affine set is also convex set. Then line segment if we just consider a line segment, line segment is a convex set, but the line segment is not affine because the entire line does not lie in that set. So, a line segment is a convex, but not affine.

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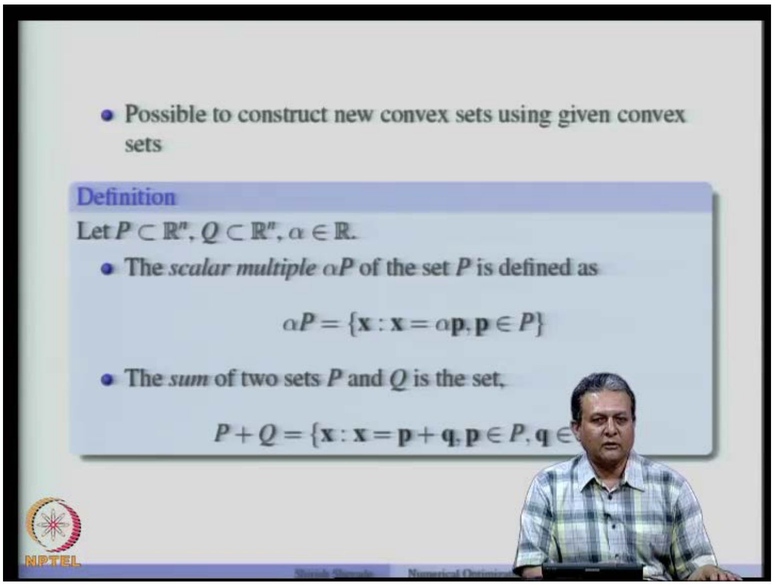

Now, it is possible to construct some new convex sets using given convex sets. So, then we will have some ways to derive new sets, new convex sets from the existing convex sets. Now, before we look in to those details, let us look at some definitions. So, let p and q be subsets of r n and let alpha be a real number. Now, the scalar multiple alpha p of the set is defined as the set of all points x, such that x is nothing but x equal to alpha p, where p belongs to p. So, we take given alpha which is the real number. We take every p from the set p and multiplied by alpha and get x.

Now, collect all such x's from all possible p's that will form the set alpha p. Now, similarly, one can also define the sum of two sets as p plus q is nothing but you take a vector p or point p from the set p and vector q from the set q and then sum them off. So, this is the vector sum of p and q. So, collect all such x's from all possible pairs of p and q where p belongs to the set, p and q belongs to the set q and that will form the sum of two sets, p plus q.

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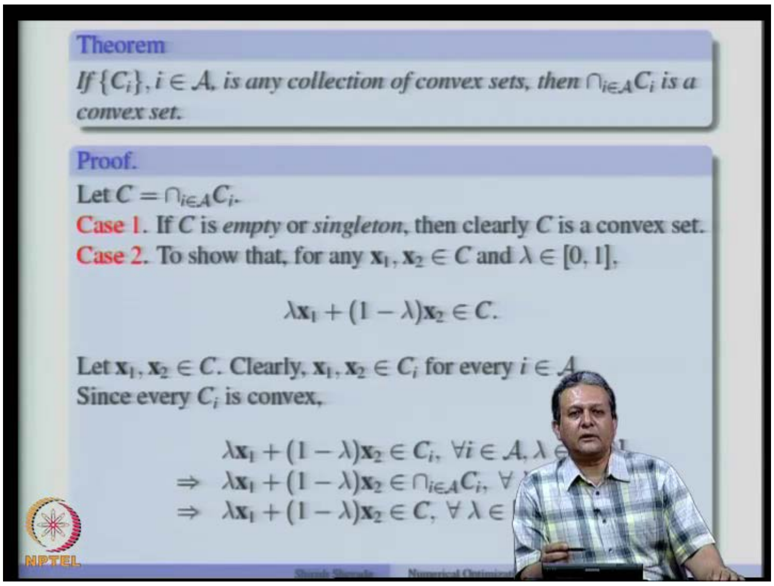

Now, with this definition, we can generate new convex sets from the given sets of convex sets. So, let us look at some results to generate new convex sets. Now, if we have collection of convex sets denoted by c i where i belongs to the set a, then the theorem says that the intersection of this convex sets is a convex set. So, let us try to proof this figure.

So, let us consider the set c to be the intersection of all c i's where i is from the set, a i is the index set, i is the index which belongs to the set a. Now, let us consider the first case where c is empty or singleton. Now, we have seen that by convention, it sees empty. It is a convex set or even if c is singleton, it is a convex set. So, we have to worry about only those cases where c is non empty and non single term. So, let us look at that case. Now, whenever we want to show that any set is a convex set, what we need to show is that for any two points in that set and lambda in the close interval 0 to 1 lambda x 1 plus 1 minus lambda x 2 should belong to c. So, this is what we need to show.

So, let us consider any two points in the set c. Now, since they are in the intersection set c, they are in the individual sets as well for every i belong to a. Now, remember that each of the c i's is a convex set, so by using the convexity of c I, what we can do is that we can say that lambda x 1 plus 1 minus lambda x 2 belongs to c i or every i belong to a, where lambda is in the close interval 0 to 1. This is because of the fact that every c i is a convex set. Now, that means that you take a line segment joining x 1 and x 2 that entirely lies between within the set c i for every i which means that if you take the line segment joining x 1 and x 2, it should also lie in the intersection of this c i's because it belongs to every c i, which means that lambda x 1 plus 1 minus lambda x 2 belongs to the intersection of all c i's for all lambda in the range 0 to 1.

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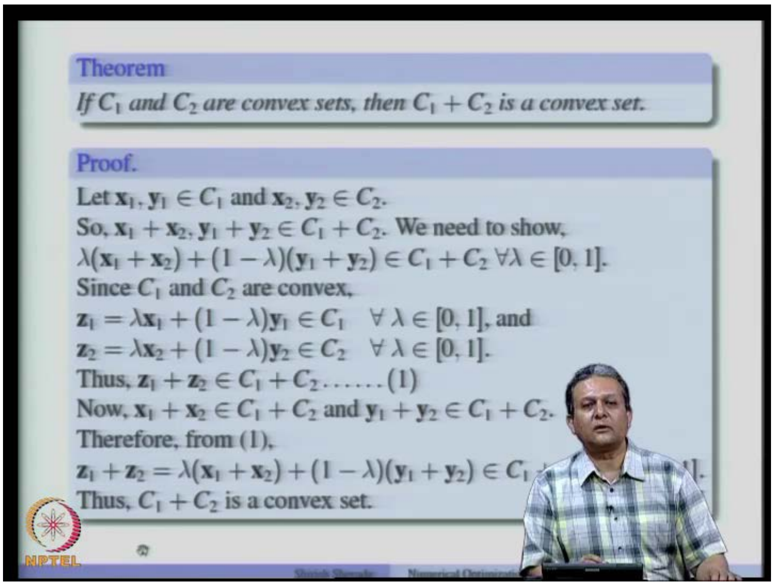

This intersection of c i belong to a is nothing but the set c. So, lambda x 1 plus 1 minus lambda x 2 belongs to the set c for all lambdas in the range 0 to 1. So, this result says that if we take a collection of convex sets, any collection of convex sets, then their intersection is the convex set. So, by using given convex sets, we can try to take their intersection and form new convex sets. So, this is the first result that we saw. Then the second result is about the sum of two sets. So, suppose if you are given c 1 and c 2 as convex sets, then c 1 plus c 2 is a convex set.

So, as I said earlier that to show that any set is the convex set, we need to take two points in that set, any two points and show that the line segment joining any two points lies entirely within the set. So, let us consider x 1 y 1 belongs to c 1 and x 2 y 2 belong to c 2. So, these are the vectors in c 1 and c 2 respectively. So, we can say that x 1 plus x 2 belongs to c 1 plus c 2 and y 1 plus y 2 belongs to c 1 plus c 2. This is by the definition of c 1 plus c 2. Now, we have got two points in the set c 1 plus c 2. What we need to show is that lambda in to x 1 plus x 2 plus 1 minus lambda into y 1 plus y 2 also belongs to c 1 plus c 2 for all lambdas in the range 0 to 1. So, this is what we need to show.

Now, remember that what is given to us is that c 1 and c 2 are convex. So, if I take two vectors, x 1 and y 1 in c 1, then the line segment joining x 1 and y 1 lies in the set c 1. So, if I write z 1 to be lambda x 1 plus 1 minus lambda y 1, so z 1 clearly belongs to c 1 for all lambda in the range 0 to 1. Similarly, if I take two vectors, x 2 and y 2 in the set c 2 and take a line segment joining x 2 and y 2, I can write z 2 to be lambda x 2 plus 1 minus lambda y 2 which belongs to c 2 for all lambdas in the range 0 to 1.

So, now we have vector z 1 which belong to c 1 and a vector z 2 which belong to c 2. So, what we can do is we can sum them up. So, z 1 plus z 2 will belong to c 1 plus c 2. So, this is the important point. The z 1 plus z 2 belongs to c 1 plus c 2. What is z 1 plus z 2? Now, if you sum the right side, what do we get? So, remember that x 1 plus x 2 belongs to c 1 plus c 2 and y 1 plus y 2 belongs to c 1 plus c 2. So, if you sum these, the right side, what we get is lambda into x 1 plus x 2 plus 1 minus lambda into y 1 plus y 2 and that is nothing but z 1 plus z 2 and since, z 1 plus z 2 belong to c 1 plus c 2 from 1, what we get is that the vector on the right side also belongs to c 1 plus c 2 for all lambda in the range 0 to 1.

So, we had two points, x 1 plus x 2 in the set c 1 plus c 2 and y 1 plus y 2 in the set c 1 plus c 2 and a line segment joining those two points which is written as lambda into x 1 plus x 2 plus 1 minus lambda into y 1 plus y 2 also belongs to the set c 1 plus c 2 for all lambda in the range 0 to 1, which means that c 1 plus c 2 is a convex set. So, if we are given two convex sets, we can generate new convex sets. For example, if you take x axis in the two-dimensional space, if we take x axis as one convex set and y axis as one convex set, then the set c 1 plus c 2 is entire r 2 and that turns out to be a convex set. So, new convex sets could be generated by taking the vector sum of the convex sets easily.

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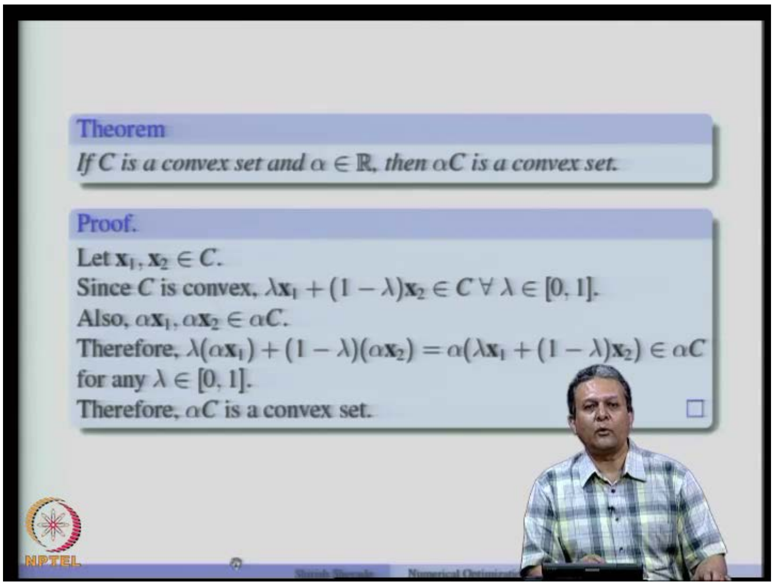

Now, here is the third result which says that if c is a convex set and alpha is a real number, then alpha c is the convex set again. It is very easy to prove that. So, let us take two points in the set c and show that the line segment joining these two points also belong to the set c. So, let us take x 1 and x 2 in c and since, c is convex which is given to us, lambda x 1 plus 1 minus lambda x 2 also belongs to the set c. Now, if you take alpha x 1 and alpha x 2, where alpha is the real number, now clearly this alpha x 1 and alpha x 2 belongs to alpha c. What we have to show is that the line segment joining alpha x 1 and alpha x 2 should also belong to the set alpha c.

So, if we take the light segment joining alpha x 1 and alpha x 2 which is represented as lambda into alpha x 1 plus 1 minus lambda into alpha x 2, we can rewrite it as alpha into lambda x 1 plus 1 minus lambda x 2 and this is nothing but a point in the set c. So, this alpha into this quantity clearly lies in the set alpha c for any lambda in the range 0 to 1, which means that alpha c is a convex set. So, this theorem tells that new convex sets can be generated by shrinking or expanding the given convex set appropriate.

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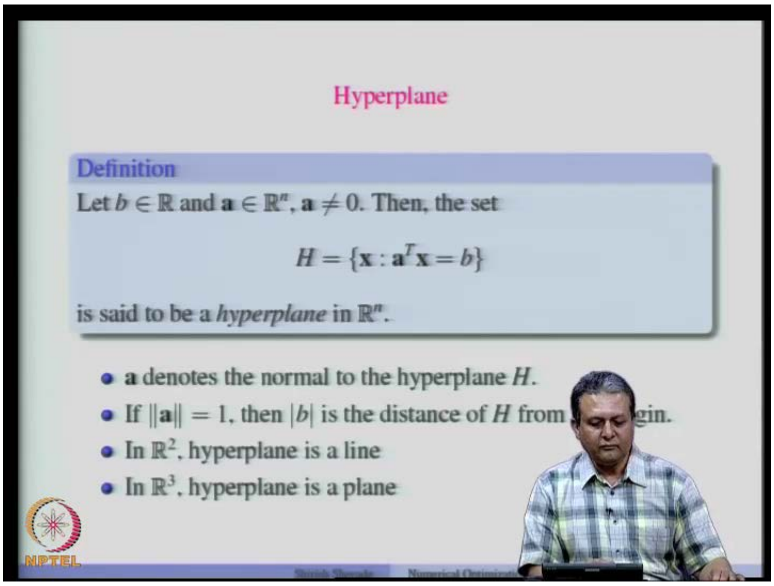

Now, let us look at the definition of hyperplane. So, suppose b is a real number and a is a non-zero vector in r n. Then the set of all h such in the set h is the set of all x, such that a transpose x equal to b is said to be a hyperplane in r n. So, let us see the significance of each of these terms in the set h. Now, the vector a, denotes the normal to the hyperplane and if the norm of a is 1, then the modules of b, that absolute value of b is the distance of h from the origin. Now, in r 2 in two-dimensional space, a hyperplane is a line and in three-dimensional space, a hyperplane is a plane and in high dimensional cases, we call this as a hyperplane.

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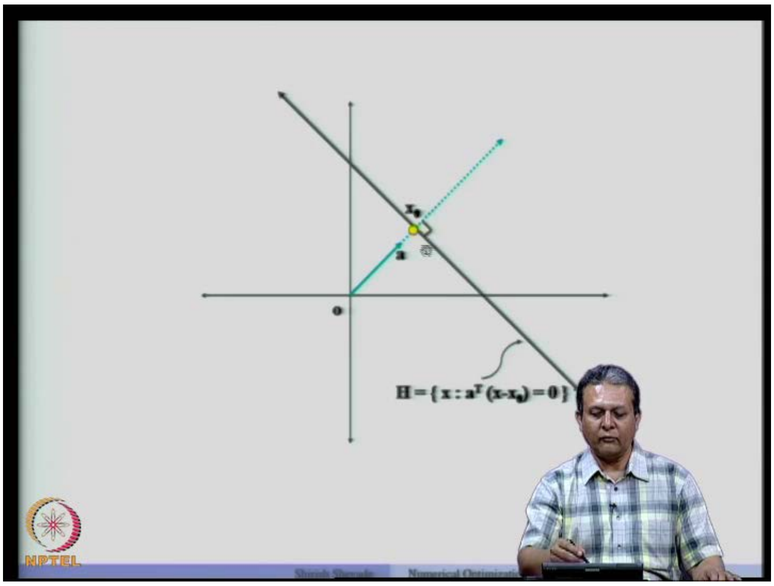

So, if you draw in two-dimensional space, it is a hyperplane. So, again let us consider a two-dimensional case, but the ideas are essentially the same problem, high dimensional cases. So, we have vector a, which is pointing in this direction and let us consider the set which is shown in by the black line as the set of all points x, such that a transpose x equal to b. So, this is a transpose x equal to b is an equation of this hyperplane and if we collect all such x's, that forms the set h and that is called the hyperplane.

So, remember that a is normal to this hyperplane. Now, if you are given a point which belongs to the hyperplane, then that this equation of the hyperplane can be written as, if x 0 is a point which lies on the hyperplane and we are still talking about the normal to the hyperplane as a, then the set of x, such that a transpose x. So, the a transpose x minus x naught equal to 0 forms the hyperplane. So, you will see that if we take any vector on this and form the vector x minus x naught and take a dot product of that with a, the dot product, they are perpendicular to any vector here and this vector a, they are perpendicular to each other or orthogonal to each other. So, their dot product is 0 and that is what is reflected in the definition of this hyperplane.

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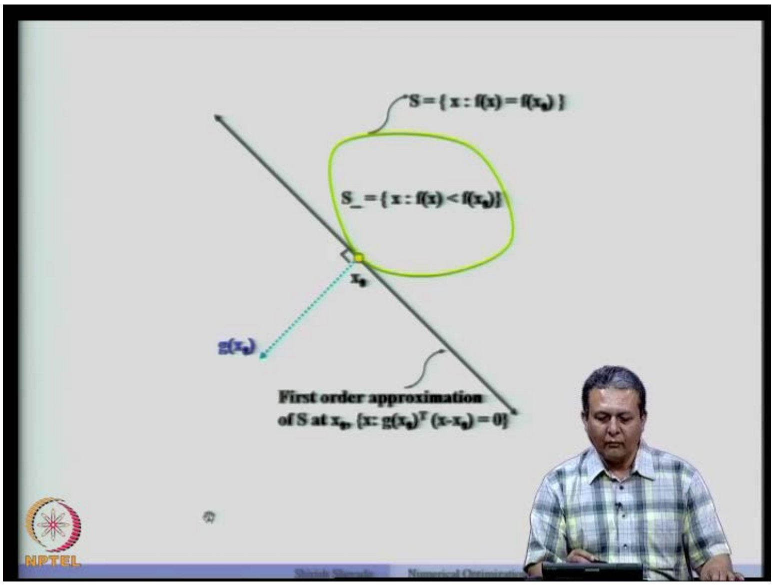

So, it is just another way to represent the hyperplane. If we know a point x 0 which belongs to the set to the hyperplane and if you know the normal a to the hyperplane, now if you are given a set s, if you are given some function f which is differentiable and the contour of that function, so the contour of that function we are denoting it by s, such that set of all x, such that f x is equal to f of x naught because the contour passes through the point x naught. So, all the points on this contour will have the value of the function to be equal to f of x naught and as you move in the interior, the value of the function increases or decreases.

So, this is denoted by the set s minus. So, it is basically the set which is in the interior of this where f of x is strictly less than f of x 1. So, as you move in the interior, the value of the function decreases. As you move away in the other side, away from this curve, then the value of the function increases. Now, if you take a point x naught which belongs to this contour, then we can approximate this contour by a hyperplane and if we use the first derivative of the function, you remember that we have assumed the function f is differentiable. So, if we use the first derivative of the function and this vector g is a gradient vector of f at x naught, so gradient of f of x naught is nothing but g of x naught.

So, then one can use the order approximation of s at x naught as the set of all points, such that we have a normal to the hyperplane and the hyperplane passing through the points x naught to set of all x, such that g of x naught transpose x minus x naught to be equal to 0. So, this is going to be the first order approximation of the function of a differentiable function f at point x naught. So, we will require this sometime later when we derive the optimalytic conditions. So, I just wanted to show that the first order approximation can be represented in the form like this.

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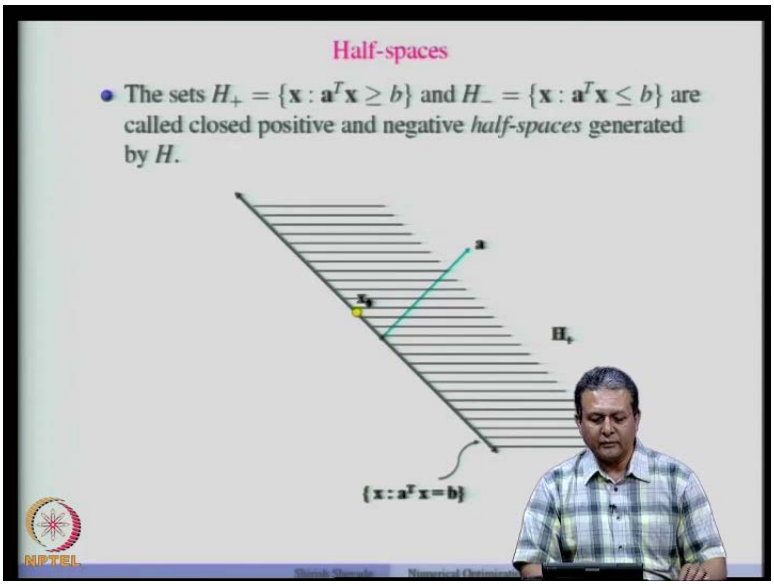

Now, let us look at half-spaces. So, the set h plus which is the collection of all x, such that a transpose x is greater than or equal to b, where a is again a non-zero vector in the space of n dimensions and b is a scalar. The set h plus is called a close positive half-space generated by the plane, hyperplane h. So, remember that the equation of the hyperplane h is set of all x, such that a transpose x equal to b. So, we are just considering the set of all points which lie on one side of this, which lies on or one side of this hyperplane.

Similarly, one can define the negative half-space set of all x, such that a transpose x less than or equal to b. Now, we will show this geometrically. So, we have hyperplane h and this a is normal to this hyperplane as this hyperplane is the set x, such that a transpose x equal to b and the points. This is the set of points which lie on this side of this hyperplane where a is pointing and that is called the positive half-space. So, the point on the hyperplane and on this side of the half of the hyperplane is called a closed half-space. Now, if we make the inequality strict that is the set of all x is that a transpose x greater than b, so these are the points which are not on the hyperplane, but are on this side of the hyperplane. So, that is called a open half-space, open positive half space.

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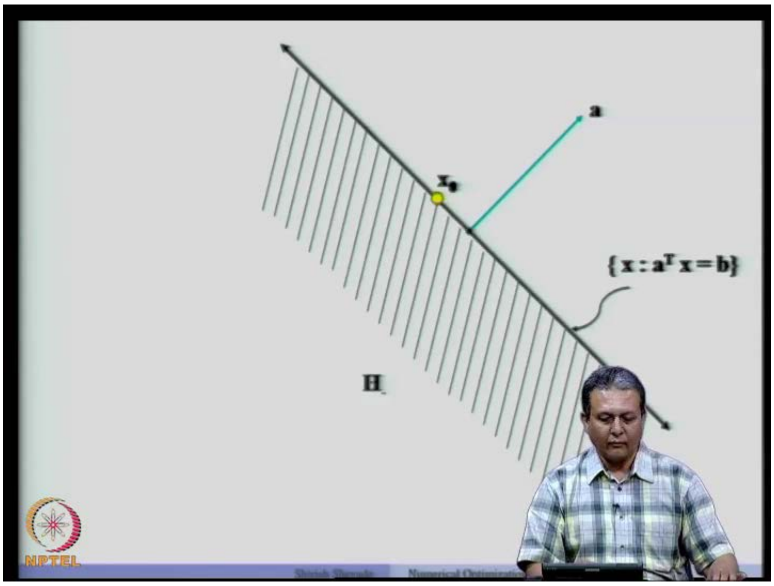

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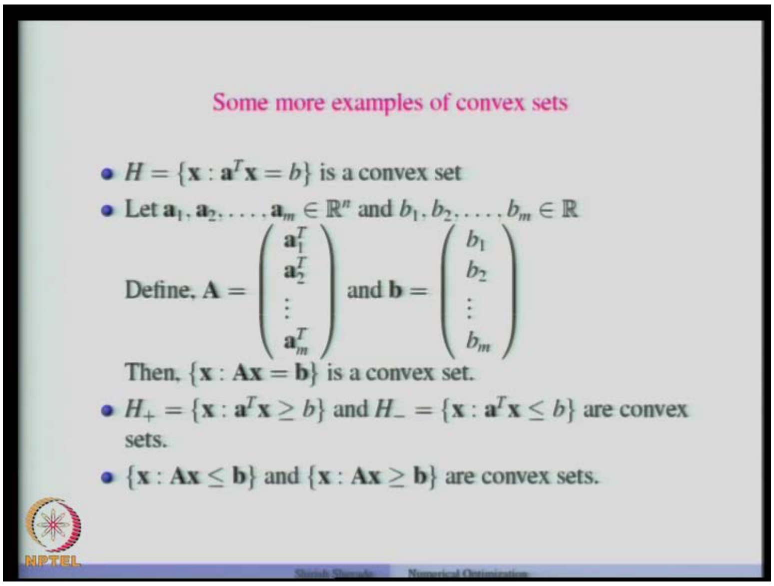

Similarly, one can talk about the closed negative half-space. So, again we are given a hyperplane whose normal is a and the points on the hyperplane and points on the other side of the normal, they form a closed negative half-space. Now, with definitions of half-space, let us look at some more examples of convex sets. So, the hyperplane is a convex sets. So, it is obvious because if you take any two points on the hyperplane ha, the line segment joining those two points always lies in the hyperplane.

Now, let us consider m vectors in n dimensional space and m scalars b 1 to b 2. So, a 1 to a m are the m vectors and suppose, we arrange them in the form of a matrix, where every row is at i throw denotes the transpose of the vector a i. So, you have m by n matrix and b is a m dimensional vector. Now, the claim is that the set of all x's, that x equal to b. So, if you want to solve this system of equations x equal to b, then the solutions of that system of equations, it forms a convex set. Now, it is obvious because a x is equal to b. This set can be written as a 1 transpose x equal to b 1, a 2 transpose x equal to b 2 and up to a m transpose x equal to b m.

Now, we have already seen that set of all x as that a transpose x equal to b is the convex set. So, each of a i transpose x equal to b i, I am going from 1 to m is a convex set and what we are doing is that we are trying to take a intersection of this convex sets and we have seen earlier that the intersection of convex sets is a convex set. So, the solution set of the system of linear equations x equal to b, where a is a matrix like this and b is a vector like this. It forms a convex set.

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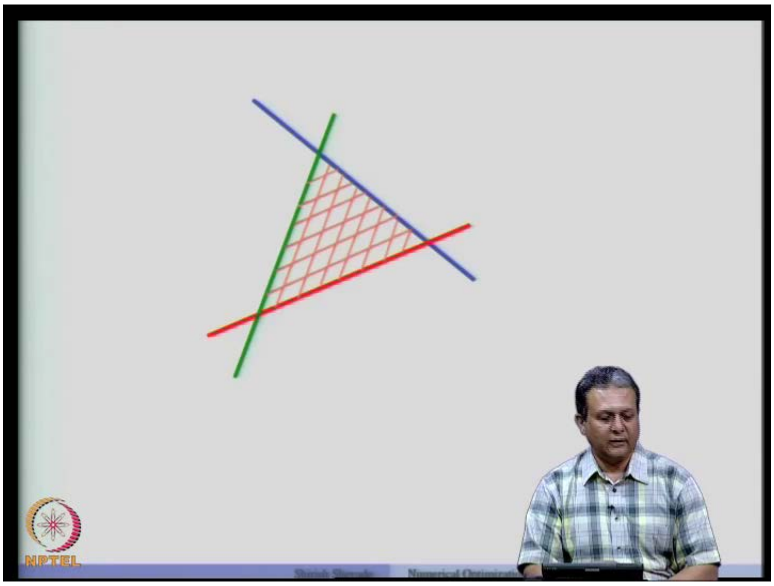

Now, if you take closed half-spaces, either positive or negative, both are convex sets. Now, again using the ideas similar to this, we can show that the set of all x's, that a x less than or equal to b or the set of all x's at x greater than or equal to b, they form convex sets. Now, here is a simple example. So, let us consider a hyperplane. Although, I have shown arrows, but this is a hyperplane and then suppose we are interested in the closed half-space of this, formed by this hyperplane, the close half-space pointed by this arrow. So, clearly this is a convex set.

Now, let us consider another hyperplane. So, this another hyperplane and we are interested in the closed half-space of this hyperplane. The half-space which is formed which is denoted by this arrow. Now, if you take the intersection of the previous half-space and the current half-space, so it is basically the area between these two is the region between these two hyperplanes and that also is a convex set. If you add one more, so we get a triangle in this two-dimensional space. So, the interior of this triangle is a convex set. So, by using the existing convex sets is possible to construct new convex sets and that is illustrated here.

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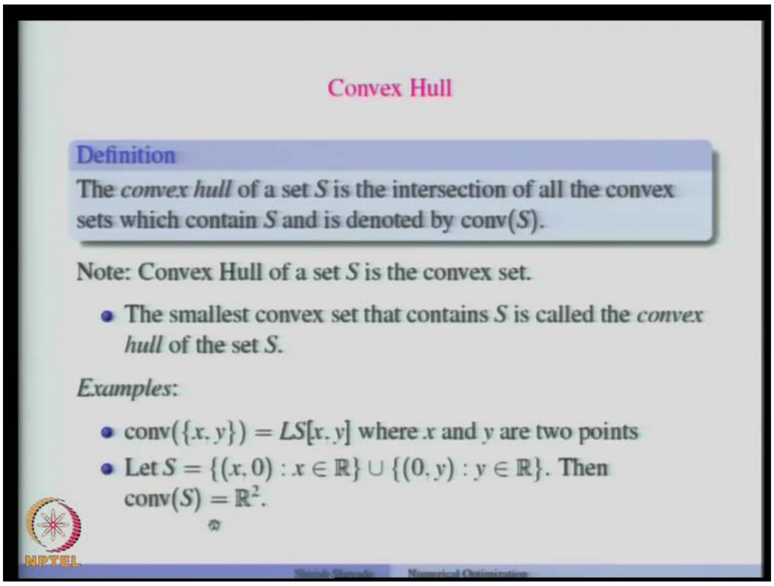

Now, let us look at the definition of a convex hull. The convex hull of a set s of a general set s is the intersection of all convex sets, which contain s and it is denoted by convex hull of x. So, it will be denoted like this. Now, by the very definition of convex hull, you will see that it is the intersection of all convex sets and we know that intersection of all convex sets is a convex set. So, every convex hull is a convex set. In fact, if you are given a set s, then sometimes we would like to form a convex set which contains s and such a convex set which is the smallest convex set is called the convex hull of the set s. So, many times this convex hulls are used for convex hull of a set s without bringing in any redundancy.

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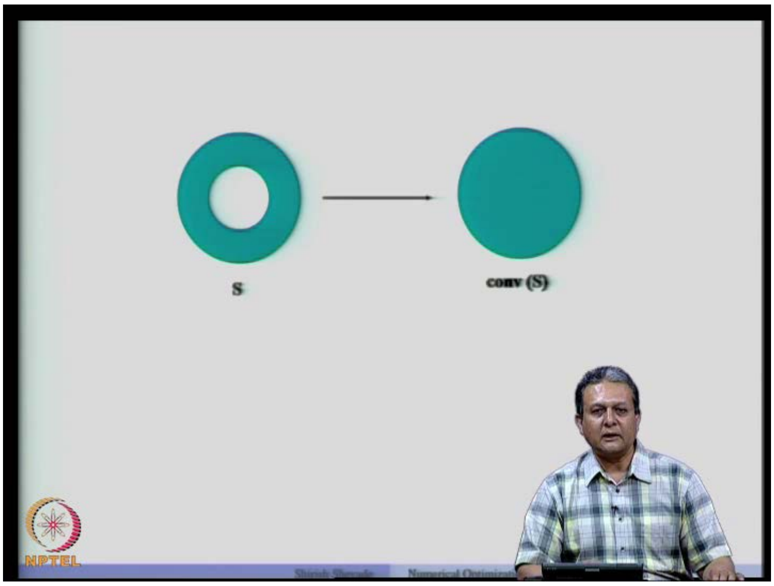

So, a convex hull can be thought of as a smallest convex set that contains the given set s, and we will see some examples. So, if you are given two points, then the line joining those two points, it is a convex hull of the set of those two points. Now, in two dimension, suppose if you are given x axis and y axis, then the convex hull of these two sets is entire two-dimensional space. Now, when we studied convex sets, we saw this example and we demonstrated that this s is not a convex set.

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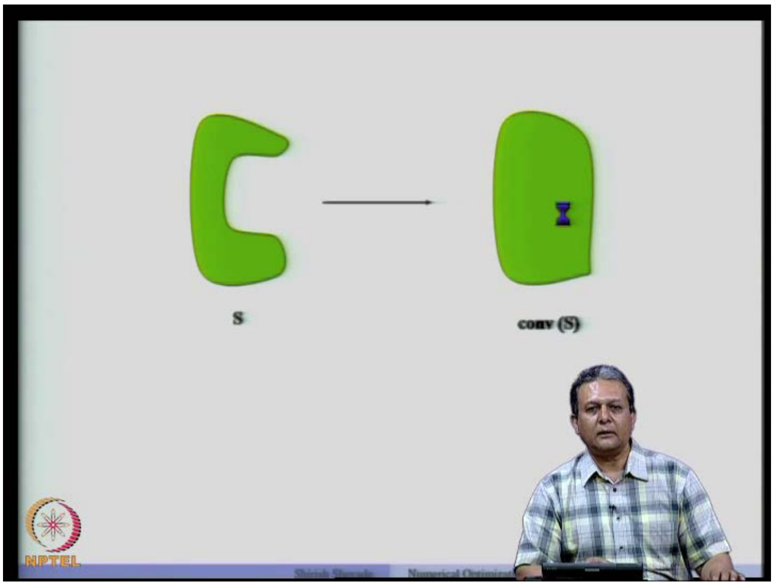

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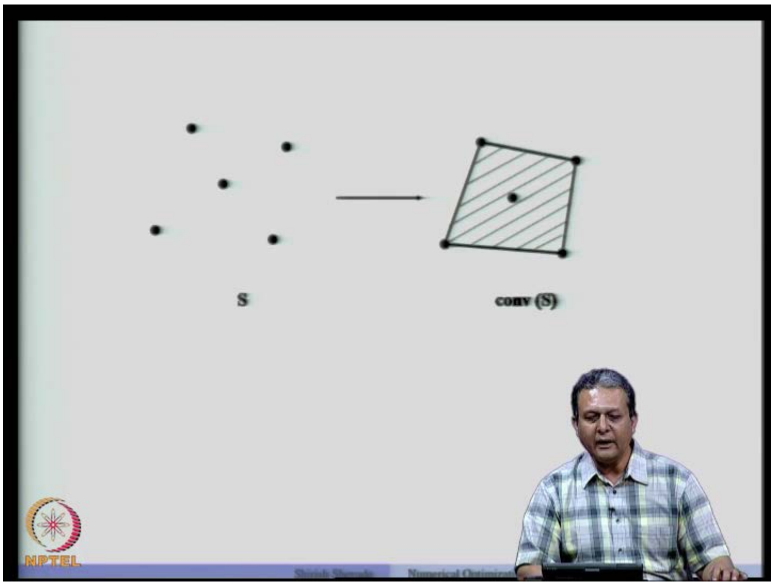

Now, one can form a convex set of this set. So, that becomes this set. Now, this set is a convex set as you will see that you take any two points and this set, it will always form a, that line segment always lies in the set. Now, here is another example which we saw that this is s is not a convex set, but then one can find the convex hull of this set and suppose, we are given a set of points and if you want to find the convex hull of this, so it is a smallest convex set that encompasses all the points. So, this is the smallest convex set which encompasses all the points.

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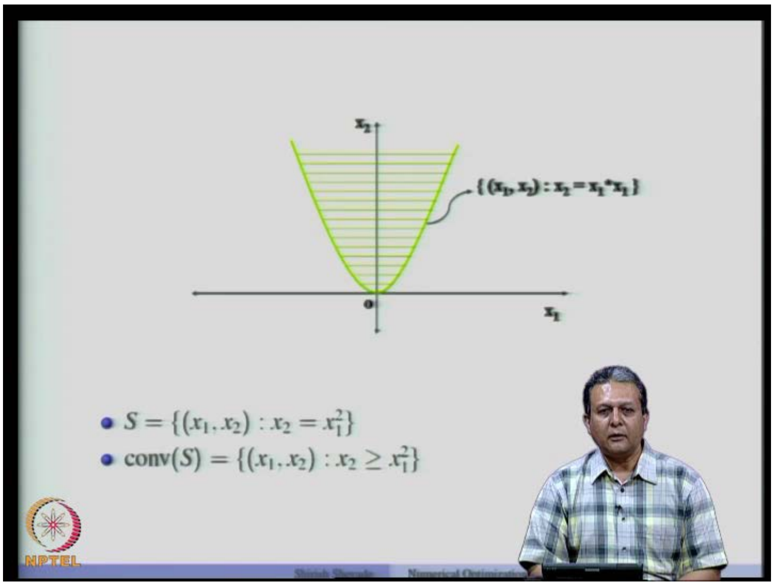

So, remember that the points in the interior, they are not used in forming the convex set or representing the boundary of the convex hull. So, here is one more example that we consider the set x 1, x 2, say that x 2 equal to a x 1 square. So, this was the given set and the convex hull of this set is the set of points on this curve, and the points above this curve. So, the shaded region here plus the points on the curve that together forms the convex hull of the set. So, this is basically the set x 1, x 1 such that x 2 greater than or equal to x 1 square. So, we will stop here and in the next lecture, we will continue with the convex sets and some more results related to the convex sets.

Thank you.