Numerical Optimization
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Lecture - 36
Interior Point Methods - Affine Scaling Method

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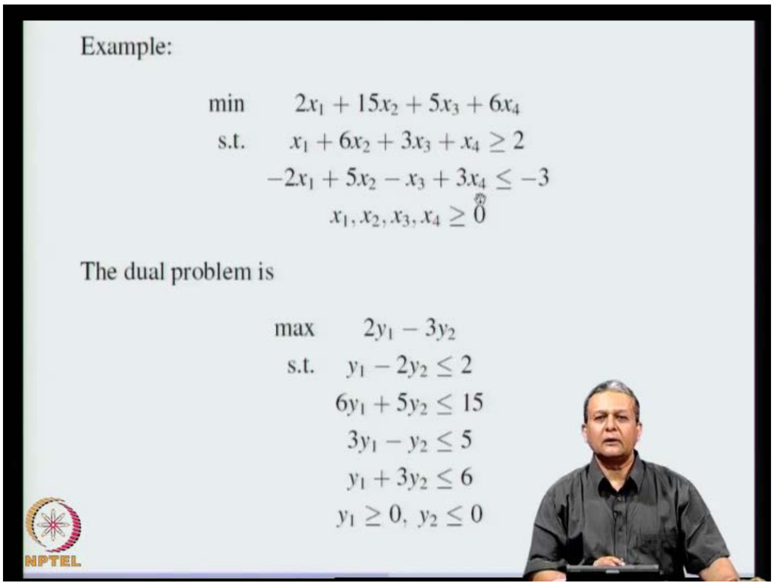

Welcome back. So, in the last class, we started discussing about duality and we are looking at this example, where we want to minimize this objective function subject to the constraints, and this example was chosen to illustrate the advantage of duality in the sense that this example has more number of variables than number of constraints. In particular, it has four variables and two constraints. So, if you write the dual, that becomes a two-dimensional optimization problem which is easy to solve graphically and we found out the solution of this dual problem using graphical method.

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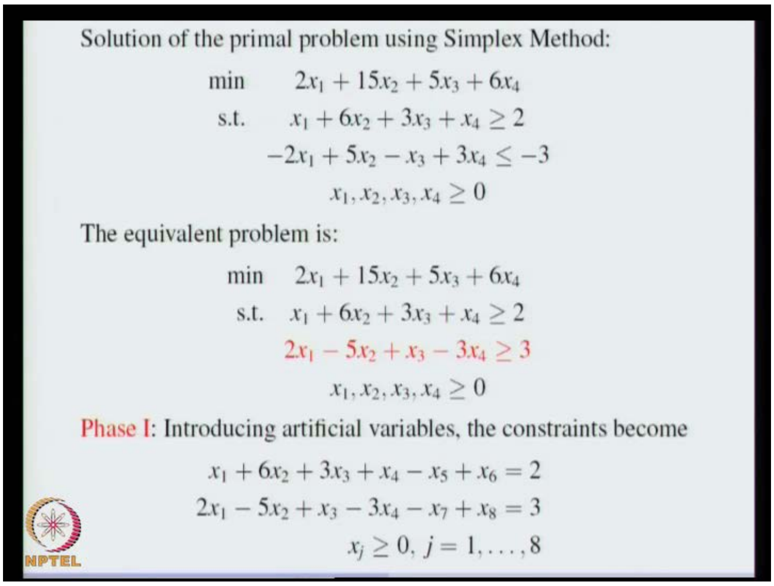

Now, this dual problem was obtained based on the relationship between the variables and constraints between the primal and dual methods, which we saw in the last class. Now, let us start looking at the primal problem. Now, this primal problem has a 1 constraint which has a negative value on the right hand side. So, to be consistent with our notations, we will convert this constraint to a form where the right hand side is positive. So, that is done by multiplying this constraint by minus 1 and now, we are in a position to start using our simplex method. Now, the basic feasible solution is not immediately obvious from these constraints. So, what we do is that we add the surplus variables or use the surplus variables and also add the artificial variables.

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Therefore, the artificial linear program is,

$$\min \quad x_6 + x_8$$

$$\text{s.t.} \quad x_1 + 6x_2 + 3x_3 + x_4 - x_5 + x_6 = 2$$

$$2x_1 - 5x_2 + x_3 - 3x_4 - x_7 + x_8 = 3$$

$$x_j \geq 0, \; j = 1, \ldots, 8$$

Initial Tableau:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS
1	6	3	1	-1	1	0	0	2
2	-5	1	-3	0	0	-1	1	3
0	0	0	0	0	1	0	1	0

Making the relative costs of basic variables 0,

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS
1	6	3	1	-1	1	0	0	2
2	-5	1	-3	0	0	-1	1	3
-3	-1	-4	2	1	0	1	0	-5

So, the surplus variables are x 5 and x 7 and the artificial variables associated with the two constraints are x 6 and x 8 and associated with artificial identity matrix in the constraint, we make use of that to start the phase one of our simplex method. So, the initial tableau will look like this, where we have x 6 and x 8 as the basic variables and then the corresponding columns denote the vectors associated with the identity matrix and then the initial basic feasible solution for this artificial linear program is x 6 equal to 2 and x 8 equal to 3 and we want to minimize x 6 and x 8.

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Using Simplex Method, final tableau for the artificial linear program:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	RHS
0	$\frac{17}{5}$	1	1	$-\frac{2}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	0	$\frac{1}{5}$
1	$-\frac{21}{5}$	0	-2	$\frac{1}{5}$	$-\frac{1}{5}$	$-\frac{3}{5}$	0	$\frac{7}{5}$
0	0	0	0	0	1	0	0	0

Basic variables for the original program: $x_1 = \frac{7}{5}, x_3 = \frac{1}{5}$

Initial Tableau (for the original program):

x_1	x_2	x_3	x_4	x_5	x_7	RHS
0	$\frac{17}{5}$	1	1	$-\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
1	$-\frac{21}{5}$	0	-2	$\frac{1}{5}$	$-\frac{3}{5}$	$\frac{7}{5}$
2	15	5	6	0	0	0

When we discuss the theory of two-phase simplex method, we saw that if the solution of this program, if the optimal objective value of this function is 0, then the original linear program has a basic feasible solution. Otherwise, it does not have a basic feasible solution. So, the first step was to make the relative cost of the basic vector variables by doing the matrix manipulations and then we can get started with our simplex method.

So, after the simplex iterations, the final tableau that one get is shown here where the variable x 3 has a value 1 by 5 and the variable x 1 has a value 7 by 5. Now, we use this solution because the optimal objective functional value for artificial linear program is 0. So, we use this basic feasible solution as the initial basic feasible solution for our linear program and these are our basic variables and in initial tableau for the original program with all the cost considered would look like this. Remember we had that 2 x 1 plus 15 x 2 plus 5 x 3 plus 6 x 4 in the objective function of the original linear program.

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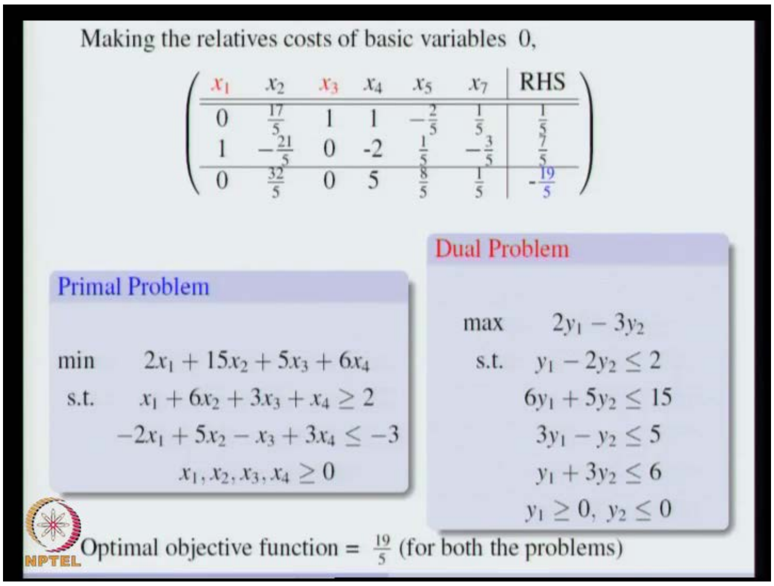

Now, if you see the relative cost of the basic variables here, they are non-zero here. So, we first make those relative costs 0 by multiplying the first row by minus 5, second row by minus 2 and then adding that to third row. So, making the relative costs of the basic variables 0, what we get is the following that the relative costs of all the variables are non-negative and in particular x 3 equal to 1 by 5 and x 1 equal to 7 by 5. The relative x 2, x 4, x 5 and x n which are non-basic variables are in fact strictly positive. Therefore, the current point is the optimal point for the original linear program and the optimal solution or optimal objective functional value is 19 by 5.

If you recall, this is the same objective functional value that we obtained for the dual program. So, this is the case where both primal and dual have optimal solutions and they are equal. So, we had this primal problem and the corresponding dual we saw in the last class, and we also found out the solution for this dual problem using a graphical method and as we saw that the optimal objective functional value is 19 by 5 for both problems. So, the important thing to notice is that at optimality, both primal and the dual linear programs in this case have the same solution.

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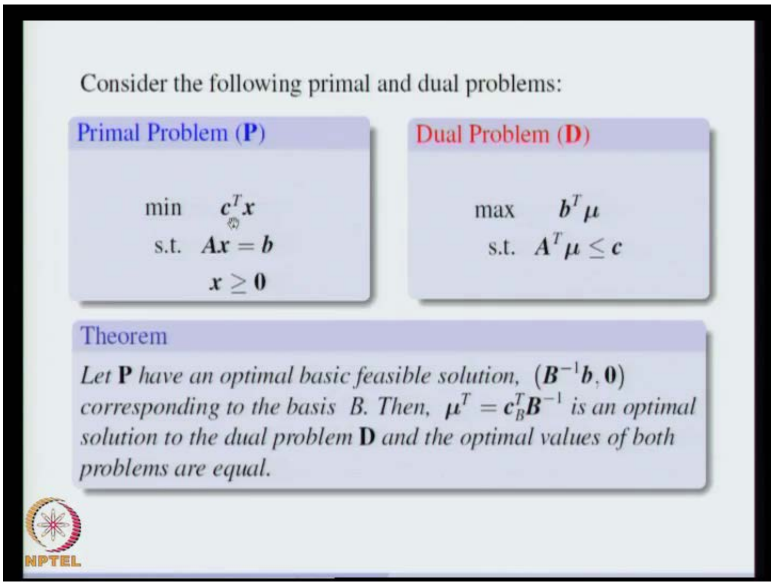

Now, let us again look at the primal dual relationship. We have the standard linear program on the left hand side and then the corresponding dual program on the right side. Now, given the optimal solution of this primal problem is there a way to get an optimal solution of the dual problem and we use simplex method that we have seen earlier to find out the optimal solution to the dual problem. So, this is going to be the next topic of our discussion. So, here is the theorem which relates the solution of the primal and the dual problem. So, suppose the problem P has an optimal basic feasible solution and since, it is basic feasible solution we have set of basis vectors and set of non-basic vectors and associated with the basis vectors is the matrix B and the basic feasible solution, if it is optimal, let us assume that it is B inverse b, 0.

So, B inverse b uses the basic vector x b at the optimal solution and 0 denotes the non-basic vector and this is associated with the basis B. Then the claim is that the vector mu whose transpose is C B, transpose B inverse is an optimal solution to the dual problem. Note that A is partitioned into two matrices B and N and similarly, the cost vector C is also partitioned into two matrices, two vectors C B and C N and C B along with the B inverse gives us the optimal solution to the dual and the optimal value of both the problems are equal. That is true because of strong duality theorem because in this case, we assume that both the solutions exist.

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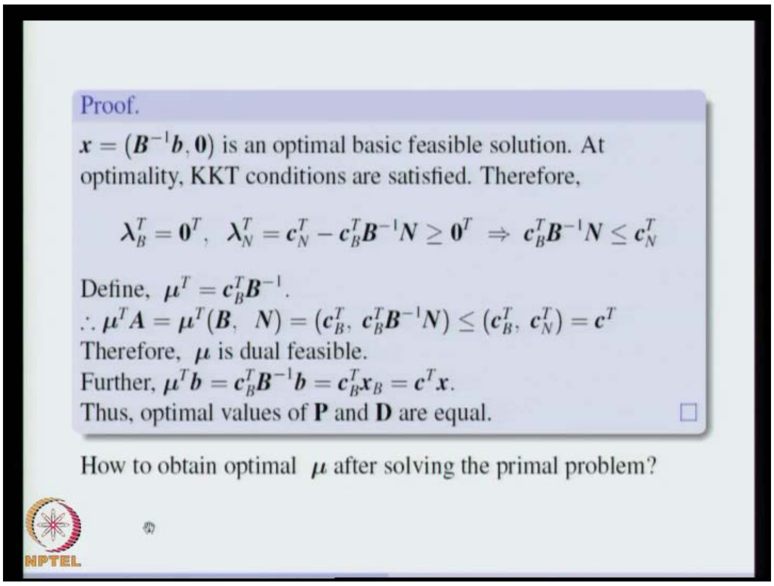

Now, let us see the proof of this theorem. Now, to see the proof, first we need to ensure that mu obtained this way is indeed a feasible mu for this problem that is a transpose mu, where mu transpose is C B transpose B inverse should be less than or equal to C. So, that is the first part and then the second part we show that if we use this mu, then transpose mu will be equal to C transpose X at optimality. So, let us show that X is an optimal basic feasible solution which is given to us. The X B component of X is B inverse b and the X N component of X is 0.

Now, we know that the KKT conditions are necessary and sufficient for linear programs. We assume that the Slater's conditions are satisfied and therefore, the (()) multipliers associated with the inequality constraints in the primal problem which we are going to denote by lambda, we have already seen that. Because of the complimentary slackness condition, if X B is greater than 0, remember that we are not talking about degenerate case. Instead, we are talking about a non-degenerate case. So, X B will be greater than 0. Therefore, because of the complimentary slackness condition, lambda B has to be 0 because lambda A i X i is equal to 0 for all variables i.

So, if X i is greater than 0 which is the case for X B, the corresponding component for lambda has to be 0 and for the non-basic variables, lambda N's are non-negative. This is from the KKT condition. So, given this solution, we know that these two conditions should hold. Now, we make use of these conditions to write that C B transpose B inverse B is greater is less than or equal to C N transpose and we make use of this fact to write to check whether a transpose mu is less than or equal to c because then we can say that mu is feasible. So, let us use the value of mu which is defined as mu transpose is equal to C B transpose b inverse A. Therefore, mu transpose A is nothing but mu transpose B and N, where A is partitioned into two matrices associated with the basic and the non-basic variables.

Now, expanding further and using the fact that mu transpose is equal to C B transpose B inverse, so if we substitute mu to be C B transpose B inverse here, so the first quantity will be C B transpose and the second quantity will be C B transpose B inverse N. Now, we have already seen that C B transpose B inverse N less than or equal to C N transpose. So, this quantity is less than or equal to C B transpose C N transpose and C B transpose. C N transpose is nothing but the transpose of the objective function cost vector which is C transpose. Therefore, mu transpose A is less than or equal to C transpose or A transpose mu less than or equal to C, means that mu is feasible to the dual problem. So, the given mu which is defined in this way is feasible to the dual problem at optimality. So, that is very important.

Now, at optimality what happens is mu transpose B can be written as C B transpose B inverse B and B inverse B is nothing but X B which is the optimal basic feasible solution for the linear primal problem. So, this will be nothing but C B transpose X B and we add C N transpose X N to this. The right hand side does not change because X N is 0 as we have seen here. So, C B, the right side we have C B transpose X B plus C N transpose X N which is nothing but C N transpose X and therefore, if we get optimal feasible solution which is X equal to B inverse B and 0, that is X B is equal to B inverse B and X N equal to 0 and if we use mu transpose to be C B transpose B inverse, then we have shown that mu is dual feasible. That means, it satisfies the feasibility conditions of the dual and moreover, the way we have found mu, mu transpose B will be equal to C transpose X. So, this is nothing but the strong duality theorem.

Now, the next question is how do we get the mu after solving the primal problem? Suppose, we have solved the primal and obtained this and here is the theorem which showed that if we substitute mu transpose to be C B transpose B inverse, we get the optimal dual objective function value, but from the simplex tableau, how do we get this C B transpose B inverse without having to invert the matrix B directly. So, let us now see that.

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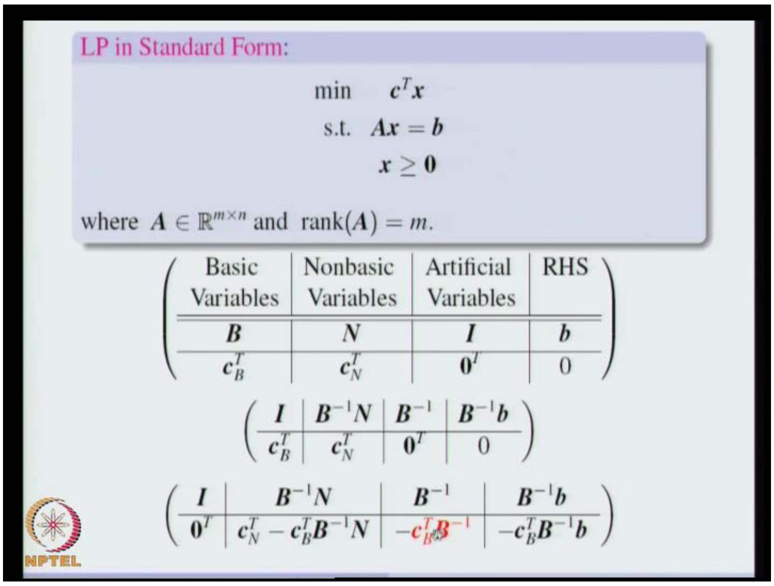

So, here is the linear program in standard form, where A is the m by n matrix and rank of A is m. Now, as we saw earlier, there is A matrix B associated with the basic variables and matrix N associated with the non-basic variables and B and N together form the matrix A. Now, let us assume that the identity matrix is not obvious from the constraints Ax equal to b. So, we introduce artificial variables. So, these artificial variables associated with them in the identity matrix in the constraint and then we have the right side.

Now, corresponding to the cost function, we have C B transpose associated with basic variables, C N transpose associated with non-basic variables and zero vector associated with artificial variables, and this last cell in this matrix is 0. Now, by doing the usual matrix transformations, for example, multiplying the first m rows by B inverse, so we get A identity matrix here, B inverse N here, B inverse appears here and B inverse B appears here and then subtracting C B transpose of the first m rows from the last row. So, we get the tableau which is like this. So, this first tableau is obtained by simply multiplying B inverse throughout the m rows. Now, we do the second operation of subtracting C B transpose. The m rows from the last row and what we get is something like this. So, here you will see that C N transpose minus C B transpose B inverse N corresponds to lambda N transpose B inverse B is the current basic feasible solution, C B transpose B inverse B is the current objective function value that we have seen earlier. Now, we have B inverse matrix which is directly available here.

So, as a part of simplex method at the solution at optimality, we will get B inverse which is the matrix below the artificial variables and we have C B transpose B inverse component whose negative value appears in this part of the row, this part of the last row associated with artificial variables. So, suppose we have solved the original problem using simplex method by introducing artificial variables, and at optimality assuming that the optimal solution exist C N transpose minus C B transpose B inverse N greater than or equal to 0, then this matrix gives us all the information that we need. We get B inverse b which is the basic feasible solution, X B the current optimal cost at A, at optimality is C B transpose B inverse b, then the columns associated with artificial variables. If you look at those columns in the final tableau, we will get the B inverse matrix and below the B inverse matrix is the negative, the optimal dual variables will appear.

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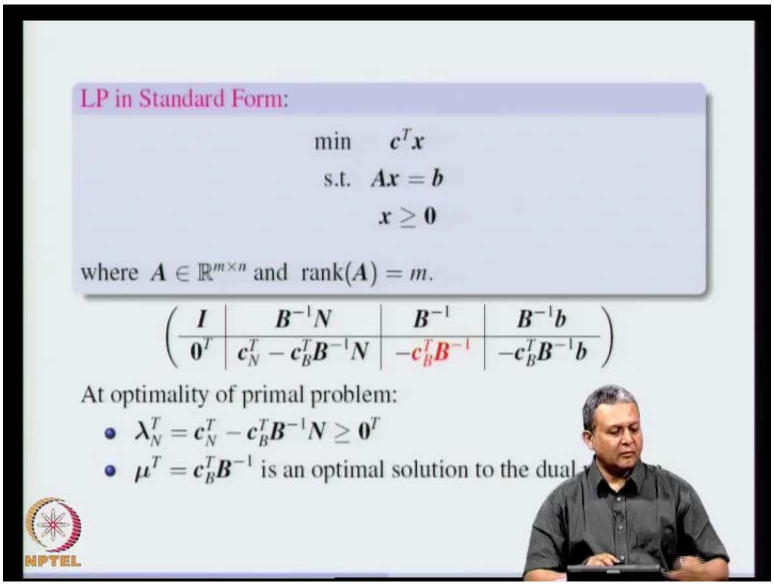

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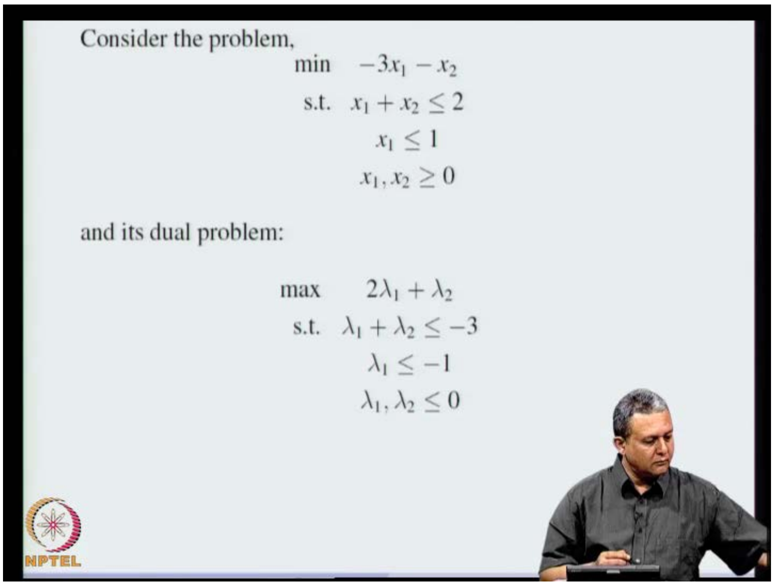

So, C B transpose B inverse can be easily read from the simplex tableau. Therefore, at optimality lambda N is non-negative and mu transpose is C B transpose B inverse b. This is shown here. The negative of entry here is an optimal solution to the dual problem. So, it is possible to get optimal solution through a dual problem by using simplex method to solve the primal problem. Now, let us see an example. This example we have already seen and so I will not repeat the solution of this example, but if you look at the dual problem, the dual problem is maximize 2 lambda 1 plus lambda 2 subject to lambda 1 plus lambda 2 less than or equal to minus 3 and lambda 1 less than or equal to minus 1.

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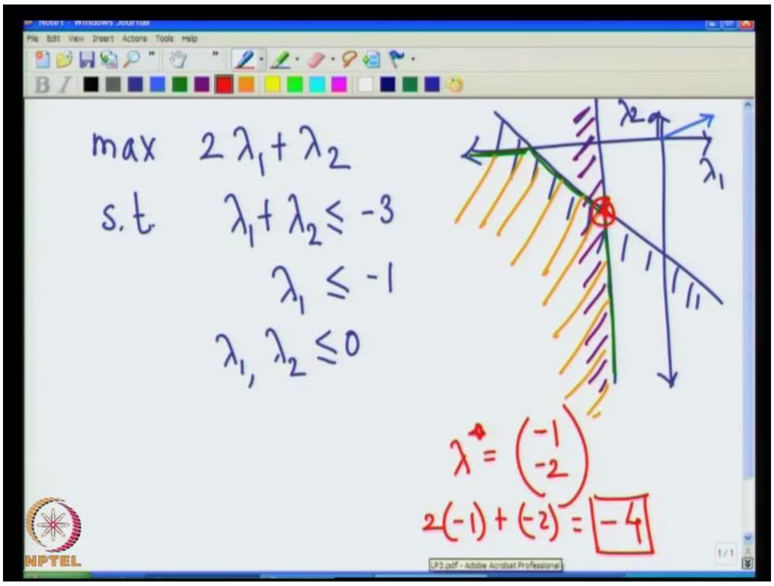

So, we have the dual problem which is maximize 2 lambda 1 plus lambda 2 subject to the constraint lambda 1 plus lambda 2 less than or equal to minus 3 lambda 1 less than or equal to minus 1 and lambda 1 lambda 2 less than or equal to 0. Now, if we solve this problem graphically, so lambda 1 and lambda 2 are non-positive. So, that means that they lie in the third quadrant. So, we have lambda 1 and lambda 2 and we are interested in the third quadrant. So, lambda 1 plus lambda 2 less than or equal to minus 3 is the region shown here and lambda 1 less than or equal to minus 1. So, it is the region shown here.

Now, if you take the intersection of these two, so the feasible region will be like this. Now, we want to maximize 2 lambda 1 pus lambda 2. So, if you take the vector 2 lambda 1 plus lambda 2, so it will be in this direction and the maximum will occur at this point and this point. So, X star, this point will be minus 1 and it is in intersection with lambda 1 plus lambda 2 equal to minus 3 and that is minus 2. What is the objective function value? So, c transpose, so we have 2 into minus 1 plus minus 2 which is equal to minus 4. So, this is the optimal objective function value for this problem. Now, remember that the value that the optimal value which is lambda star, not x star. It is minus 1 minus 2. So, lambda 1 star is minus 1 and lambda 2 star is minus 2 and the optimal objective function value is minus 4 and if you recall that we got the same objective function value when we solved the corresponding primal problem earlier.

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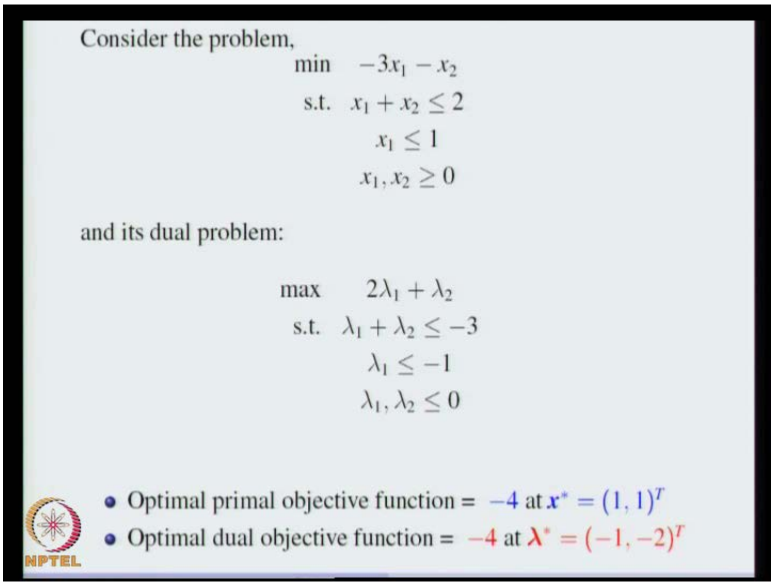

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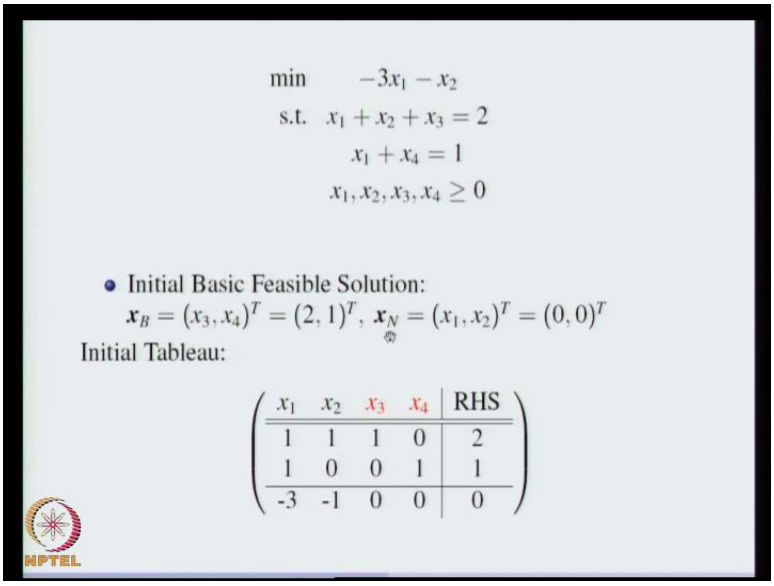

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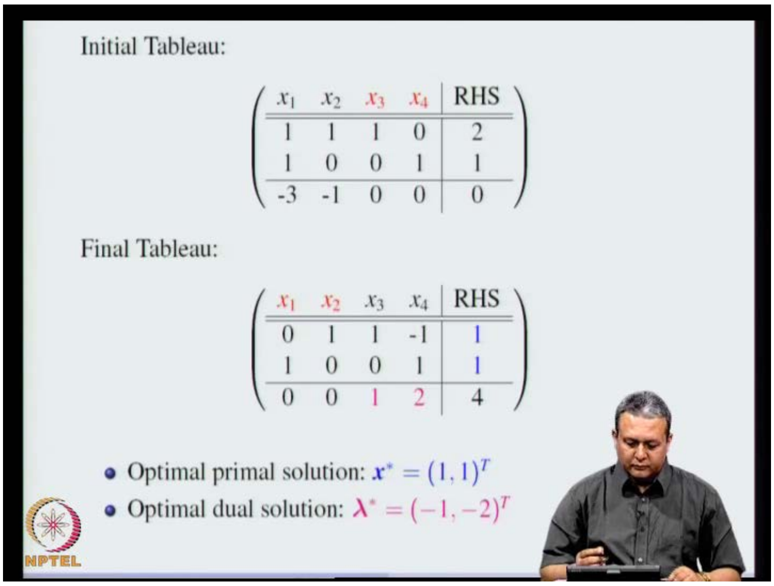

So, optimal primal objective function value that we saw earlier was minus 4 and by solving this problem graphically that the dual problem also has the optimal objective function value to be minus 4. Now, let us solve this problem using the simplex tableau. So, by introducing artificial variables by introducing slack variables, we get x 3 and x 4 to be the initial basic feasible variables and x 1 x 2 as the non-basic variables and this is the initial tableau and then the final tableau would look like this. Now, the optimal primal solution is x 2 is equal to 1 and x 2 is equal to 1. The current objective function value which is also optimal is minus 4 and if you look at the columns associated with slack variables, you will see that the last row contains the negative of the lambdas.

So, if you recall that the lambda 1 star was minus 1 and lambda 2 star was minus 2, so the negative of those values appear here. So, from the simplex tableau, we are able to find out the optimal primal solution as well as the optimal dual solution. So, simplex method is thus very useful in getting lot of important information about the problem. For example, we can get the B inverse b which is the X B. The current basic feasible solution C B transpose B inverse b which gives us the current objective function value, then the relative cost or the lagrangian multipliers associated with the non-basic variables can be used to check whether optimality is reached and then the columns associated with the artificial variables, they give us the B inverse at optimality.

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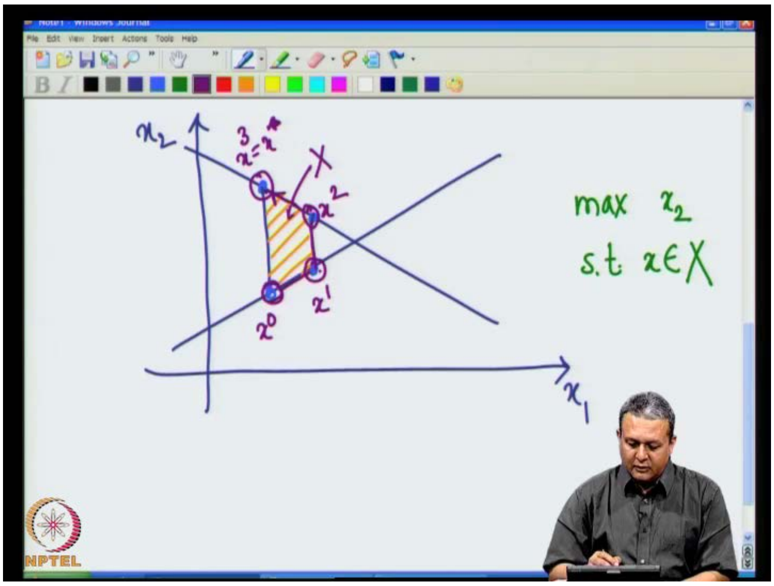

So, at optimality there is basic variable associated with the solution and the corresponding B inverse is easily available and also, the optimal dual objective. Optimal dual variables can be obtained using the simplex tableau. So, lot of information can be obtained using simplex algorithm by solving a given problem. So, simplex method became very popular because of its nice structure and also simplicity to use. Now, let us consider an example. Suppose, we want to solve a problem and then the constraint set is shown by the shaded region here and the vertices of this constraint set are given here. So, this is a two-dimensional problem. Let us call this region as x. So, suppose this is a feasible region for a program. The given program is the following. So, we want to maximize x 2 subject to the vector x belongs to the set x.

Now, we have already seen that the solution to a linear program lies at the extreme point if the solution exists. So, these four extreme points are standard for the solution. Now, suppose we start with this point as our initial point. So, let us call this as x 0. Now, there are two neighboring points for this vertex. Suppose, the simplex algorithm chooses to move along this and go to the point x 1. We have already seen that when simplex method moves from one vertex to another, if the solutions are non-degenerate, then there exists the objective function value certainly decreases. So, since we have moved from this to this point, the objective function value has improved.

In this case, we are talking about maximization problem which can be written as the minimization problem. So, in going from x 0 to x 1, the objective function has moved, that is the b value of x 2 or the second variable has increased. Now, from x 1, there is only way to go to improve the objective function value which is here. So, this is the point x 2 and from x 2, again there is only one way in this case to improve the objective function, and it is to move along this direction and this x 3 which is also equal to x star because as far as this feasible region is concerned, this vertex has the maximum x 2 variable. So, the feasible region x in this case has four vertices and simplex method has to traverse through all the four vertices to reach the solution.

So, this example can be extended to a high dimensional case and it is possible to construct examples, where starting for a given problem starting from a given point, it is possible for simplex method to obtain the solution only after visiting all the vertices. So, in this case, there are four possible candidate solutions and if you started from here and followed this path, we had to visit all the vertices to get the solution. So, if the numbers of vertices are very large which is typically the case in many practical problems, there may be exponentially large in number. So, simplex method may have to traverse through exponentially large number of vertices to get the optimal solution and therefore, simplex method is not a polynomial time algorithm for linear programs because one can always construct some examples, where it would require to visit the exponential number of vertices before reaching the solution.

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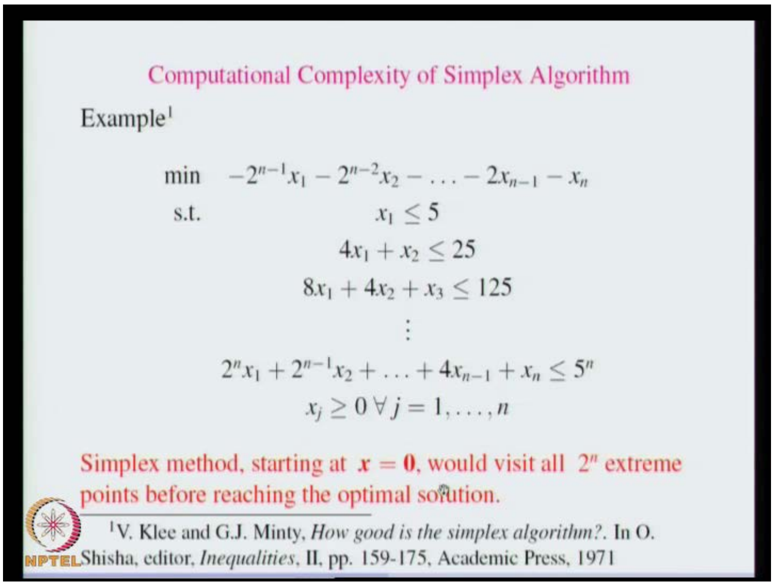

So, in 1972, Klee and Minty gave an example where they showed that in n dimensional space, simplex method will have to visit all the vertices of a feasible region before reaching the solution. So, let us start discussing about that. So, as I mentioned that in 1971, Klee and Minty wrote a paper on the goodness of the simplex algorithm. So, the title of the paper is how good is the simplex algorithm and in particular, they gave this example to minimize an optimization, a linear optimization function subject to the constraints and this showed that for this example if we start from x equal to 0, then simplex method would have to visit all the vertices before he finds out the optimal solution.

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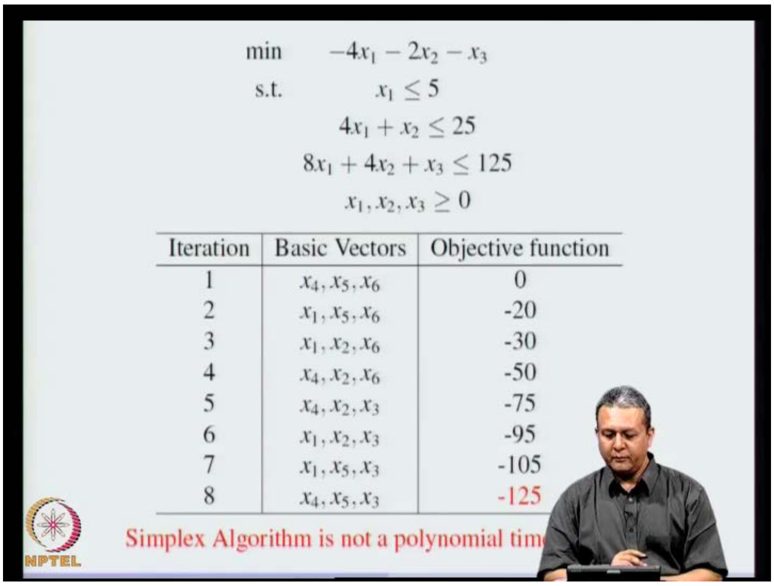

So, it would visit all 2 the power n extreme points. So, this is the problem in n variables and n constraints, but it has 2 to the power n extreme points and if you start from x equal to 0 and use simplex method, it will have to visit all the extreme points before it finds the optimal solution. So, let us take a simple case of that example with n equal to 3. So, the same example which Klee and Minty gave is given here. In case of n equal to 3, so we have three variables and eight vertices for the feasible region and if we write down the simplex initial tableau for this by introducing the slack variables x 4, x 5 and x 6, then initially the slack variables are the basic vectors.

So, x 4, x 5, x 6 are the basic vectors and since, x 1, x 2, x 3 are 0, the objective function value is 0. So, in the next iteration by using simplex method, by bringing out the basic variable x 4 and by bringing in the non-basic variable x 1, the objective function value is minus 20 and so on and so forth. You will see that at the end of eight iterations that means after visiting all the eight vertices, simplex method gives us the optimal solution which is x 4, x 5 and x 3 which has x 4, x 5 and x 3 as the basic vectors and the optimal objective function value is minus 25. So, this example clearly shows that simplex method or simplex algorithm is not a polynomial type algorithm and because this simplex method cannot be used for large scale linear programs because if the initial point is such that the method will have to visit all the possible vertices of the feasible region before it enters the optimal basis or it finds out the optimal basic feasible solution.

So, this non polynomial time complexity of the simplex algorithm, lot of researchers got interested in solving linear programs in different ways. So, the problem with simplex method is that every time it moves from one vertex to the neighboring vertex so as to optimize the objective or so as to improve the objective function. So, the idea which became very popular among the researchers was not to move from one vertex to another, but rather start from some point which is in the interior of the feasible region, and every time make sure that one does not cross the boundary of the feasible region. So, every time one stays in the interior of the feasible region, then as the iterations progress, the algorithm would finally converge to the optimal solution which is a vertex.

Now, once when one starts talking about the points which are in the interior of the feasible region, so locally the problem would look an unconstraint problem. So, many non-linear programming techniques can be used to solve simplex method because when one is in the interior region, it is easy o use the non-linear programming ideas and that was the motivation for developing the interior point methods for linear programs. So, the earlier works in this direction was done by Kutchian in 70's and then there was a work by Karmakar who developed a polynomial time algorithm for solving simplex, for solving linear programs and there was an extension of that which is called the affine scaling method which also became very popular.

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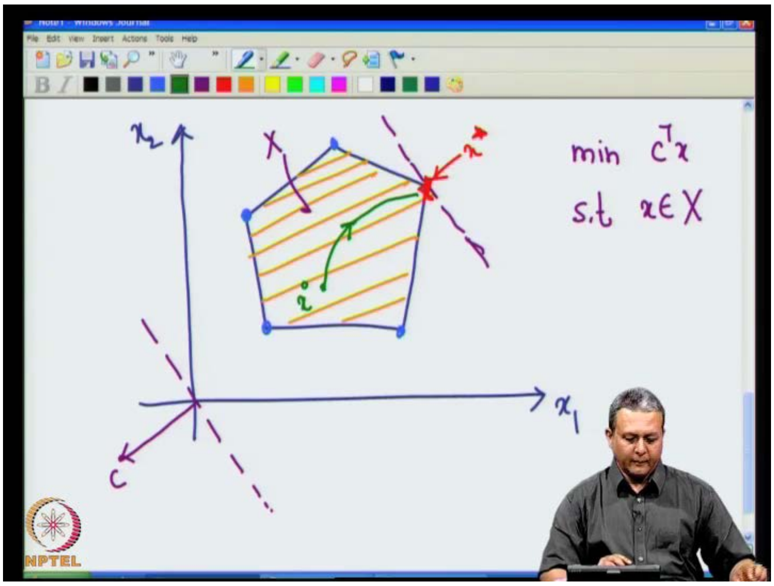

So, next we are going to discuss interior point methods for linear programming. So, as the name suggests, these methods rely on generating a sequence of points which are in the interior of the feasible region. So, rather than restricting oneself only to the vertices, these methods restrict themselves to the interior of the feasible region. So, let us see an example. Suppose that…

Suppose that the feasible region is shown here and let us assume that the vector c is pointing in this direction, and this is the feasible region. Now, this feasible region has these vertices and if you take a hyper-plane which is perpendicular to c or the hyper-plane for which c is normal vector, then we are interested in minimizing c transpose x subject to x belongs to x. So, minimization of c as we saw earlier will occur in this direction. So, we will take this parallel hyper-plane to this hyper-plane which supports this set x from below. So, it will be and therefore, this will be the solution point. So, this will be x star. Now, the interior point methods they work in the following fashion. So, one starts from a point which is say in the interior of the set. So, let us call this as our initial point and then using non-linear programming ideas, one can move to a new point. Then the new point is also, it is also ensured that the new point also lies in the interior of the feasible region.

So, if one traces the path generated by such methods, it may look so since at any point of time, the point generated by the inter point methods has to be in the interior. One may get very close to the solution, but not find the exact solution, but to given accuracy, one can reach close to the solution x star. So, this will be the path. This could be one of the paths generated by interior point methods for linear programs. So, you will see that none of these points which are part of this method is outside the feasible region. So, it is very important to make sure that the points generated by the algorithm are always in the interior of the feasible region.

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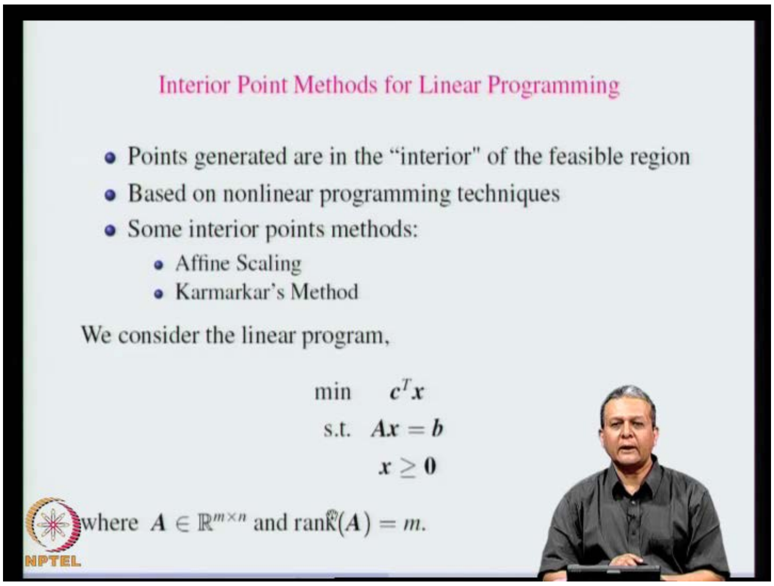

So, this is the characteristic of the interior point method that the point generated are in the interior of the feasible region and the ideas for this interior point methods are based on non-linear programming techniques. So, the steepest decent method can be applied or can be modified to solve linear program. So, when the objective function is linear, steepest decent method is a popular choice as far as use of non-linear programming technique is for solving linear programs is concerned. Now, Karmarkar's method was the first method which was developed in 1984 which gave the polynomial time algorithm for solving linear programs.

So, in that sense, this work was the first work to make to find the solutions of linear programs in polynomial time. Now, affine scaling method which was developed sometime in 86 used some of the ideas proposed by Karmarkar and made some simple modifications and that method came later. Of course, there were some other methods like path following algorithms or potential function based approaches. We will not go in to the details of those approaches. So, in this course, we will concentrate mainly on Karmarkar's method and affine scaling method.

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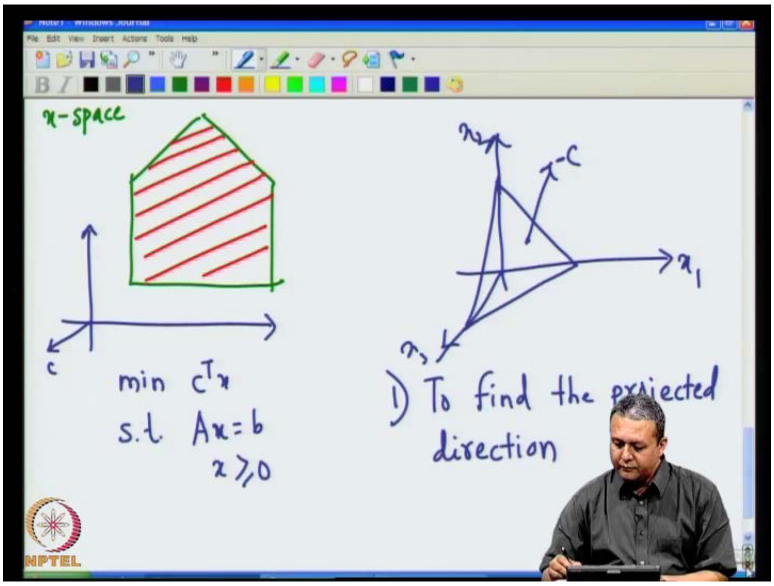

So, we will first start discussing about affine scaling method and then move on to Karmarkar's method. Although Karmarkar's method was invented first and then the affine scaling method. So, we continue working with standard linear program of this type and we assume that the rank of the matrix A is m and A is m by matrix. So, it is a full row rank matrix. Now, let us see the idea of affine scaling. So, let us take a simple example. So, let us assume that this is our feasible region in the input space. So, let us call this as x space and this is the feasible region. Now, let us assume that this vector c is this direction and we want to minimize c transpose x. So, minimize c transpose x subject to Ax equal to b. So, this feasible region is chosen just to illustrate the ideas.

Now, one important point is that suppose if you want to use the steepest decent direction, so if you take the gradient of this objective function which is minus c, which is c and take the negative gradient which is minus c. So, sum of given feasible point, one needs to move along the direction of steepest decent, but this steepest decent direction may not always lie in the feasible region. So, what we need to do is that we want to find out the direction which is projection of steepest decent direction on the feasible region. For example, suppose the feasible set in the three-dimensional space is like this. So, this is the intersection of the hyper-plane with first (( )).

Now, if suppose this is the feasible region and then the current point is here and then the negative gradient direction minus c is in this direction. Now, the moment we start moving along the steepest decent direction, we will move away from the feasible set. So, what we need to do is that we need to project this steepest decent direction on to the feasible region. So, the first step of all this interior point methods is to project the required direction on to the feasible set, so that we can make a moment in the feasible region and this is necessary, because if you start moving along the required direction, we may leave the feasible region.

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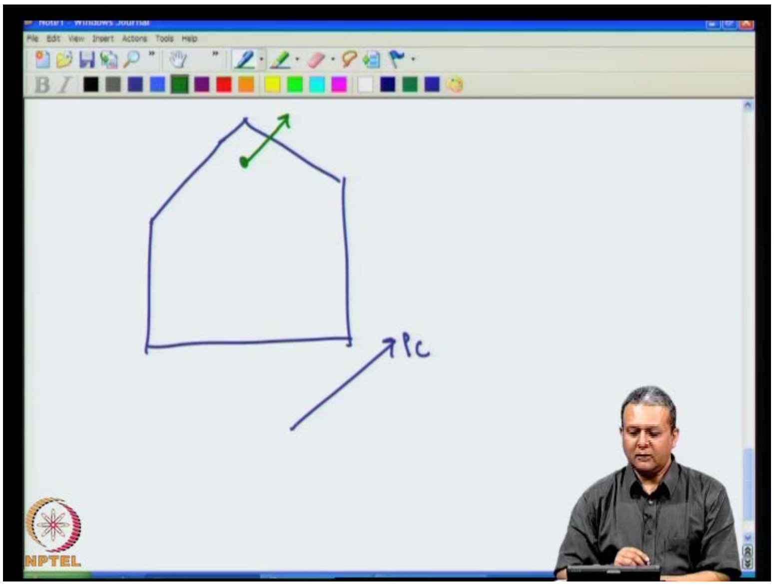

So, the first step is to find the projected direction. So, in particular, it could be projected steepest decent direction. Now, the second point which is again important is that suppose we consider the feasible region that we had. Now, suppose that the projected direction; let us call that projected direction as Pc, the projected direction of the cost vector on to the sub-space. That means, we need to make a moment from the current point along the direction Pc. Now, let us consider a point which is here and suppose, this is our current point and we make a moment along Pc. So, we need to move along this direction which is parallel to Pc.

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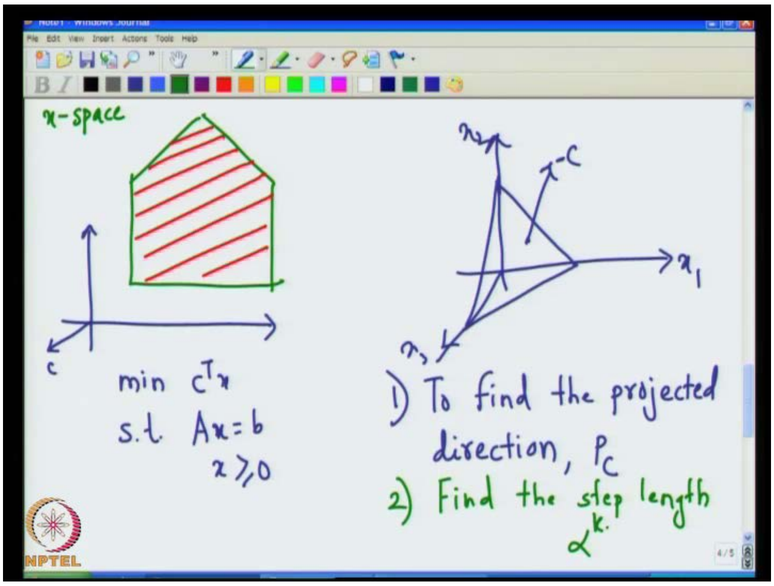

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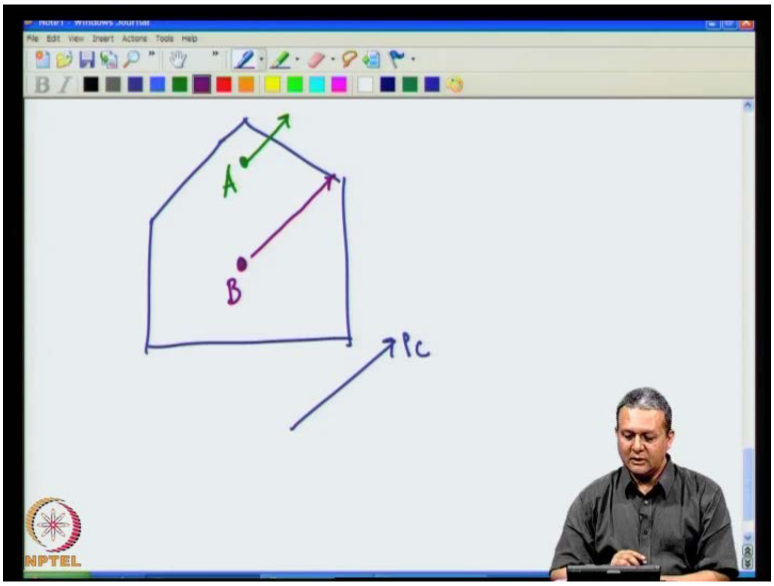

Now, at some point of time, we will leave the feasible region. Therefore, we have to make sure that we do not leave the feasible region because otherwise the constraints will be violated. So, like a non-linear program, we also need to find out the step length. So, find the projected direction Pc and the second point is, find the step length. So, in our earlier nomenclature, this was called alpha k. So, the step length is chosen, so that the feasibility is ensured. Now, suppose the initial point was somewhere here and if we move along the direction Pc, between these two points you would see that starting from this, there could be a significant improvement in the objective function if we move along the direction Pc compared to starting with the point which is shown here.

So, if we call this point as A, and this point as B, so as far as making a considerable improvement in the objective function is concerned, B seems to be a better choice than A because from B, there is a significant amount of step which is taken, so that the objective function has improved reasonably. Therefore, given a direction along which to move it may be a good idea to start from a point which is close to the center of the feasible region. Now, Karmarkar gave a very noble solution to this problem. So, what he suggested was to use the transformation to convert the feasible region in to a new space, such that the feasible point when mapped to the new space lies close to the center of the feasible region.

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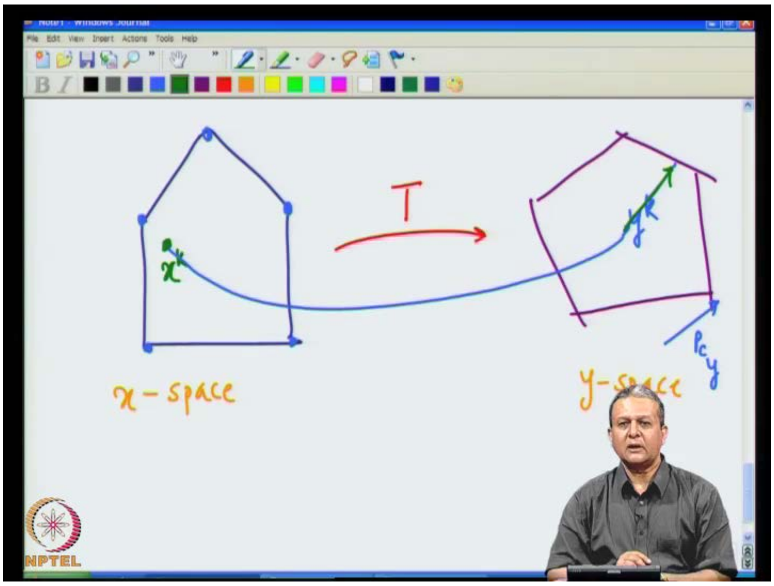

So, suppose that this is our initial feasible region and these are the vertices. Suppose that the current point is here, let us call it as x k, the current point which is in the interior of the set. So, what Karmarkar suggested was the following. He suggested that you use a transformation t to amp this feasible region in to some other region in the new space. So, let us call this as x space and let us call this as y space. So, the feasible region in the x space is transformed to some feasible region in the y space, such that this point, the point x k is mapped to a point y k in the new space, where y k is close to the center of the feasible region.

So, any point x k would be able to, would be able to be mapped to the new space, but in particular x k will be mapped to the point y k which is close to the center of the feasible region in the y space, and there if we now use the direction, let us call it Pc in the y space, then you will see that one can make a significant improvement in the objective function by moving along the direction Pc by starting from a point which is close to the center, and after having gone to this point, one needs to come back to the original space. Therefore, we need this transformation t to be invertible transformation. So, we will see more about this in the next class.

Thank you.