Numerical Optimization
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Lecture - 33
Optimality Conditions and Simplex Tableau

Hello, welcome back to this series of lectures on Numerical Optimization. In the last class we saw the equivalence between an extreme point of a feasible set, and the vertex basic feasible solution of a linear program. Now, in today's class we will discuss about the solution or an algorithm to get a solution of a linear program.

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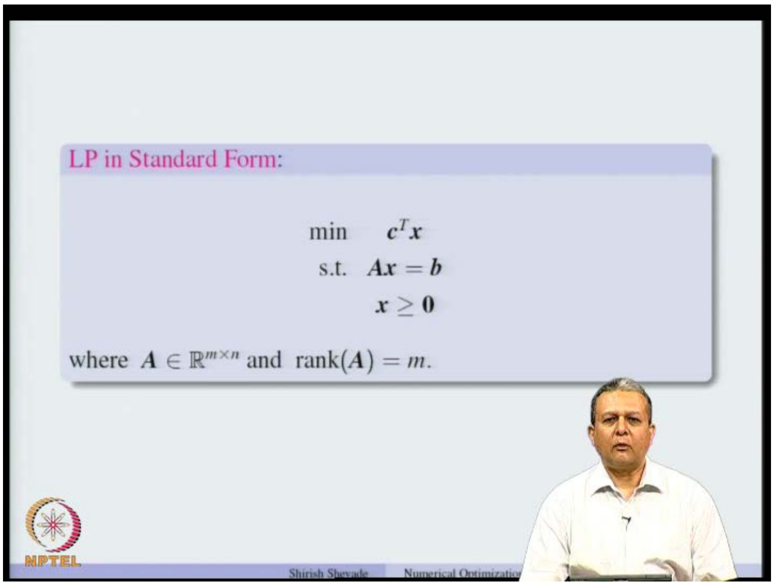

So, let us quickly relook at some of the topics that we discussed in the last class. So, we started with the linear programming standard form, which is of the type minimize c transpose x subject to x equal to b and x non negative, and A is rank m matrix of size m by n.

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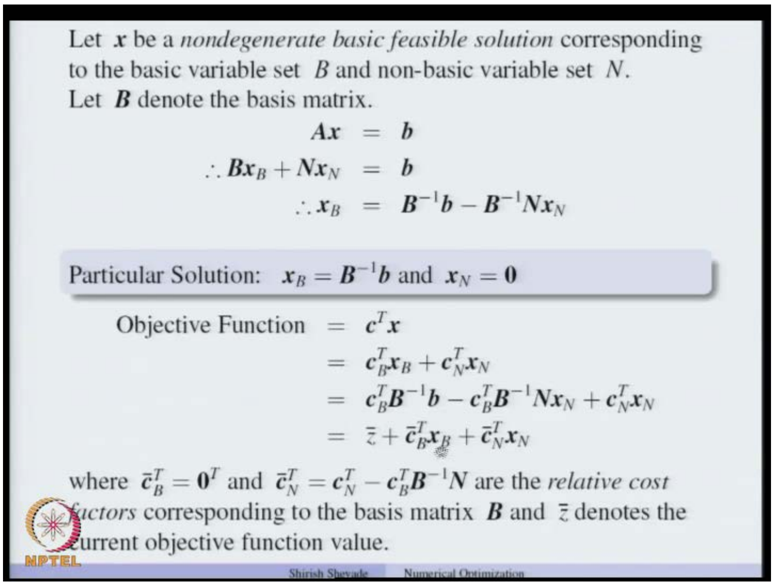

And we assume that x is a non degenerate basic feasible solution, which corresponds to the basic variable set B, and the non basic variable set N. And if B denotes the basis matrix, then A x equal to b can be written in the form B x B plus N x N equal to b. So, A is split in to two sub matrices B and N, and the corresponding components of x are x B and x N. And they put together will give us this equation B x B plus N x N equal to b, and the since b denotes the basis matrix that means, the columns of b are linearly independent.

And is of the size m by N, we can always invert it and get a solution x B which is of this form, x B equal to B inverse b minus B inverse x N, and a particular solution is piloting x N to be 0 in this solution, we get x B equal to B inverse b and x N equal to 0. And the objective function c transpose x can be written as the current objective function, which is c B transpose B inverse b, which we are going denote by z bar plus c B transpose x B plus c N bar transpose x N.

Now, c B bar is 0 and c N bar is nothing but c N transpose minus c B transpose inverse N, these are called the relative cost factors, associated with basis matrix B. And the current objective function value is c B transpose B inverse b, because c b bar is 0 and x N give 0. So, the current objective function is nothing but m z bar which is c B transpose inverse b.

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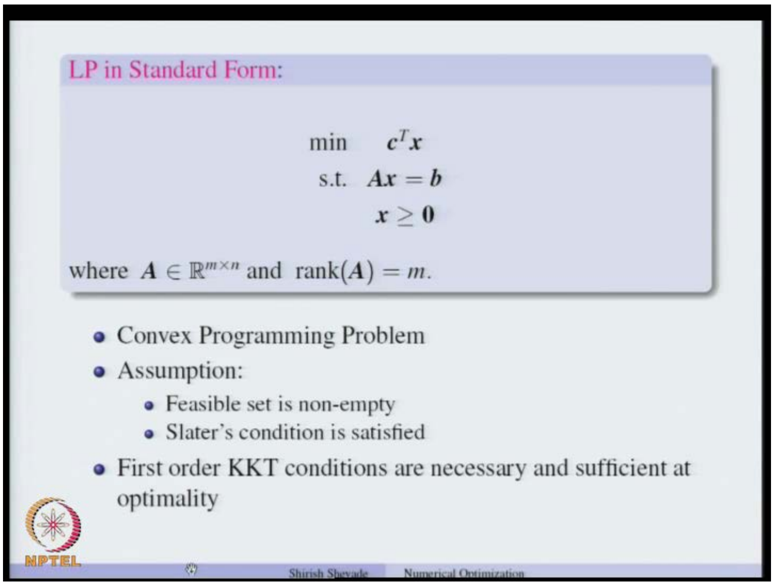

So, we also assume that the feasible set of this linear program is non empty, and also Slater's condition is satisfied that means, the feasible set has non empty interior. Then under this condition, we had already seen that first order KKT conditions, are necessary and sufficient at optimality.

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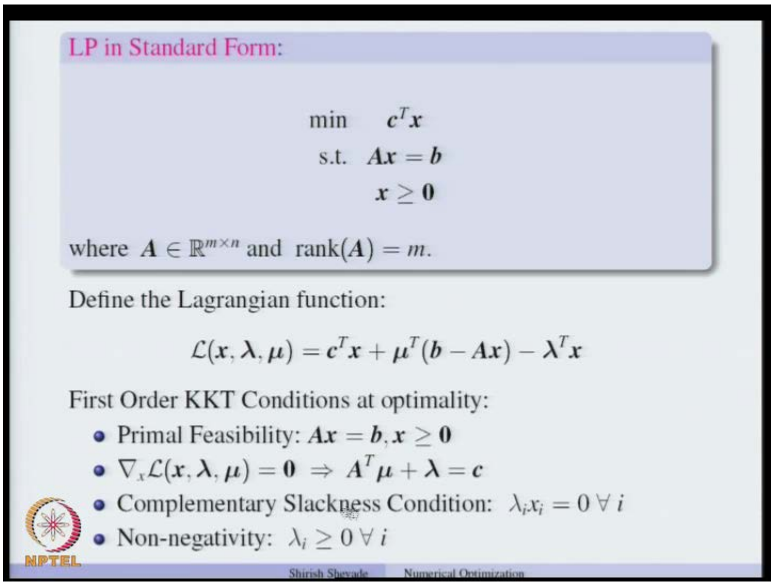

So, we write down the Lagrangian of this objective this optimization problem, which is nothing but the L of x lambda mu. Now, lambda's are the Lagrangian multipliers associated with the inequality constraints, and mu's are the Lagrangian multipliers corresponding to the equality constraints. So, the Lagrangian function is the objective function c transpose x plus mu transpose b minus A x minus lambda transpose x. Now, if you write down the KKT conditions, we will see that, the first order KKT conditions at optimality or that the primal x is primal feasible.

So, which means it should satisfy x equal to b and x non negative, then the gradient of the Lagrangian with respect to x should vanish at optimal at optimal point x lambda mu. So, with that implies that c minus A transpose mu by minus will be 0, so which gives us A transpose mu plus lambda equal to c, and complimentary slackness condition lambda i x i equal to 0 for all i. Now, there are also a non negativity constraint that is lambda ash non negative, for all ash. So, all these conditions should be satisfied by any optimal point x lambda mu. And in today's class, we will see how to get a solution of the linear program by making use of this optimality conditions.

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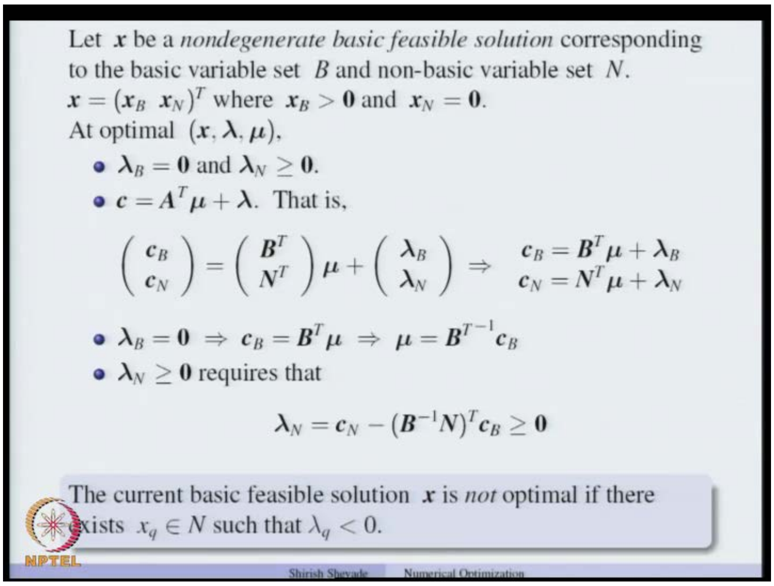

Now, if x is non degenerate basic feasible solution, so let us write x as x B and x N, as two parts of x, where x B is strictly greater than 0, and x N equal to 0. So, as we saw at optimal x lambda mu, since at any feasible x, basic feasible x, we are assuming that there is non-degenerate that means, that x B is greater than 0. So, if that particular x is optimal, then what we want is that the lambda B should be 0, because of the complimentary slackness condition; and since, x N is 0, lambda N can be greater than or equal to 0.

So, if we are given x, we will first find out the value of mu and see whether lambdas satisfies the KKT conditions, so if they do satisfy KKT conditions, then we can claim that we have found the solution. Now, remember that mu’s are the Lagrangian multipliers associated with equality constraints and therefore, they are unrestricted in sign. On the other hand, lambdas are the Lagrangian multipliers associated with the inequality constraints of the type x greater than or equal to 0.

And therefore, lambdas are restricted in sign, and we make use of that too check whether a given point x lambda mu is a optimal point. And therefore, we started with a basic feasible point or basic feasible solution which is x B x N, which if it is optimal lambda B has to be 0, and lambda N non negative. Now, c is equal to A transpose mu plus lambda, now we will make use of this equation to get mu or in other words, if we as we split x into two parts x B and x N, we split c also into two parts c B and c N.

And the A matrix we have already split it into two parts B and N, so that the equation c equal to A transpose mu plus lambda, can be written as c B equal to b transpose mu plus lambda B, and c N equal to N transpose mu plus lambda N. Now, now if lambda B is equal to 0 that means, that we can write c B is equal to b transpose mu and mu is equal to B transpose inverse c B, so that gives the value of mu. So, for a given x we have found out mu, we have also we also want lambda B to be 0 so that, KKT conditions would be satisfied.

And the m is to check whether lambda N is are 0, so by forcing lambda B to be 0, if we can get lambda N to be greater than or equal to 0, then the point x lambda mu would satisfy the optimality conditions. So, by forcing lambda B to be 0, we have got mu and now the next step is to get lambda N, now lambda N is nothing but c N minus N transpose mu, and if you plug in this value of mu what we get is that, lambda N is equal to c N minus N transpose c B. And that should be greater than or equal to 0, so which means that the the Lagrangian multipliers corresponding to the non basic variables N, they should be non negative.

So, if we look at of the steps that we followed, so we started with the basic feasible solution x which consisted of x B and x N, where x B is greater than 0, and x N equal to 0. Now, in order to check this solution is optimal, what we need to is that since x B is greater than 0, we can force lambda B to be 0, because complimentary slackness condition needs to be satisfied. And x N equal to 0 and therefore, lambda N had to be non negative and by using the fact that lambda B equal to 0 we got the value of mu, and we use that value of mu to be find lambda N.

And now it is just a matter of checking whether lambda N is greater than or equal to 0 for all non negative variables or non basic variables. So, the current basic feasible solution x is not optimal, if there exists a non basic variable say x q such that, the corresponding Lagrangian multiplier is less than 0 or strictly negative. So, in such a case we can definitely say that, the current solution current basic feasible solution is not optimal.

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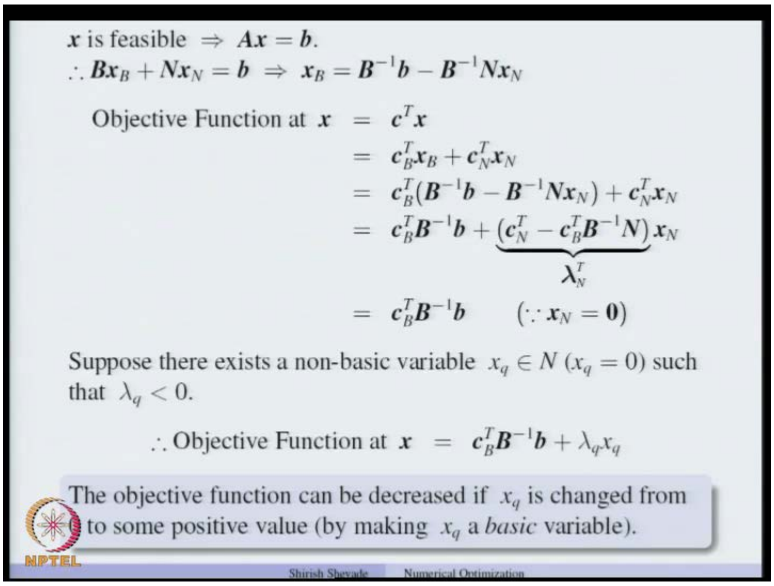

Now, there is another way to look at this, and that depends that justification is based on the cost of the objective function, so if you can show that by making one of the non basic variable, a basic variable we can improve the objective function. So, that means, that there is a possibility of reduction in the objective function value, which means that the current basic feasible solution is not an optimal solution, so let us see how to do that. So, x is feasible implies that x equal to b and x nonnegative, so by making use of the components of A as B and N, we can write this as B x B plus N x N equal to b.

And therefore, B is x B is equal to B inverse b minus B inverse N x N, we have already seen this earlier, now let us look at the objective function of at x which is c transpose x. Now, as the variables are split in to two components basic and non basic, the objective function vector c is also split in to two components c B and c B c N, so transpose x becomes c B transpose x B plus c N transpose x N. And by using this value of x B, we can write this as c B transpose B inverse b minus B inverse N x N plus c N transpose x N. And this is nothing but c B transpose B inverse plus c N transpose c B transpose B inverse N x N, and this is nothing but the lambda N transpose that we had seen earlier.

So, these are these are also called the relative cost factor corresponding to the non basic variables, and remember that the relative cost factor corresponding to the basic variables was 0, that c B bar transpose that we saw earlier. And if x N equal to 0 then current objective functions is c B transpose B inverse b, now this equation gives the clue about optimality. Now, if there exists some non basic variable say x q, so that lambda q is less than 0, note that all the non basic variables are 0, so x q also is 0, but if lambda q happens to be negative, then there will be a way to improve the objective function by making x q to be positive.

So, let us see, so suppose there exists a non basic variable x q in the set N such that, lambda q is less than 0, now since x q is in the non basic variable set the current value of x q is 0. Now, if you look at the objective function, objective function at x will be c B transpose B inverse b plus lambda q x q, so let us assume that, we are not interested in the remaining non basic variables, their lambda could be negative, positive or 0. But, we are at the moment interested only in one non basic variable which is x q or which lambda q is less than 0.

So, the objective function can be written c B transpose B inverse b plus lambda q x q, and remember that lambda q is less than 0, so currently the value of x q is 0, now if we increase x q to 0 to some positive quantity, the objective function value is going to decrease. Of course, we will move from x to some other new point in the process, but there is a scope for improvement in the objective function or decrease in the objective function, if x q is increased from 0 to some positive quantity. And therefore, if x q is changed from 0 to some positive quantity, we can say that x q is made a basic variable.

Now, we have seen earlier that, a solution of a linear program lies at the boundary point and in a particular, if it lies at the boundary point, and if the solution set is convex and compact, then the solution lies at the extreme point also. So, from a current extreme point, if we want to move to an adjacent extreme point by making x q as our basic variable, then one of the existing basic variables has to be eliminated or made non basic.

So, there will be a swap of a basic and a non basic variable between the sets B and N. So, that we can move from one vertex to an adjacent vertex, and in the process we will decrease the objective function value.

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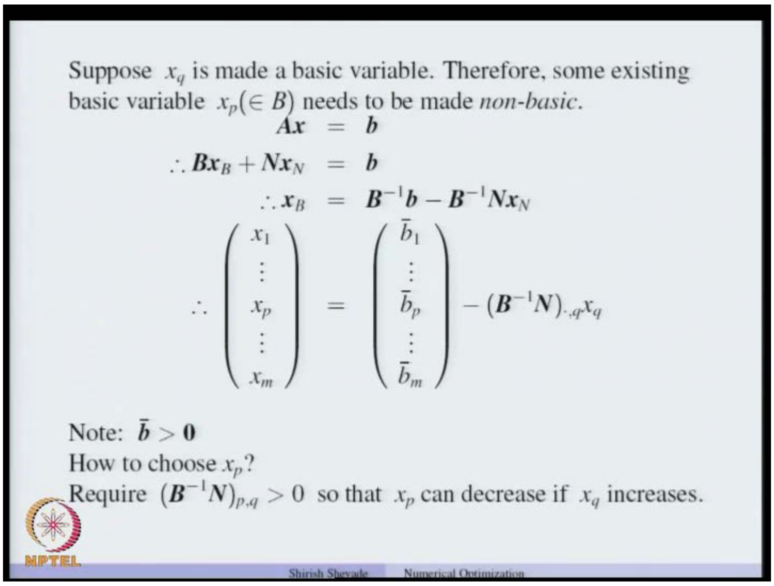

Now, how to find out a variable which can or a basic variable which can become non basic, now let us assume that x q is made a basic variable. And we are interested in finding out some existing basic variable x B which belongs to the set B, that needs to be made non basic. Because, we know that any at every vertex for a given linear program, there are m variables corresponding to the basis set, and N minus m variables corresponding to the non basic set.

So, when we move to when we want to bring in a non basic variable in to the basis vector set, one of the existing basic variables has to be made non basic, and how that can be done that is the question that we would like to address now. So, again let us relook at the system A x equal to b, and B x B plus N x N equal to b, so we have x B is equal to b inverse b minus B inverse N x N. Now, we are not interested in the entire B inverse N matrix at this moment, because we are concentrating mainly on the on one variable which is x q, so x q is currently non basic.

So, we are interested in the q th column of this b inverse N matrix, so we can write x B as B inverse b minus B inverse N dot, I mean the q th column of B inverse N x N, so which is given here. So, you have x 1 to x m as the basic variables without loss of generality, now b one bar to b m bar denote b inverse b, and since it is a non degenerate basic feasible solution, we can say that b bar is greater than 0. And what we are interested in is the q th column of B inverse N, so that is denoted by B inverse N dot slash q dot comma q.

So, which means that we consider all the rows in the q th column of B inverse M multiplied by x inverse q. So, at the moment we are interested only in the vary x q, variable of the non basic vector variable set, now let us look at this equation.

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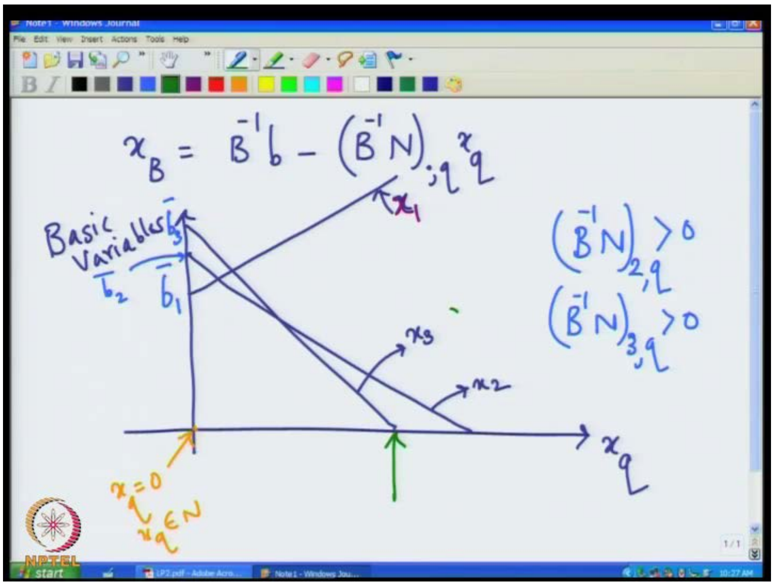

So, we have x B is equal to B inverse b minus B inverse N (( )), so this is the equation that we have, now so what we are looking at now is dependence of the basic variables x B on the variables x q which is currently 0. So, let us take x q and on the vertical axis, we will consider all basic variables, now remember that currently x q is equal to 0, because it is a non basic variable, so x q belongs to N and it is non basic variable currently.

Now, there will be a variation in x B, if x q is a made a basic variable or in other words, if x q is changed from the current value of 0 to some positive value, there will be a change in x B. And that depends on the corresponding entry in the B inverse N matrix q th column, so for example, one possibility is that for a particular variable, so let us call this variable as x 1. So, so here we have b 1 bar, b 1 bar is nothing but the first entry in the vector B inverse b, and that corresponds to the first basic variable and without loss of generative erosion that the first m variables are basic variables.

So, what it means is that as x q is increases from 0 to some positive value, the basic variable x 1 also increases remember that, these are the basic variables x B that we are plotting on the y axis, so as x q increases x 1 also increases. Now, there could be another case where so for a variable 2, so this is corresponding to variable 2, so what it means is that as x q increases the variable x 2 decreases, and comes to 0 at this value of x q. Now, there could be a situation where, suppose that this corresponds to the variable x 3, now x 3 behaves in the similar way to x 2 in the same sense that, as x q increases the variable x 3 decreases.

So, this is b 3 bar and this value is b 2 bar, so you will see that as x q is made a basic variable by increasing it is value from 0 to some positive quantity, x 1 increases and both x 2 and x 3 decrease. Now, and both become 0 at certain value of x 2, so for example, x 3 becomes 0 when x q achieves this value, now among all this variables which are the candidate variables to be made non basic. Now, certainly x 1 is going x q increases, so x 1 cannot be a candidate variable to be included to be included in the set N or to be excluded from the basic variable set.

Because, it is not going to become 0 as x q increases on the other hand, x 2 and x 3 they become candidate variables as x q becomes a basic variable, so they are the candidate variables to be made non basic. Now, out of out of x 1, x 2, x 3 you will see that these are the candidates where b minus B inverse, so B inverse N q was greater than 0, so which means that the line, if you consider this as a line where x 2 denotes the horizontal axis, and the x 2 denotes the vertical axis, then that line has a negative slope. So, which means that B inverse N has to be positive, so similarly B inverse N that was also greater than 0.

So, the candidate basis vectors which possibly could be made non basic, is determined based on whether the corresponding entry in the B inverse N matrix is positive or negative. So, for the variable x 1 the B inverse 1, q are the first entry in the q th column of b inverse N matrix is negative, so which means that, the line x B is equal to x 1 is equal to b 1 bar plus some quantity in to x 1 has a positive slope. On the other hand for these two variables x 2 and x 3, this line has a negative slope and therefore, these are the candidates which can be made non basic, if x q is made basic; now out of this candidates, there could be several more candidates like this, so out of all these candidates which one do we choose.

So, clearly we will see that, we will choose that basic variable which reaches the objective function which reaches the 0 value first, so clearly in this case where we have only three variables x 3 reaches 0 first compare to x 2, because if x 2 increase further then x 3 becomes non positive or negative. And therefore, it will not remain feasible and therefore, we should choose this point where x q or x q is increased up to this point, so that x 3 becomes 0. And since, x q is now positive, x q can become a basic variable and x 3 which was in the previous iteration a basic variable, since it has attained a value of 0, it will become a non basic variable.

So, we are going to see this now, so the x p is chosen based on the, q th the entries in the q th column of the B inverse N matrix, in particular x p can be chosen such that, B inverse N p q has to be greater than 0, so that x p can decrease if x q increase. So, which means that if x q increased from 0 to some positive value, x p can be decreased from a current positive value to a value 0 which means that, x p can be made non basic while making x q a basic variable.

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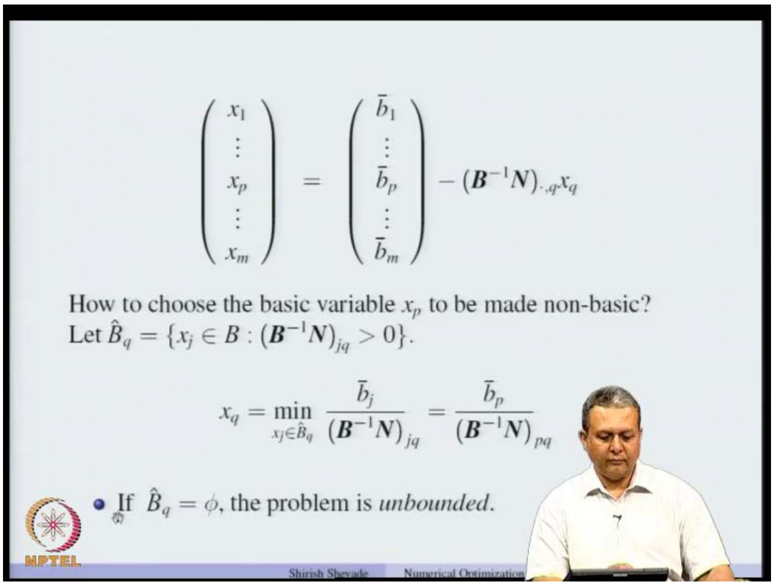

So, the natural question is that how to choose the basic variable, x p to be made non basic. So, we saw earlier that we choose all the entries in they B inverse N matrix which are positive, and collect all those variables let us denote them by b hat q. So, b hat q denotes the set where denotes set of candidate basis vectors which could be made non basic. Now, among all those variables which are candidate basic vectors, we will try to choose that basic vector which attains the value of 0 first or in other words, x q will be assigned to the minimum B bar z minus B inverse N z q.

So, if you substitute x p to be 0, we will get a value of x q and among all those possible values of x q will choose that value of x q such that, the one of the basic variables becomes 0 first, that we saw in the illustration earlier. So, x q will be B p bar minus divided B inverse N p q, so now, if you do this there are some possibilities that if you cannot get a non empty set B hat q, then the problem is unbounded. So, which means that we can increase the value of x q without removing any of the basis vectors, and the objective function value goes to minus infinity or the problem is unbounded. So, if the problem is bounded, then x q could be found using this formula or in other words if B hay q is non empty, then we can get a variable to be made non basic.

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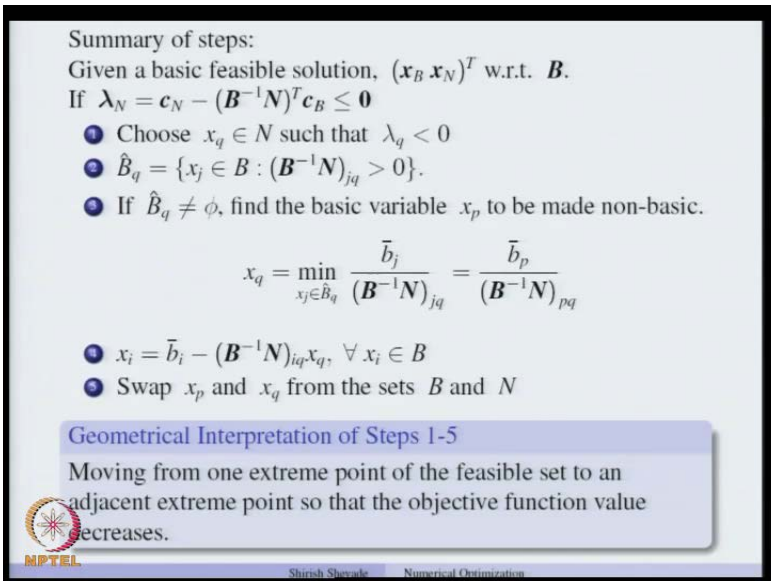

So, now, let us summarize the steps that we have seen earlier remember that, we started with the basic feasible solution which is x B x N with respect to the basic matrix B. Now, if lambda N which is c N minus B inverse N transpose c B, if it less than or equal to 0 that means, the relative cost associated with the non basic variables are negative for at least one non basic variable. Then we choose one non basic variable x q such that, lambda q is less than 0 remember that, when we say lambda less than or equal to 0, so which means that there exists at least one non basic variable whose relative cost is negative, among all those variables whose relative cost are negative we choose one and denote it by lambda q.

Now, having chosen a variable or having chosen a non basic variable to be made basic the next step is to determine the variable or the basic variable to be made non basic. And for that purpose, we find out the matrix B hat q sorry, the set B hat q, and that set is obtained by finding out all the basic variable for which B inverse N th N 3 in the q th column is greater than 0. Or for all the basic variables x is such that, the z entry in the q th column of B inverse N matrix is greater than 0, and if that set is non empty, then the basic variable to be made non basic is obtained by finding out the minimum of b bar divided by B inverse N j q, where x j belongs to B hat q.

And that minimum value is given by the p th basic variable which will be made non basic, and which is a the value of 0 first and x q is increased from 0 to some positive quantity. Now, once we do this what we need to do is that, we need to update all the basic variables and that is done by for every basic variable x i, we update is b i minus B inverse N i q into x q. So, if you this you will realize that when I is equal to p, we will be updating x p to be b p bar minus B inverse N p q into b p bar minus B inverse N p q, and which will make x p to be 0.

So, this update will automatically make x p to be 0, this update will make x q to be positive, and the remaining basic variables get updated using this, and remember that since we have chosen x p to be made non basic. That means, that there will be a point where x p becomes 0 and as x p increases, and that is the first point where x p becomes 0 and the rest of the non basic variables will remain 0. So, in the entire step what we did was, we increased x q to a positive quantity, note that both the numerator and denominator are positive.

So, x p increase, x p to a positive quantity made it a basic variable by this update we made x p 0, so that it becomes a non basic variable, and other basic variables they either increase or decrease depending upon the sign of B inverse N. But, none of them will go to 0, the next step that we want to do is swap x p and x q from the sets B and N, just to indicate that now x p has become a non basic variable, and x q which was non basic earlier, has now become a basic variable. And this way of bringing in a basic a non basic variable leaving out one basic variable has a nice geometrical interpretation, and what it denotes is that, is basically denotes how to move from one extreme point from the feasible set to an adjacent extreme point, so that the objective function value decreases.

So, remember that, we have chosen a variable x q to be brought in to the basis vector set, based on the fact that c B transpose B inverse b plus lambda q x q can be made negative, if x q is made positive or it is a variable for which the relative cost factor lambda q is negative. So, so we have ensured that the objective function is going to decrease, if we make sure that x q is increased, but then to move to the adjacent extreme points, one of the extreme points what we need to do is that, we need to make one of the basic variables are non basic variable; and that is done using this formula. So, this summary of steps has a nice geometric interpretation also.

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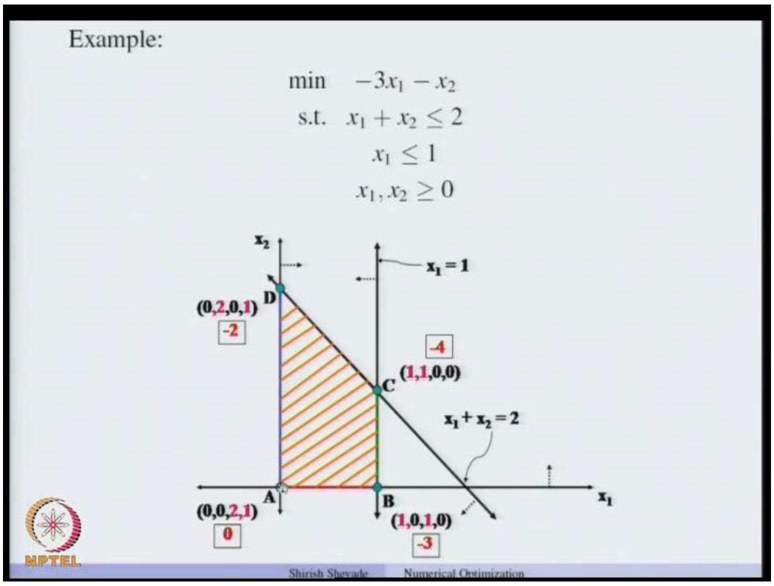

And if you look at our example, which we saw in the last class, the problem is to minimize minus 3 x 1 minus x 2, subject to x 1 plus x 2 less than or equal to 2, x 1 less than or equal to 1, and x 1 and x 2 non negative. So, you will see that this is a compact convex set, the feasible set is a compact convex set, the solution exists at one of the extreme points. Now, there are four extreme points or four vertices for this feasible set, and if you consider the point B suppose, then from B if we go to point A then the objective function value increases.

On the other hand, from B if we go to point C the objective function value decreases, and at C both the adjacent points are higher objective function value than the objective function value at C which was which is minus 4. So, therefore, the solution to this problem is x 1 equal to 1 x 2 equal to 2, so these are the basic variables and this is also an optimal solution. Now, compare this optimal solution with the neighboring vertices objective function values, and also the basis vectors, so we will see that in moving from B to C the variable x 2 which was non basic here is made basic.

And variable x 3 which was basic is made non basic, similarly from this that the variable x 1 which was non basic is made basic while moving from D to C.

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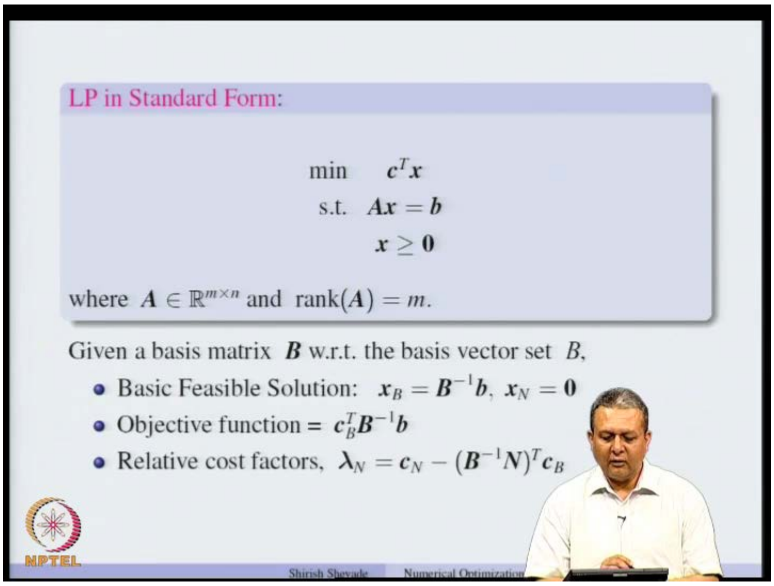

Now, let us try to get an algorithm to solve this linear program in standard form, now let us assume that we are given a basis matrix B, which is associated with the basis vector set B. And the basic feasible solution x B is equal to B inverse b and x N equal to 0, and assume that the basic feasible solution is nondegenerate, we assume that x B is greater than 0. Now, if any of the B is negative, then by making some appropriate transformation, one can make sure that the initial basic feasible solution x B is always greater than 0.

Now, we know that the objective function value is c B transpose B inverse b, because x N 0, so c N transpose x N is 0, so at a basic feasible solution c B transpose B inverse b gives us an objective function value. Then the relative cost factors corresponding to the basic variables are 0, and the corresponding cost factors for the non basic variables are c N minus B inverse N transpose c B. Now, can we use this information and get away or find a way, so that this the entire information is compactly available, in the form say matrix.

So, by looking at the matrix we should be able to find out what is B inverse b, what is or what is the current basic feasible solution, what is the objective function value which is c B transpose B inverse b, and what is the relative cost factors associated with the non basic variables. And remember that, the relative cost factors associated with the basic variables have to be 0, so if we have this information available, then that easily tells us whether we have reached an optimal solution.

If we have reached the optimal solution, then lambda N has to be greater than or equal to 0 for all non basic variables, and the if that is the case, then the current basic variables are x B equal to B inverse b. So, this gives us the optimal solution, and the optimal cost is c B transpose B inverse b, so at a time if all this three pieces of information are available, then we can easily find out whether the given feasible solution is an optimal solution.

And this also would possibly help us to find out, if suppose lambda N is in not greater than or equal to 0 that means, there exists a non basic variable for which the Lagrangian multiplier is negative, or which has a negative cost factor. Then we should also be able to find out what is the basic variable that needs to be made non basic, so that the q th basic non basic variable can be made basic, so let us see how to do that.

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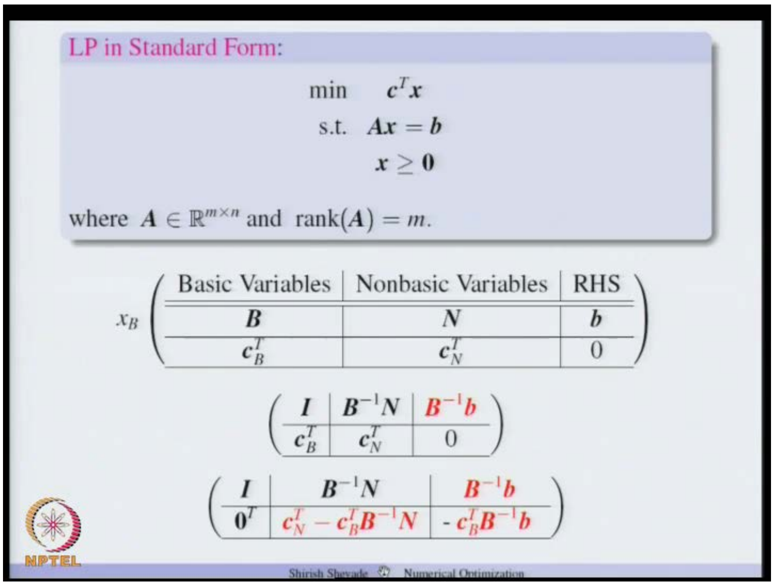

So, let us arrange the basic and non basic variables in the form of a matrix, so again without loss of generality we assume that the first m variables are basic variables, the next N minus m are non basic variables. And then we have the column to indicate the right hand of this, now first we write the matrix B associated with the basic variables, matrix N associated with the non basic variables; and then the corresponding vectors in the objective function. So, c transpose is related c B transpose and c N transpose, and this entry currently kept 0.

Now, you will see that this set of rows correspond to the basic variables, and what we want to do is that, we want to find out if the current feasible solution is a basic feasible solution that is x B greater than 0 and x equal to 0. So, from this matrix we should be able to get what is the current basic feasible solution, and that is x B equal to B inverse b and we know that x N equal to 0, so we do not have to worry about that part. So, how do we get B inverse b, so let us do some matrix operations, so multiply this first m rows or the matrix B by B inverse, so which will make the identity matrix.

Then this matrix will become B inverse N matrix, and this will be this entry will be B inverse b, so so if the first m rows correspond to the that denote the basic variables, also refers to m columns denote the basic variables, so x B is B inverse b. So, the current solution, current basic feasible solution is easy to obtain using this equal to B inverse b and x N equal to 0. Now, we also want the basic variables, cost factors, so that is c N transpose minus B inverse minus c B transpose B inverse N. So, what we do that, we multiply the m by N matrix I the m by N matrix here, by c B transpose and subtract it from this.

So, what we get is c B transpose minus c B transpose I, which is nothing but c B transpose minus c B transpose and which is 0, and if pre-multiply B inverse N by c B transpose, and subtract it from c N transpose. Then what we get is c N transpose minus c B transpose B inverse N, and here we get minus c B transpose B inverse b. So, now, this is the information that we are looking for, so this matrix uses all the information that is needed to check whether a given feasible point is optimal, given basic feasible point is optimal.

So, for example, as we saw earlier that x B is equal to B inverse b, so this the last column the last column denotes the value of x B, and as I mentioned earlier c N transpose minus c B transpose c B transpose is the nothing but lambda N transpose. So, this allows us to check whether, the relative cost factors of non basic variables are nonnegative, now this 0 relates relative cost factor of the basic variables. Remember that, the first m columns here are associated with the basic variables, so the relative cost factors of basic variables are 0, relative cost factors of non basic variables is c N transpose minus c B transpose B inverse N.

And then in the last entry in this matrix is minus c B transpose B inverse b, now from this last entry we can easily find out what is c B transpose B inverse b, which is basically the negative of the entry in this particular row and column; and that gives us the current objective function value. So, the basic feasible solution, the relative cost factors associated with non basic variables, and the objective function value, they all current objective function value, they all can be obtained by using this matrix.

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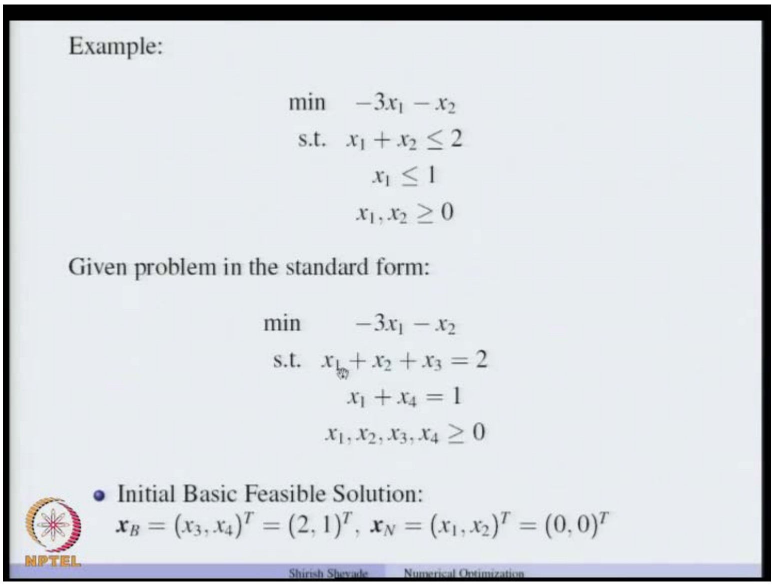

So, let us see an example, again the same example that we saw to get a graphical solution to the problem. So, let us first write the linear program in standard form by introducing the slack variables for the two constraints, so now we have x 1 plus x 2 plus x 3 equal to 2 and x 1 plus x 4 equal to 1. So, this is the linear program in standard form, because it of the type minimize c transpose x subject to A x equal to B x greater than or equal to 0. Now, one can easily determine it is initial basic feasible solutions, so what we need to is that, you can just take x 3 to be 2 and x 4 equal to 1 or if you write in the form A x equal to b, then we take a sub matrix of matrix A which is an identity matrix.

So, if you write the matrix A for this system of equations, so it will have two columns and four rows, so the first row is 1 1 1 0, and the second row is 1 0 0 1, so the last two columns of the matrix A they correspond to or the last 2 by 2 matrix of the matrix A, 2 by 2 sub matrix of A happens to be an identity matrix. So, we can easily get the initial basic feasible solution to be x 3 equal to 2 and x 4 equal to 1 by setting x 1 and x 2 to 0 or making x 1 and x 2 to be non basic.

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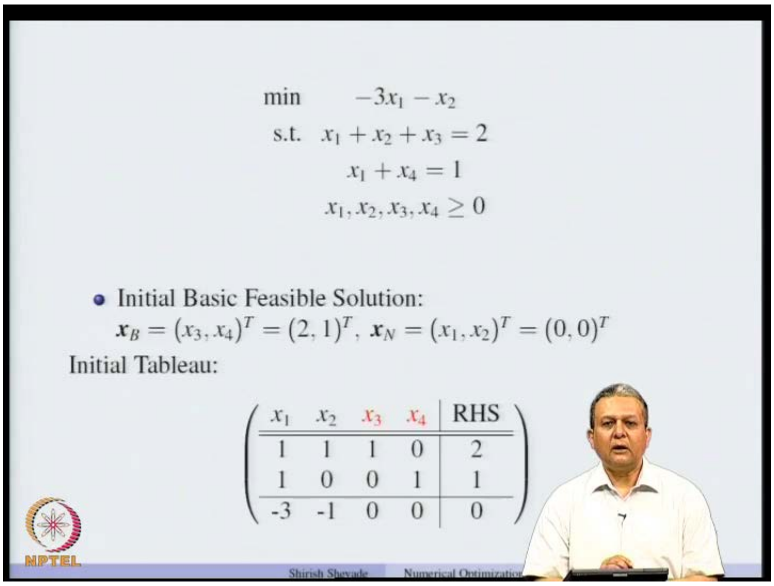

So, as I mentioned that this is the A matrix that we have got, associated with this constraint set, and the last two columns basically denote the identity matrix, so last two by 2 by 2 matrix of A, and that gives us the basic feasible solution. So, this row corresponds to x 3, and this row corresponds to x 4 in this particular matrix, this is also called tableau. So, x 3 equal to 2 and x 4 equal to 1 is the current basic feasible solution, x 1 and x 2 are 0 and here, the relative cost factors of basic variables is 0; and the relative cost factors of the non basic variables are negative, so which means that the current feasible solution is not an optimal solution.

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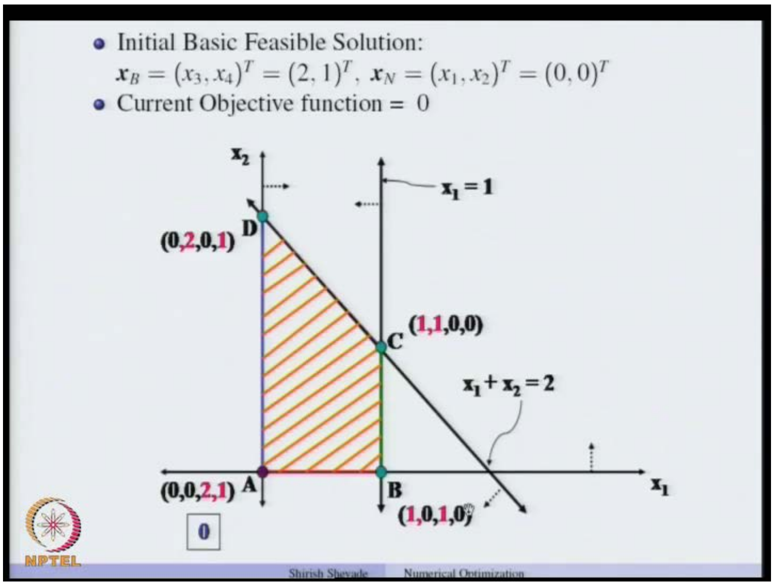

Now, to show it geometrically we have the point A, where x 1 and x 2 are 0 and x 3 and x 4 are 2 1, so the basic variables are x 3 and x 4, and the non basic variables are x 1 and x 2, and the objective function value is 0 and that is what we saw from the tableau.

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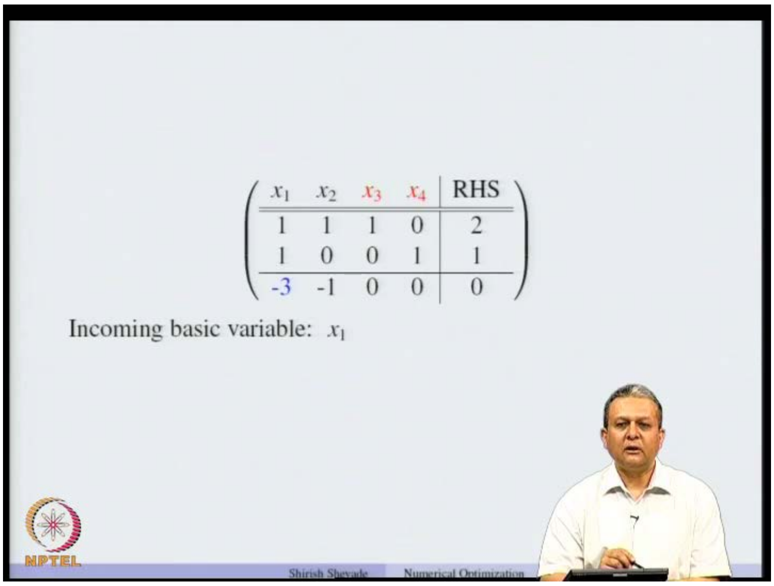

Now, the relative cost factors associated with the basic variables are negative, so we can choose any of these two variables to be made basic, now suppose we choose x 1 to be made basic; so that is indicated here by incoming basic variable x 1, because it has a negative cost factor. Now, one we choose x 1 to be a incoming basic variable, we need some basic variable to be made non basic. Now, currently x 3 and x 4 are basic variables, so if you look at the column associated with this minus 3, then you will see that all the entries are positive. So, this is the column in the matrix B inverse N that is what we are looking at, so what we do is that, we determine the value b bar by B inverse N. So, 2 by 1 and 1 by 1, and among them we choose the one which has the lowest value this is also called ratio test.

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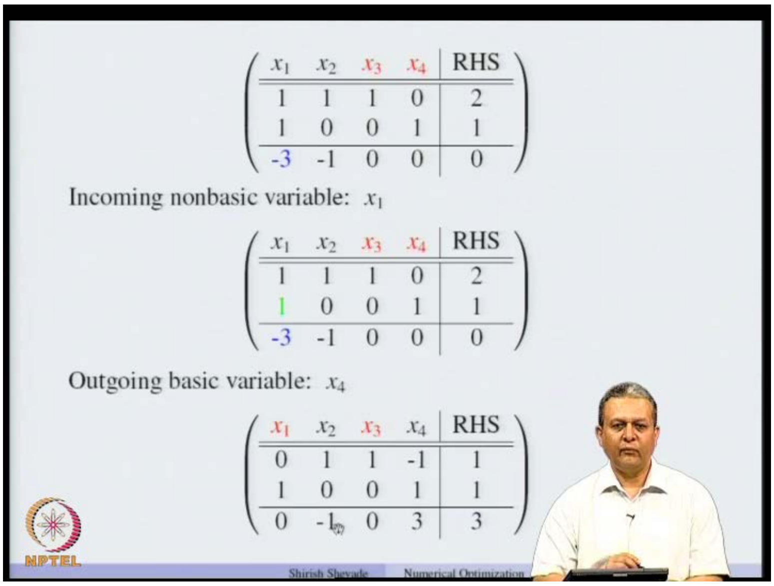

So, between the two ratios 2 by N 2 by 1 and 1 by 1, this ratio is the least which is shown by green, so the outgoing variable is corresponding to this row, and this row corresponds to the variable x 4, so x 4 becomes an outgoing basic variable. So, we have incoming non basic variable which is x 1, outgoing basic variable which is x 4, now the next step is to make sure that, this column entire this column corresponds to the should be made similar to the column in the x 4 th variable case, so that means, that this entry should be made 0, and this entry is 1.

And then the relative cost of that variable, because it is going to become basic variable should be made 0, so this is done using some matrix transformations, so for example, what we can do is that subtract the first row from the second row. So, we get 0 and then the remaining quantities, then subtract three times this row to the last row, so 3 into 1 plus minus 3 will make it 0; so which means that the basic variables cost, the relative cost if the basic variable is made 0.

And now there will be corresponding changes in the column of x 4, so now now if take if you look at the identity sub matrix, in this we will see that we will get x 1 and x 3 as the identity sub matrix of this. And which means that, x 1 and x 3 are basic variables, the value of x 3 is 1, value of x 1 is 1, and x 2 and x 4 are non basic. So, x 3 remains basic, x 4 which was basic earlier is now made non basic, and x 1 non basic earlier is made basic; and it was just ensured that the identity sub matrix of the this matrix is corresponding to the basic variables x 1 and x 3. Now, this entry is 3, so which is which means that, the current objective function value is minus 3, so we have the basic feasible solution which is x 3 equal to 1 and x 1 equal to 1. The corresponding objective function value is minus 3, and the relative cost of the non basic variable x 2 is negative.

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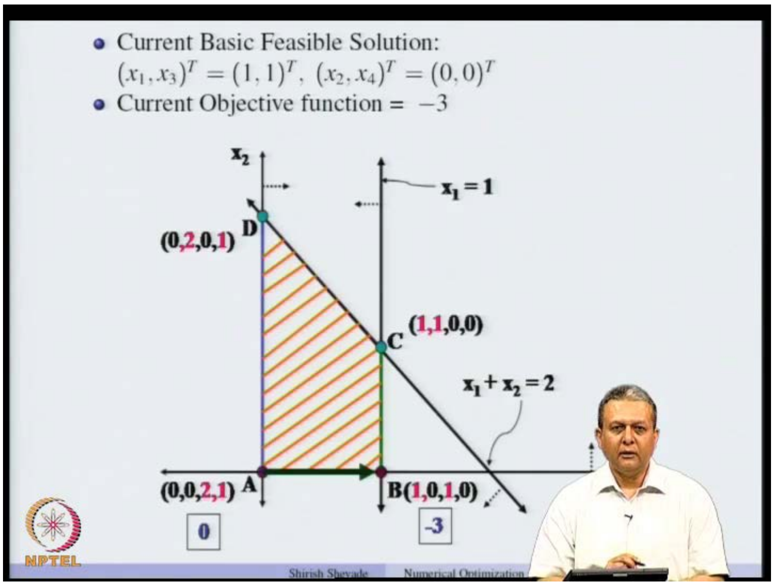

So, let us see this geometrically, so we have the current basic feasible solution which is (x 1, x 3) to be 1, 1, and (x 2, x 4) to be (0, 0) current objective function value is minus 3, and this is indicated here. So, which means that by making x 1 to be basic, and removing x 4 from the basic variable set, we were able to move from the point A to the point B in this feasible region. And not only that, we also decrease the objective function value from 0 to minus 3 in moving that, so you will see that now x 1 and x 3 are the basic variables, and x 2 and x 4 are non basic. Now, the same procedure needs to be followed, and we will see that in the next class.

Thank you.